Portfolio Management
2010-2011

QUESTION 1

a) Suppose that current one year interest rate is 5% per annum. Also assume that the 1-year forward interest rate $(f_{1,1})$, is 6%. This forward rate means that you are able to commit to investing $X one year from now and be certain of receiving $x $(1+f_{1,1})$, two years from now. How much money will you have in two years if you invest $100 in the current one-year rate (the spot rate) and commit to investing the proceeds of a one-year forward rate? Assume that interest is calculated annually.

b) Assume that investing $100 at the current 2 year interest rate will leave you with the same amount of money that you calculated in part (a) at the end of two years. What is the current 2-year rate?

QUESTION 2

The following table records current spot rates for zero-coupon bonds of various maturities

Bond	Maturity Years	Spot Rate (%)
A	1	5.0
B	2	5.5
C	3	6.0

Use the Spot rates to derive (a) the forward rate between year 1 and 2, (b) the forward rate between year 2 and 3, and (c) the forward rate between year 1 and 3. The forward rates should be expressed in annual basis.

QUESTION 3

A three-year coupon bond has payments as follows:

Bond Cash Flows ($) by Year		
Year 1	Year 2	Year 3
8	8	108

This 8% coupon bond is currently trading at par ($100).

a) What is the annually compounded yield of the bond?
b) Compute the Macaulay Duration and the modified Duration of the bond.

QUESTION 4

A Financial Institution has raised $1 million by selling a number of 2-year zero-coupon bonds to individuals. These bonds have a yield-to-maturity of 6%. The institution has used the proceeds to

buy a number of long-term coupon bonds. These bonds have a Macaulay duration of 12 years and a yield-to-maturity of 7%. Use the concept of Duration to explain how this institution is exposed to changes in the interest rates. In particular, what happens to the value of the zero-coupon bonds, the long-term bonds, and the value of the firm as a whole, if the yields on these bonds change by 50 basis points (half of one percent)? (*Hint*: Use the duration as an approximation for percentage price changes).

QUESTION 5
A portfolio manager made the following statement: "To immunize a portfolio in order to satisfy a single liability, all that is necessary is that (1) the market value of the assets is equal to the present value of the liability and (2) the duration of the portfolio is equal to the duration of the liability. There are absolutely no risks except for the risk that any of the bonds in the portfolio will default or decline in value due to credit downgrades."
Explain whether or not you agree with this statement.

QUESTION 6
An investor purchases a 3-month put option, which has a strike price of $30 and an option market price of $0.50, when the stock is initially priced at $31.50 per share. If the stock's price is $27 when the option expires, what is the expiration-day value of the put?

QUESTION 7
ABC Corp. stock is currently trading at $65. Assuming an investor buys a call option on 100 shares of ABC Corp. stock, which includes a call premium of $2.5 per share and a strike price of $68, what is the profit or loss on the expiration day of the call if the stock closes at $72?

QUESTION 8
Suppose that you are asked to evaluate the performance for two mutual funds, X and Y, over a recent 5-year period with the following characteristics:

	Fund X	Fund Y
Average return	14%	16%
Standard Deviation	5.7	7.1
Beta	1.20	0.91

The risk free rate is 6.5%.

a) Determine which fund performed better, using Sharpe's and Treynor's index.

b) Use Jensen's index to determine how mutual Funds X and Y performed. The average return on the market for the period was 13.5%.

c) Which is a better performance index, Sharpe's index or Treynor's index? Explain your answer carefully.

QUESTION 9

An investor is considering the purchase of an option-free corporate bond with a coupon rate of 7.25% (coupons are paid semiannually) and 15 years remaining to maturity. The price of the bond is 106.1301 and the yield to maturity is 6.6%. Assume that the Treasury yield curve is flat at 6% and that the credit spread for this issuer is 60 basis points for all maturities. Compute:

a) The 1-year total return on a bond-equivalent basis[1]

b) The 1-year total return on an effective rate basis.

Assume:

i) the reinvestment rate is 4%.

ii) at the horizon date, the treasury yield curve is flat at 5.65%.

iii) at the horizon date, the credit spread for this issuer is 50 basis points for all maturities.

QUESTION 10

An investor is considering the purchase of an option-free-high-yield corporate bond with a coupon rate of 10% (coupons are paid semiannually) and 9 years remaining to maturity. The price of the bond is 95.7420 and the yield to maturity is 10.75%. Assume that the Treasury yield curve is flat at 7.5% and that the credit spread for this issuer is 325 basis points for all maturities. Compute:

a) the 1-year total return on a bond equivalent basis.

b) the 1-year total return on an effective rate basis.

Assume:

i) the reinvestment rate is 5%.

ii) at the horizon date, the treasury yield curve does not change and therefore remains flat at 7.5%

iii) at the horizon date, the credit spread for this issuer declines to 200 basis points for all maturities.

QUESTION 11

In order to neutralize two positions against a small parallel shift in interest rates, the effective dollar durations of the two positions in a trade must be matched. It is not sufficient to match the effective durations of the two positions. Explain

QUESTION 12

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[1] ***Bond Equivalent Yield – BEY:*** for an investment is a calculated annual yield for an investment, which may not pay out yearly. It is not to be confused with a bond's coupon rate. This allows investments which payout with different frequencies to be compared.The bond equivalent yield formula helps to restate the yield on a T-bill or any other debt instrument in terms of semiannual investment yield, in order to facilitate direct comparison with an interest bearing coupon security, viz. T-bond, T-note.

Mr. X is a portfolio manager who owns $15 million par value of bond ABC. The market value of the bond is $13 million and the effective dollar duration for a 100 basis point change in rates is $1.2 million. Mr. X is considering a swap out of bond ABC and into bond XYZ. The market price of bond XYZ is $75 per $100 of par value and the effective duration is 7. How much par value of bond XYZ would Mr. X have to purchase in order to maintain the same exposure to small parallel shift in interest rates as he has with bond ABC?

QUESTION 13

Suppose that the actual return for a portfolio and the returns on the benchmark index, are as shown below. Calculate the actual tracking error for this portfolio.[2]

Month in 2010	Portfolio Return	Benchmark Index Return
Jan	0.75	1.65
Feb	0.40	0.89
March	1.79	0.52
April	-0.89	-0.47
May	0.50	0.65
June	0.72	0.33
July	3.20	2.31
August	1.95	1.10
Sept	0.23	1.23
Oct	1.20	2.02
Nov	-1.90	-1.38
Dec	-0.25	-0.59

QUESTION 14

What is the problem with using the actual tracking error of a portfolio relative to a benchmark index to assess the potential future tracking error of the portfolio?

QUESTION 15

In assessing historical performance of a portfolio, a portfolio manager should use forward looking tracking error. Comment

QUESTION 16

a. What is the duration for the following portfolio?

Bond	Market Value	Duration
1	$10 million	7.2
2	$8 million	6.1

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[2] Tracking error: $T.E. = \omega = \sqrt{Var(d-b)}$ where d-b is the difference between the portfolio return and the index return. Index funds are expected to have minimal tracking error.

| 3 | $4 million | 1.1 |
| 4 | $12 million | 4.8 |

b. What is the contribution of bond 1 to portfolio duration.

SOLUTIONS
QUESTION 1
a)

Current one year interest rate $= 0r_1 = 0.05$

One year forward interest rate $= f_{1,1} = 0.06$

Invest @ 1 year spot and reinvest @ current 1 year forward

$100 \times (1.05) \times (1.06) = \$111.30$ at the end of two years

b)

Assume that $\$100 \times (1+0r_2)^2 = \$111.30$

So, $0r_2 = (111.30/100)^{0.5} - 1 = 5.4988\%$

Double check your result. Investing $100 at the rate of 5.4988%, gives:

$100 \times 1.054988^2 = \$111.30$

Hence, investing at 5.4988% for two years yields the same as investing at 5% now for a year, and investing forward at 6% for the following year.

QUESTION 2

Bond	Maturity Years	Spot Rate (%)
A	1	5.0
B	2	5.5
C	3	6.0

a)

The forward rate between year 1 and 2 is given by: $\dfrac{(1+r_2)^2}{1+r_1} - 1 = \dfrac{1.055^2}{1.05} - 1 = 6.00\%$

b)

The forward rate between year 2 and 3 is given by: $\dfrac{(1+r_3)^3}{(1+r_2)^2} - 1 = \dfrac{1.06^3}{1.055^2} - 1 = 7.01\%$

c)

The forward rate between year 1 and 3 is given by: $\dfrac{(1+r_3)^3}{(1+r_1)^{0.5}} - 1 = \left(\dfrac{1.06^3}{1.05}\right)^{0.5} - 1 = 6.50\%$

Bond	Maturity Years	Spot Rate (%)	Forward Rate (in %per year)	
A	1	5.0		
B	2	5.5	6.00	6.50%

	C	3	6.0	7.01

QUESTION 3

a)

The bond is selling at par. This means that the coupon rate equals the current yield of the bond. Hence the annually compounded yield is 8%.

b)

The Macaulay duration equals 2.78 and the modified Duration 2.58 (computed according to the table below).

Period	Time	Payment (in $)	Yield (in % per year)	PV	Time ×PV
1	1	8	8	$\frac{8}{(1.08)^1} = 7.41$	$7.41 \times 1 = 7.41$
2	2	8	8	$\frac{8}{(1.08)^2} = 6.86$	$6.86 \times 2 = 13.72$
3	3	108	8	$\frac{108}{(1.08)^3} = 85.73$	$85.73 \times 3 = 257.20$
Sum				100	278.33
Macaulay Duration					278.33/100 = 2.78
Modified Duration					$\frac{1}{(1.08)} \times 2.78 = 2.58$

QUESTION 4

The Macaulay duration of zero-coupon bonds is equal to their maturity (2 years) because all cash-flows occur at the same time. The duration of the coupon bonds is given in the question (12 years). If interest rates change by 50 basis points, the value of the zeros will change by:

$$\frac{\text{Duration}_{\text{(2-year bonds)}}}{1+\text{yield}_{\text{(2-year bonds)}}} \times \text{change in the yield}, \text{ that is}$$

$$\frac{2}{1.06} \times \frac{50}{100} = 0.9434\%$$

Which is $9,434. Similarly, the value of the coupon bonds will change by:

$$\frac{\text{Duration}_{\text{(Long term bonds)}}}{1+\text{yield}_{\text{(Long term bonds)}}} \times \text{change in the yield}, \text{ that is}$$

$$\frac{12}{1.07} \times \frac{50}{100} = 5.6075\%$$

Which is $56,075.

In both cases the value of the bonds changes in the opposite directions to the interest rates.
In particular, if interest rates increase by 50 basis points, the value of the firms' liabilities decreases by $9,434 and the value of its assets decreases by $56,075.

QUESTION 5
The statement is incorrect. There are in fact two other risks. First, even if the duration of the portfolio is matched to the duration of the liability, the portfolio is still exposed to a nonparallel shift in the yield curve. This risk is commonly referred to as immunization risk. Moreover, if there are callable securities in the portfolio, the portfolio is exposed to call risk.

QUESTION 6
The expiration value of a put option is equal to Max $(0, X - S)$ = Max $(0, \$30 - \$27)$ = $3.00

QUESTION 7
Total cost of the acquiring the option is $100 \times \$2.50 = \$250$
Value of the call at expiration is $\$72 - \$68 = \$4 \times 100 = \$400$
Profit/loss equals the value of the call less the option premium paid:
$\$400 - \$250 = \$150$

QUESTION 8
a) Sharpe's index
   Fund X = $(14\% - 6.5\%) / 5.7 = 1.31$
   Fund Y = $(16\% - 6.5\%) / 7.1 = 1.34$

   Fund Y achieved higher returns, and also outperform Fund X on a risk-adjusted basis

   Treynor's index
   Fund X = $(14\% - 6.5\%)/1.20 = 6.25\%$
   Fund Y = $(16\% - 6.5\%)/0.91 = 10.44\%$

   Fund Y outperformed Fund X according to Treynor's index.

b) Jensen's index
   Fund X = $14\% - (6.5\% + (13.5\% - 6.5\%) (1.2) = -0.9\%$
   Fund Y = $(16\% - (6.5\% + (13.5\% - 6.5\%) (0.91) = 3.13\%$

   Fund Y performed better than Fund A (higher Jensen's index). Fund Y also outperform the market (which has an index of zero).

c) Which index is better depends on each investor's situation. Sharpe's ratio uses standard deviation as a measure of risk, while Treynor's ratio uses beta to measure risk. Recall that standard deviation measures the total risk of an asset and beta measures the systematic risk. Therefore, Sharpe's index is more relevant for investors who do not hold any other portfolios, whereas Treynor's index is more relevant for investors who hold many other assets apart from the mutual fund.

QUESTION 9

Given the assumptions about the treasury yield curve and the credit spread, the horizon yield is 6.15%. This means that the yield for this corporate bond declines over the 1-year investment horizon from 6.6% to 6.15%. the calculation of the 1-year total return is shown below:

Step 1: Compute the total coupon payments plus reinvestment income assuming an annual reinvestment of 4% per year or 2% every six months. The semiannual coupon payment is $3.625. The future value is:

$$\begin{array}{rl}
\text{First coupon payment reinvested for six months} = \$3.625\,(1.02) = & \$3.6975 \\
\text{Second coupon payment (not reinvested)} = & \underline{\$3.6250} \\
\text{Total} = & \$7.3225
\end{array}$$

Step 2: Next, compute the horizon price. The horizon yield is 6.15% (5.65% Treasury + 50 bp corporate spread). The 7.25% coupon 15-year corporate bond now has 14 years to maturity. The price of this bond, when discounted at 6.15% (a flat yield curve is assumed), is $110.2263.

Step 3: Add the amounts in Steps 1 and 2. The total future value equals $117.5488.

Step 4: Compute the following:

$$\$106.1301 \times (1+r)^2 = \$117.5488 \Leftrightarrow \left(\frac{\$117.5488}{\$106.1301}\right)^{1/2} - 1 = 5.24\%$$

Step 5: The total return on a bond-equivalent basis and on an effective rate basis are:

$2 \times 5.24\% = 10.48\% \ (\text{BEY})$

$(1.0524)^2 - 1 = 10.76\% \ (\text{effective rate basis})$

QUESTION 10

Given the assumption about the Treasury yield curve and the decline in the credit spread, the horizon yield is 9.5%. the calculation of the 1-year total return is shown below.

Step 1: Compute the total coupon payments plus reinvestment income assuming an annual reinvestment of 5% per year or 2.5% every six months. The semiannual coupon payment is $5. The future value is:

$$\begin{array}{rl}
\text{First coupon payment reinvested for six months} = \$5\,(1.025) = & \$5.1250 \\
\text{Second coupon payment (not reinvested)} = & \underline{\$5.0000} \\
\text{Total} = & \$10.1250
\end{array}$$

Step 2: Next, compute the horizon price. The horizon yield is 9.5%. The 10% coupon 9-year corporate bond now has 8 years to maturity. The price of this bond, when discounted at 9.5% (a flat yield curve is assumed), is $102.7583

Step 3: Add the amounts in Steps 1 and 2. The total future value equals $112.8833.

Step 4: Compute the following:

$$\$95.7420 \times (1+r)^2 = \$112.8833 \Leftrightarrow \left(\frac{\$112.8833}{\$95.7420}\right)^{1/2} - 1 = 8.58\%$$

Step 5: The total return on a bond-equivalent basis and on an effective rate basis are:

$2 \times 8.58\% = 17.17\%$ (BEY)

$(1.0858)^2 - 1 = 17.90\%$ (effective rate basis)

QUESTION 11

If the two positions have the same effective duration, then their respective percentage price change will be equal for a small change in rates. However, if the two positions have different dollar values, the dollar price changes will not be equal. In a trade that does not attempt to benefit from changes in interest rates the objective is the neutralization of the trade against interest rate risk. Matching effective dollar durations accomplishes this for small changes in rates.

QUESTION 12

The market value of bond XYZ that must be purchased is computed as follow (where duration refers to effective duration):

$$\text{Dollar Duration} = - \text{DE} \times (\$\text{Market Value}) \times \Delta r$$

$$\text{Market Value of Bond XYZ} = \frac{\text{Dollar Duration of Bond ABC}}{\text{Duration of Bond XYZ}/100}$$

Since:

Dollar Duration of Bond ABC = \$1.2 million
Duration of Bond XYZ = 7
Then:

$$\text{Market Value of Bond XYZ} = \frac{\$1,200,000}{7/100} = \$17,142,857.14$$

The par value of bond XYZ that must be purchased is equal to:

$$\text{Par value of Bond XYZ} = \frac{\text{Market Value of Bond XYZ}}{\text{Price of XYZ per \$1 of par value}}$$

Since the price of XYZ is \$75 per \$100 of par value, the price per \$1 of par value is 0.75 and therefore:

$$\text{Par value of Bond XYZ} = \frac{\$17,142,857.14}{0.75} = \$22,857,142.86$$

QUESTION 13

The actual tracking error is 77.23 basis points as shown below:

Month in 2010	Portfolio Return	Benchmark Index Return	Active Return	Deviation from mean	Squared Deviation
Jan	0.75	1.65	-0.90	-0.85	0.7282
Feb	0.40	0.89	-0.49	-0.44	0.1965
March	1.79	0.52	1.27	1.32	1.7336
April	-0.89	-0.47	-0.42	-0.37	0.1394
May	0.50	0.65	-0.15	-0.10	0.0107
June	0.72	0.33	0.39	0.44	0.1907
July	3.20	2.31	0.89	0.94	0.8773
August	1.95	1.10	0.85	0.90	0.8040
Sept	0.23	1.23	-1.00	-0.95	0.9088
Oct	1.20	2.02	-0.82	-0.77	0.5980
Nov	-1.90	-1.38	-0.52	-0.47	0.2240
Dec	-0.25	-0.59	0.34	0.39	0.1495
Sum			-0.56	0.00	6.5607
Mean			-0.0467		
Variance					0.5964
Standard Deviation = Tracking Error					0.7723
Tracking Error (in basis points)					77.23

Notes: Active Return = Portfolio Return – Benchmark Return

Residual Return = Portfolio Return – βp (Benchmark Return)

Variance = (Sum of square deviations from the mean)/11

Division by 11 which is number of observations minus 1

Tracking error = Standard Deviation = Square root of variance

QUESTION 14

If tracking error is measured historically, it is called 'realised' or 'ex post' tracking error. If a model is used to predict tracking error, it is called 'ex ante' tracking error. The former is more useful for reporting performance, whereas ex ante is generally used by portfolio managers to control risk. Various types of ex-ante tracking error models exist, from simple equity models which use beta as a primary determinant to more complicated multi-factor fixed income models.

One problem with using actual tracking error as a measure of future tracking error is that actual tracking error does not reflect the impact of the portfolio manager's current decisions on the future active returns. The manager can significantly alter key risk exposures of the portfolio (such as the portfolio's effective duration or sector allocations). To the extent that the actual tracking error calculated using data from prior periods would not accurately reflect the current portfolio factor risks. As a result, the actual tracking error has little predictive value and can be misleading with regard to future portfolio risks.

QUESTION 15

Forward looking tracking error is inappropriate for measuring historical performance because it reflects the current composition of a portfolio rather than the composition of the portfolio that resulted in the realized tracking error (called actual tracking error).

QUESTION 16

a.

Bond	Market Value	Percent	Duration	Percent ×Duration
1	$10 million	29.41176%	7.2	2.1176
2	$8 million	23.52941%	6.1	1.4353
3	$4 million	11.76471%	1.1	0.1294
4	$12 million	35.29412%	4.8	1.6941
Total	34 million	100.000%	7	5.3765

Portfolio Duration = 5.4

b. the contribution of bond 1 to portfolio duration is

$$\frac{\text{Market Value of bond 1}}{\text{Market value of Portfolio}} \times \text{duration of bond 1} = \frac{\$10\text{million}}{\$34\text{million}} \times 7.2 = 2.1176$$