Derivatives
Revisions 2
Solutions

B. Questions

B1. The investor's broker borrows the shares from another client's and sells them in the usual way. To close out the position, the investor must purchase the shares. The broker then replaces them in the account of the client from whom they were borrowed. The party with the short position must remit to the broker dividends and other income paid on the shares. The broker transfers these funds to the account of the client from whom the shares were borrowed. Occasionally the broker runs out of places from which to borrow shares. The investor is then squeezed and has to close out the position immediately.

B2. The forward price of an asset today is the price at which you would agree to buy or sell the asset at a future time. The value of a forward contract is zero when you first enter into it. As time passes the underlying asset price changes and the value of the contract may become positive or negative.

B3. The forward price is $30e^{0.12\times0.5} = \$\mathbf{31.86}$

B4. The futures price is $350e^{(0.08-0.04)\times0.3333} = \$354.7$

B5.

a. The forward price, F_0, is given by $F_0 = 40e^{0.1\times1} = 44.21$
or, $44.21. The initial value of the forward contract is zero.

b. The delivery price K in the contract is $44.21. The value of the contract, f, after six months is given by, $f = 45 - 44.21e^{-0.1\times0.5} = 2.95$

i.e., it is $2.95. The forward price is: $45e^{0.1\times0.5} = 47.31$ or $47.31

B6. The six month futures price is, $150e^{(0.07-0.032)\times0.5} = 152.88$ or $152.88

B7. The theoretical futures price is $0.8000e^{(0.05-0.02)\times2/12} = 0.8040$

The actual futures price is too high. This suggests that an arbitrageur should buy Swiss francs and short Swiss francs futures

B8. The present value of the storage costs for nine months are $0.06 + 0.06e^{-0.10\times0.25} + 0.06e^{-0.10\times5} = 0.176$ or $0.176.
The futures price is given by F_0 where, $F_0 = (9.000 + 0.176)e^{0.1\times0.75} = 9.89$, i.e. it is $9.89 per ounce.

B9. In total the gain or loss under a futures contract is equal to the gain or loss under the corresponding forward contract. However the timing of the cash flows is different. When the time value of money is taken into account a futures contract may prove to be more valuable or less valuable than a forward contract. Of course the company does not know in advance which will work out better. The long forward contract provides a perfect hedge. The long futures contract provides slightly imperfect hedge.
a. In this case the forward contract would lead to a slightly better outcome. The company will make a loss on its hedge. If the hedge is with a forward contract, the whole of the loss will be realized at the end. If it is with a futures contract the loss will be realized day by day throughout the contract. On a present value basis the former is preferable.

b. In this case the futures contract would lead to a slightly better outcome. The company will make a gain on the hedge. If the hedge is with forward contract the gain will be realized at the end. If it is with futures contract the gain will be realized day by day throughout the life of the contract. On a present value basis the latter is preferable.

c. In this case the futures contract would lead to a slightly better outcome. This is because it would involve positive cash flows early and negative cash flows later.

d. In this case the forward contract would lead to a slightly better outcome. This is because, in the case of the futures contract, the early cash flows would be negative and the later cash flow would be positive.

C. Problems

C1.
a. The present value, I, of the income from the security is given by:

$$I = 1 \times e^{-0.08\times2/12} + 1 \times e^{-0.08\times5/12} = 1.9540$$

The forward price, F_0, is given by:
$$F_0 = (50 - 1.9540)e^{0.08\times0.5} = 50.01, \text{ or } \$50.01$$

The initial value of the forward contract is (by design) zero. The fact that the forward price is very close to the spot price should come as no surprise. When the compounding frequency is ignored the dividend yield on the stock equals the risk-free rate of interest.

b. In three months, $I = e^{-0.08\times2/12} = 0.9868$

The delivery price, K, is 50.01. The value of the short forward contract, f, is given by

$$f = -\left(48 - 0.9868 - 50.01e^{-0.08\times\frac{3}{12}}\right) = 2.01$$

And the forward price is:
$$(48 - 0.9868)e^{0.08\times3/12} = 47.96$$

C2.

Suppose that F_0 is the one-year forward price of gold. If F_0 is relatively high, the trader can borrow \$550 at 6%, buy one ounce of gold and enter into a forward contract to sell gold in one year for F_0. The profit made in one year is:

$$F_0 - 550 \times 1.06 = F_0 - 583$$

If F_0 is relatively low, the reader can sell one ounce of gold for \$549, invest the proceeds at 5.5%, and enter into a forward contract to buy the gold back for F_0. The profit (relative to the position the trader would be in if the gold were held in the portfolio during the year).

$$549 \times 1.055 - F_0 = 579.195 - F_0$$

This shows that there is no arbitrage opportunity if the forward price is between \$579.195 and \$583 per ounce.