Derivatives
Revisions 4
Questions

Trading Strategies involving Options

1. What is meant by a protective put? What position in call options is equivalent to a protective put?

A protective put consists of a long position in a put option combined with a long position in the underlying shares. It is equivalent to a long position in a call option plus a certain amount of cash. This follows from put-call parity:

$$p + S_0 = c + Ke^{-rt} + D$$

2. Explain two ways in which a bear spread can be created.

A bear spread can be created using two call options with the same maturity and different strike prices. The investor shorts the call option with the lower strike price and buys the call option with the higher strike price.
A bear spread can also be created using two put options with the same maturity and different strike prices. In this case, the investor shorts the put option with the lower strike price and buys the put option with the higher strike price.

3. When is it appropriate for an investor to purchase a butterfly spread?

A butterfly spread involves a position in options with three different strike prices (K_1, K_2 and K_3). A butterfly spread should be purchased when the investor considers that the price of the underlying stock is likely to stay close to the central strike price, K_2.

4 Call options on a stock are available with strike prices of $15, $17.5, and $20, and expiration dates in 3 months. Their prices are $4, $2, and $0.5, respectively. Explain how the options can be used to create a butterfly spread. Construct a table showing how profit varies with stock price for the butterfly spread.

An investor can create a butterfly spread by buying call options with strike prices of $15 and $20 and selling two call options with strike prices of $17.5. The initial investment is ($4+$0,5 – 2 ×$2=$0.5
The following table shows the variation of profit with the final stock price:

Stock Price S_T	Profit
$S_T < 15$	-0.5
$15 < S_T < 17.5$	$(S_T – 15) – 0.5$
$17.5 < S_T < 20$	$(20-S_T) – 0.5$
$S_T > 20$	$– 0.5$

5. A call option with a strike price of \$50 costs \$2. A put option with a strike price of \$45 cost \$3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

A stangle is created by buying options. The pattern of profits is as follows:

Stock Price S_T	Profit
$S_T < 45$	$(45 - S_T) - 5$
$45 < S_T < 50$	-5
$S_T > 50$	$(S_T - 50) - 5$

6. Use put-call parity to relate the initial investment for a bull spread created using calls to the initial investment for a bull spread created using puts.

A bll spread using calls provides a profit pattern with the same general shape as a bull spread using puts. Define p_1 and c_1 as the prices of a put and call with strike price K_1 and p_2 and c_2 the prices of a put and call with strike price k_2. From put-call parity:

$$p_1 + S = c_1 + k_1 e^{-rt}$$

$$p_2 + S = c_2 + k_2 e^{-rt}$$

Hence,

$$p_1 - p_2 = c_1 - c_2 - (k_2 - k_1)e^{-rt}$$

This shows that the initial investment when the spread is created from puts is less than the initial investment when it is created from calls by an amount $(k_2 - k_1)e^{-rt}$. In fact as mentioned in the text the initial investment when the bull spread is created from puts is negative, while the initial investment when it is created from calls is positive.

The profit when calls are used to create the bull spread is higher than when puts are used by $(k_2 - k_1)(1 - e^{-rt})$. This reflects the fact that the call strategy involves an additional risk-free investment of $(k_2 - k_1)e^{-rt}$ over the put strategy.

This earns interest of $(k_2 - k_1)e^{-rt}(e^{rt} - 1) = (k_2 - k_1)(1 - e^{-rt})$

The Greek Letters

1. What does it mean to assert that the delta of a call option is 0.7? How can a short position on 1,000 options be made delta neutral when delta of each option is 0.7?

A delta of 07 menas when the price of the stock increases by a small amount, the price of the option increases by 70%of this amount. Similarly, when the ptice of the stock decreases by a small amount, the price of the option decreases by 70% of this amount. A short positon in 1,000 options has a delta of -700 and can be made delta neutral with the purchase of 700 shares.

2. Calcualte the delta of an at-the-money six-momth european call option on a non-dividend-paying stock when the risk-free rate is 10% per annum and the stock price volatility is 25% per annum.

In this case S_0 = K, r = 0.1, σ = 0.25, and T=0.5. Also

$$d_1 = \frac{\ln(S_0/K) + (0.1 + 0.25^2/2) \times 0.5}{0.25\sqrt{0.5}} = 0.3712$$

The delta of the option is $N(d_1)$ or 0.64

3. What does it mean to assert that the theta of an option position is -0.1 when time is measured in years? If a trader feels that neither a stock price nor its implied volatility will change, what type of option is appropriate?
A theta of -0.1 means that if Δt units of time pass with no change in either the stock price or its volatility, the value of the option declines by $0.1\Delta t$. A trader who feels that neither the stock price nor its implied volatility will change should write an option with as high a negative theta as possible. Relatively short-life at-the-money options have the most negative thetas.

4. What is meant by the gamma of an option position? What are the risks in the situation where the gamma of a position is highly negative and the delta is zero?

The gamma of an option position is the rate if change of delta of the position with respect to the asset price. For example, a gamma of 0.1 would indicate that when the asset price increases by a certain small amount delta increases by 0.1 of this amount. When the gamma of an option writer's position is large and negative and the delta is zero, the option writer will lose significant amounts of money if there is a large movement (either an increase of a decrease) in the asset price.