Derivatives
Solutions

Question 1

Explain carefully the difference between hedging, speculation, and arbitrage.

A trader is hedging when has an exposure to the price of an asset and takes a position in a derivative to offset the exposure. In a speculation the trader has no exposure to offset. He is betting on the future movements in the price of the asset. Arbitrage involves taking a position in two or more different markets to lock a profit.

Question 2

What is the difference between entering into a long forward contract when the forward price is $20 and taking a long position in a call option with a strike price of $20?

In the first case the trader is obligated to buy the asset for $20 (the trader does not have alternative). In the second case the trader has an option to buy the asset for $20 (the trader does not have to exercise the option).

Question 3

What kind of swap should a financial institution enter into if it will be adversely affected by increasing interest rates over the next 2 years? Should the bank pay fixed and receive floating, or vice versa?

A bank that fears increasing interest rates would like to replace obligatory payments based on the floating rate with payments based on a fixed rate. Therefore, the bank should pay fixed and receive floating interest rate swap.

Question 4

What is the major distinction between a forward contract and an options contract?

A forward contract obligates the buyer to pay for (and accept delivery of) the underlying asset and obligates the seller to deliver (and accept cash for) the underlying asset. The buyer of an option contract cannot be forced to exercise the option. An investor who owns a call option has the right, but not the obligation, to buy (accept delivery of) the underlying asset. An investor who owns a put option has the right, but not the obligation, to sell (deliver) the underlying asset.

Question 5

Hedgers, speculators, and arbitrageurs are three types of futures traders. For each type, explain what would motivate the trader to open a *long* futures position.

Hedgers open long futures positions if they are short the cash asset. This would occur, for example, if the investor were obligated to buy the underlying asset in the future. Hedgers take offsetting positions in the cash and futures markets to reduce risk. Speculators try to profit by guessing which way the futures price will move. A speculator would open a long futures position if he or she *believed that the futures price was going to increase. Arbitrageurs exploit price imbalances to collect risk-free profit. An arbitrageur buys futures contracts when the futures price is "too low" with respect to the spot price.*

Question 6

A trader enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of 50,000 pounds. How much does the trader gain or lose if the cotton price at the end of the contract is:
a. 48.20 cents per pound
b. 51.30 cents per pound

a. The trader sells for 50 cents per pound something that is worth 48.20 cents per pound.
Gain = ($0.50 - $0.4820) × 50,000 = $900
Loss = ($0.5130 - $0.50) × 50,000 = $650

Question 7

Suppose you manage a portfolio of $100 million. You estimate that there is a probability of 20% that the portfolio value will go to $80 million next year and a probability of 80% that it will go to $120 million. If the value is less than $90 million, you will be fired. How can you protect yourself with put options?

Go long in 1-year put options on the underlying portfolio of $100 million with a total strike price of $90 million.

Question 8

What is the role of credit derivatives?

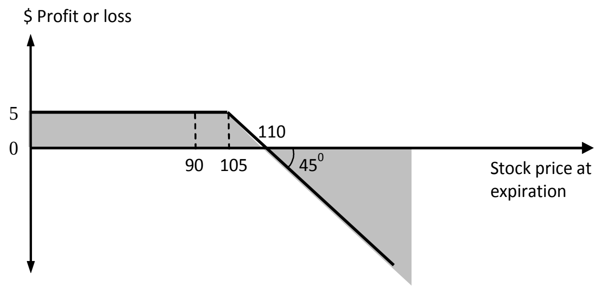
Credit derivatives can be used to:

- *facilitate access to, or to hedge credit exposure*
- *transfer credit risk*
- *enhance yield by generating leverage*
- *separate risks embedded in securities*
- *manage regulatory capital requirements*
- *Separating credit risk from underlying cash market*

Question 9

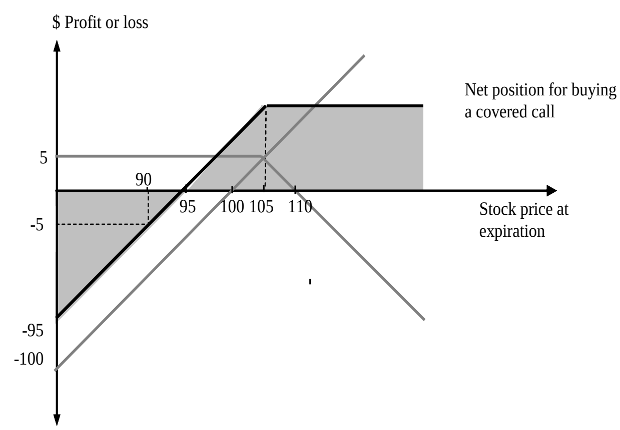
Suppose the stock price is $100 and the call price is $5 with a strike price of $105. What is the profit or loss on the following two strategies when the stock price goes up to $110 and when the stock goes down to $90?
a. Write a call option.
b. Write a covered call option; that is, write a call and buy the stock.

__a.__ In case stock price rises to $110, the holder of the call will exercise it, thus the profit/loss of the option writer will be zero, he or she will lose $5, the same amount he has gained/paid before as option premium. (See payoff diagram below).



If the stock price goes down to $90, the call won't be exercised, and the writer will retain the $5 option premium.

***b.** Write a covered call option; that is, write a call and buy the stock.*



The combination of long call and short position on stock creates the payoff diagram similar to a short position in a put.

Period t_0: At the moment when an investor writes a call and buys the stock his or her cash flow is: -$100 + $5 = - $95.

Period t_1: If the stock price rises to $110, writer's counterparty exercises the call at the strike price of $105. The writer's payoff on the option is zero (as described above), however the writer also holds the stock, bought for $100, which is now $110. Total net payoff can be calculated as: -$95 - $5 + $110 = $10

If the stock prices goes down to $90, the option is not exercised, but the writer suffers a loss from the decrease of the price of the stocks he or she holds.

Thus total net payoff is: - $95 + $90 = - $5.

In comparison with the strategy of selling a call only, there is a chance to get a greater payoff, but also a larger loss.

Question 10

When first issued, a stock provides funds for a company. Is the same true of a stock option? Discuss.

A stock option provides no funds for the company. It is a security sold by one trader to another. The company is not involved. By contrast, a stock when it is first issued is a claim sold by the company to investors and does provide funds for the company.

Question 11

A 2-year LIBOR-based swap will have four future semiannual payments. If the fixed rate is 5% and the principal is $3,000,000, the future cash flows will depend on the differences between 5% and the current LIBOR rate. If the LIBOR rate is 7% after 1 year, what semiannual payment must the fixed counterparty pay the receiver floating counterparty?

Semi-annual payment = (0.07 – 0.05)(3,000,000)(180/360) = $30,000.

Question 12

Suppose σ = 35%, S_0 = $100, X = $100, r = 5%, q = 0 and t = 1/2, $N(d_1)$ = 0.58892 and $N(d_2)$ = 0.49094. Calculate the Black-Scholes call and put option prices.

Call $= 100e^{-(0*0.5)} * 0.58892 - 100e^{-(0.05*0.5)} * 0.49094 = 58.892 - 47.8819 = \mathbf{11.01}$
Put $= 100e^{-(0.05*0.5)} * [1- 0.49094] - 100e^{-(0*0.5)} * [1- 0.58892] = 49.6491 - 41.108 = \mathbf{8.54}$

Pricing the call using the PCP:
$C_0 = 100 - \frac{100}{(1.05)^{0.5}} + 8.54 = \mathbf{10.95}$

Pricing the put using the PCP:
$P_0 = \frac{100}{(1.05)^{0.5}} - 100 + 11.01 = \mathbf{8.60}$

Question 13

Under what circumstances are appropriated:
a. a short hedge
b. a long hedge

a. *A short hedge is appropriated when a company owns an asset and expects to sell that asset in the future. It can also be used when the company does not currently own the asset but expects to do so at some time in the future.*
b. A long hedge is appropriated when a company knows it will have to purchase an asset in the future. It can also be sued to offset the risk from an existing short position.

Question 14

Explain why an FRA is equivalent to the exchange of a floating rate of interest for a fixed rate of interest.

A FRA is an agreement that a certain specified interest rate, R_K, will apply to a certain principal, L, for a certain specified future time period. Suppose that the rate observed in the market for the future time period at the beginning of the time period proves to be R_M. If the FRA is an agreement that R_K will apply when the principal is invested, the holder of the FRA can borrow the principal at R_M and them invest it at R_K. The net cash flow at the end of the period is then an inflow of $R_K L$ and an outflow of $R_M L$. If the FRA is an agreement that R_K will apply when the principal is borrowed, the holder of the FRA can invest the borrowed principal at R_M. The net cash flow at the end of the period is then an inflow of RML and an outflow of $R_K L$. In either case we see that the FRA involves the exchange of a fixed rate of interest, R_K, on the principal of L for the floating rate of interest observed in the market, R_M.

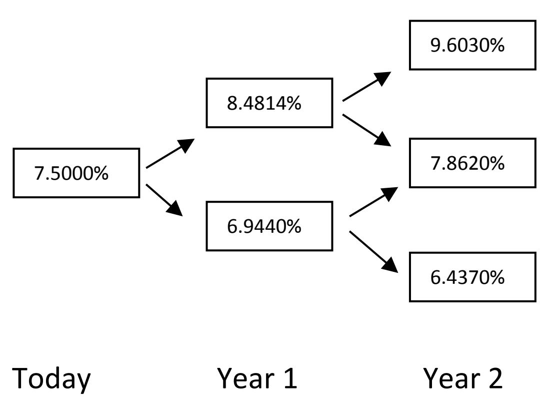
Question 15

Shaun has entered a pay fixed plain vanilla interest rate swap with quarterly settlement starting on the 1st July 2013, based on a fixed rate of 3.55% for LIBOR. LIBOR is 3.79% at the start of the swap and falls by 20bp by 1st October 2013 and a further 35bp by 1st January 2014. If the swap has a notional principal of £13,000,000 calculate the payment Shaun will make/receive on 1st January 2014. Show your calculations

Payment received is based on START of period value of LIBOR.
Thus, payment on 1st January based on 1st October value of (3.79% - 0.2% = 3.59%)
So, total payment = £13,000,000 x 3/12 x (3.59%-3.55%) = £1,300
As Shaun is the pay fixed, he will RECEIVE £1,300

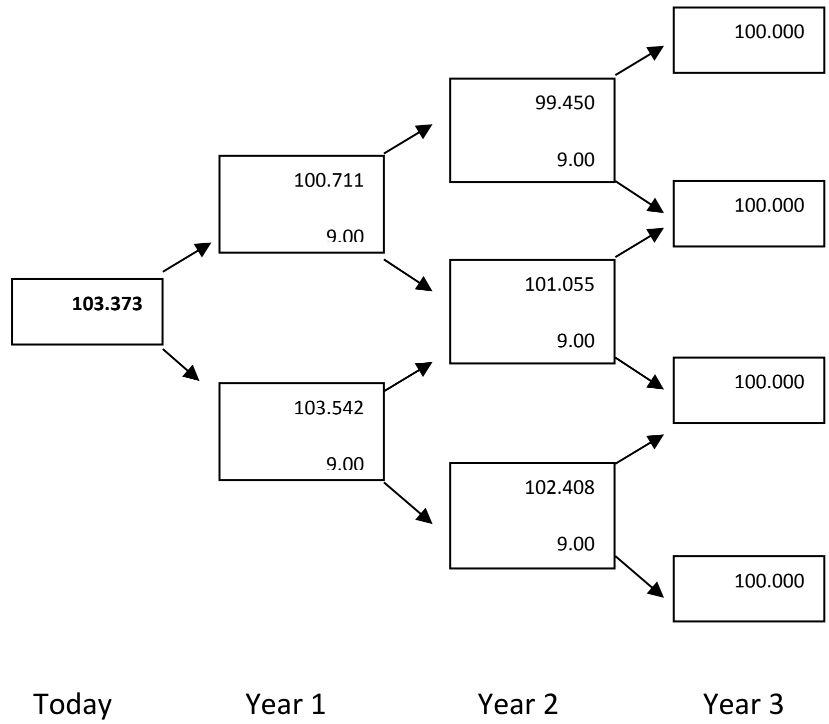
Question 16

Use the following arbitrage-free binomial interest rate tree to answer the questions that follow:

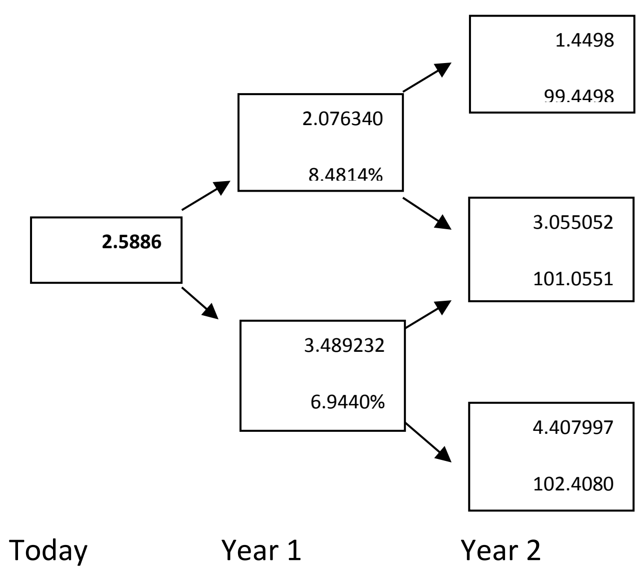


a. What is the value of a 3-year Treasury bond with a 9% coupon rate?
b. What is the value of a 2-year call option on a bond that currently has three years to maturity and coupon rate of 9% if the strike price is 98? Assume in this calculation that the current price of the 3-year Treasury bond is the value found in part a.

a. *The value of the 3-year Treasury bond is 103.373, as shown below*



b. *The current price of the bond is 103.373 as found in part i) and the price assumed in the question. The value of the 2-year call option is $2.5886, as shown below:*



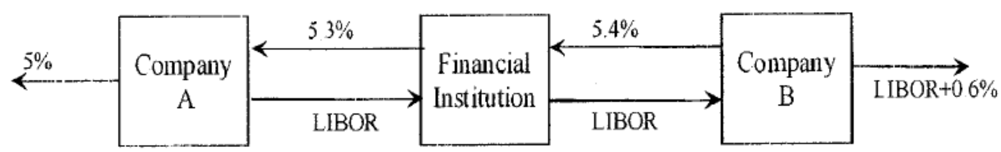
Question 17

Companies A and B has been offered the following rates per annum on a $20 million five-year loan:

	Fixed rate	Floating rate
Company A	5.0%	LiBOR + 0.1%
Company B	6.4%	LIBOR + 0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

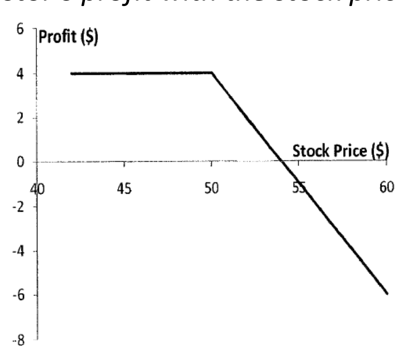
A has an apparent comparative advantage in fixed-rate markets but wants to borrow floating. B has an apparent comparative advantage in floating-rate markets but wants to borrow fixed. This provides the basis for the swap. There is a 1.4% per annum differential between the fixed rates offered to the two companies and a 0.5% per annum differential between the floating rates offered to the two companies. The total gain to all parties from the swap is therefore 1.4 – 0.5 = 0.9% per annum. Because the bank gets 0.1% per annum of this gain, the swap should make each A and B 0.4% per annum better off. This means that it should lead to A borrowing at LIBOR – 0.3% and to B borrowing at 6.0%. The appropriate arrangement is therefore shown in the figure below.



Question 18

An investor sells a European call on a share for $4. The stock price is $47 and the strike price is %50. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option.

The investor makes a profit if the price of the stock is below $54 on the expiration date. If the stock price is below $50, the option will not be exercised, and the investor makes a profit of $4. If the stock price is between $50 and $54, the option is exercised and the investor makes a profit between $0 and $4. The variation of investor's profit with the stock price is shown in the figure below.



Question 19

Explain why an American option is always worth at least as much as a European option on the same asset with the same strike price and exercise date.

The holder of an American option has all the same rights as the holder of a European option and more. It must therefore be worth at least as much. If it were not, an arbitrageur could short the European option and take a long position in the American option.

Question 20

What is meant by a protective put? What position in call options is equivalent to a protective put?

A protective put consists of a long position in a put option combined with a long position in the underlying shares. It is equivalent to a long position in a call option plus a certain amount of cash. This follows from put-call parity:

$$p + S_0 = c + K_e^{-rT} + D$$

Question 21

A call option with a strike price of $50 costs $2. A put option with a strike price of $45 costs $3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle?

A strangle is created by buying both options. The pattern of profits is as follows:

Stock Price S_T	Profit
$S_T < 45$	$(45 - S_T) – 5$
$45 < S_T < 50$	$– 5$
$S_T > 50$	$(S_T – 50) - 5$

Question 22

Three put options on a stock have the same expiration date and strike prices of $55, $60, and $65. The market prices are $3, $5, and $8, respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?

A butterfly spread is created by byuying the $55 put, buying the $65 put and selling two of the $60 puts. This costs 3 + 8 – 2 × 5 = $1 initially. The following table shows the profit/loss from the strategy.

Stock Price	Payoff	Profit
$S_T \geq 65$	0	-1
$60 \leq S_T < 65$	$65 - S_T$	$64 - S_T$
$55 \leq S_T\ 60$	$S_T – 55$	$S_T - 56$
$S_T < 55$	0	-1

The butterfly spread leads to a loss when the final stock price is greater then $64 or less than $56

Question 23

Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European option using a one-step binomial tree.

In the no-arbitrage approach, we set up a riskless portfolio consisting of a position in the option and a position in the stock. By setting the return on the portfolio equal to the risk-free interest rate, we are able to value the option. When we use risk-neutral valuation, we first choose probabilities for the branches of the tree so that the expected return on the stock equals the risk-free interest rate. We then value the option by calculating its expected payoff and discounting this expected payoff at the risk-free interest rate.

Question 24

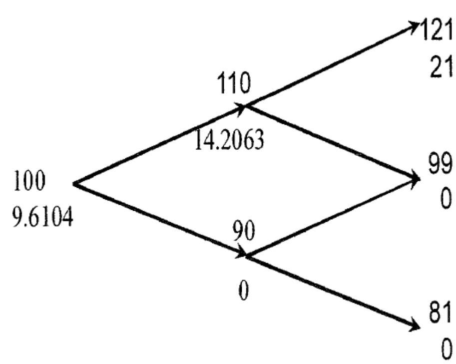
A stock price is currently \$100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of \$100?

In this case u = 1.10, d = 0.90, Δt = 0.5, and r = 0.08, so that

$$p = \frac{e^{0.08\times 0.5} - 0.90}{1.10 - 0.90} = 0.7041$$

The tree for stock price movements is shown in the figure below. We can work back from the end of the tree to the beginning, as indicated in the diagram, to give the value of the option as \$9.61. The option value can be calculated as:

$$[0.7041^2 \times 21 + 2 \times 0.7041 \times 0.2959 \times 0 + 0.2959^2 \times 0]e^{-2\times 0.08\times 0.5} = 9.61 \; or \; \$9.61$$



Question 25

The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one trading day?

The standard deviation of the percentage price change in time Δt is $\sigma\sqrt{\Delta t}$ where σ is the volatility. In the problem σ = 0.3 and, assuming 252 trading days in one year, Δt = 1/252 so that $\sigma\sqrt{\Delta t} = 0.3\sqrt{0.004} = 0.019 \; or \; 1.9\%$

Question 26

Explain the principle of risk-neutral valuation.

The price of an option or other derivative when expressed in terms of the price of the underlying stock is independent of risk preferences. Options therefore have the same value in a risk-neutral world as they do in the real world. We may therefore assume that the world is risk neutral for the purposes of valuing options. This simplifies the analysis. In a risk-neutral world all securities have an expected return equal to risk-free interest rate. Also, in a risk-neutral world, the appropriate discount rate to use for expected future cash flows is the risk-free interest rate.

Question 27

Calculate the price of a three-month European put option on a non-dividend paying stock with a strike price of $50 when the current stock price is $50, the risk-free interest rate is 10% per annum, and the volatility is 30% per annum.

In this case S_0 = 50, K = 50, r = 0.1, σ = 0.3, T = 0.25, and

$$d_1 = \frac{\ln(50/50) + (0.1 + 0.09/2)0.25}{0.3\sqrt{0.25}} = 0.2417$$

$$d_2 = d_1 - 0.3\sqrt{0.25} = 0.0917$$

The European put price is

$$50N(-0.0917)e^{-0.1\times 0.25} - 50N(-0.2417)$$
$$= 50 \times 0.4634e^{-0.1\times 0.25} - 50 \times 0.4045 = 2.37 \; or \; \$2.37$$

Question 28

What is implied volatility? How can be calculated?

The implied volatility is the volatility that makes the Black-Scholes price of an option equal to its market price. It is calculated using an iterative procedure

Question 29

Consider a position consisting of a $100,000 investment in Asset A, and a $100,000 investment in asset B. assume that the daily volatilities of both assets are 1% and that the coefficient of correlation between their returns is 0.3. What is the 5-day 99% VaR for the portfolio?

The standard deviation of the daily change in the investment in each asset is $1,000. The variance of the portfolio's daily change is:

$$1{,}000^2 + 1{,}000^2 + 2 \times 0.3 \times 1{,}000 \times 1{,}000 = 2{,}600{,}000$$

The standard deviation of the portfolio's daily change is the square root of this or $1,612.45.

The standard deviation of the 5-day change is:

$$1{,}612.45 \times \sqrt{5} = \$3{,}605.55$$

From the tables of N(x) we see that N(-2.33) = 0.01. this means that 1% of a normal distribution lies more than 2.33 standard deviations below the mean. The 5-day 99 percent value at risk is therefore 2.33 × 3,605.55 = $8,401

Question 30

A stock price is currently $40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding?

a. What is the value of a six-month European put option with a strike price of $42?

A tree describing the behavior of the stock price is shown below. The risk-neutral probability of an up move, p, is given by:

$$p = \frac{e^{0.12\times3/12} - 0.90}{1.1 - 0.9} = 0.6523$$

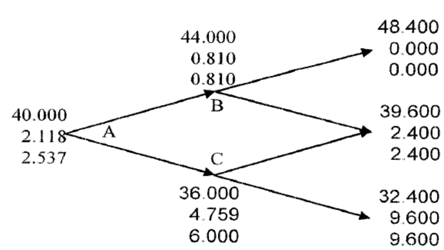
Calculating the expected payoff and discounting, we obtain the value of the option as:

$$[2.4 \times 2 \times 0.6523 \times 0.3477 + 9.6 \times 0.3477^2]e^{-0.12\times6/12} = 2.118$$

The value of the European option is 2.118. this can also be calculated by working back through the tree as shown below. The second number at each node is the value of the European option.

b. What is the value of a six-month American put option with a strike price of $42?

The value of the American option is shown as the third number at each node on the tree. It is 2.537. This is greater than the value of the European option because it is optimal to exercise early at node C.



Note: At each node, upper number is the stock price; next number is the European put price; final number is the American put price.