## SOLUTIONS

## A. Short Answer Questions

A1.
To answer this question; think about the relationship between the variables in the P/E ratio formula:
$\frac{P_{0}}{E_{1}}=\frac{D_{1}}{E_{1}\left(K_{e}-g\right)}=\frac{D_{0}(1+g)}{E_{1}\left(K_{e}-g\right)}$
a. The rise in g implies that the $\mathrm{P} / \mathrm{E}$ ratio would rise.
b. The rise in the retention rate implies the $\mathrm{P} / \mathrm{E}$ ratio will fall.
c. The rise in the earnings per share would result in the $\mathrm{P} / \mathrm{E}$ ratio falling.

## A2.

a. This director is confused. In the context of the constant growth model [i.e., $\mathrm{P}_{0}=\mathrm{D}_{1} /(\mathrm{k}-\mathrm{g})$ ], it is true that price is higher when dividends are higher holding everything else including dividend growth constant. But everything else will not be constant. If the firm increases the dividend payout rate, the growth rate $g$ will fall (given that the company will have to forego retained earnings for increased dividends), and stock price will not necessarily rise. In fact, if ROE $>k$, price will fall.
b. (i) An increase in dividend payout will reduce the sustainable growth rate as less funds are reinvested in the firm. The sustainable growth rate
(i.e., $\mathrm{ROE} \times$ plowback) will fall as plowback ratio falls.
(ii) The increased dividend payout rate will reduce the growth rate of book value for the same reason -- less funds are reinvested in the firm.

## B. Problems

B1.
a. $\quad k_{e}=r_{f}+\beta\left[E\left(r_{M}\right)-r_{f}\right]=6 \%+1.25(14 \%-6 \%)=16 \%$
$\mathrm{g}=$ Retained earnings $(\mathrm{b}$ OR $(1-\alpha)) \times$ Return on Equity (ROE)
$\mathrm{g}=2 / 3 \times 9 \%=6 \%$
$\mathrm{D}_{1}=\mathrm{E}_{0}(1+\mathrm{g})(1-\mathrm{b})=\$ 3(1.06)(1 / 3)=\$ 1.06$
$P_{0}=\frac{D_{1}}{k_{e}-g}=\frac{\$ 1.06}{0.16-0.06}=\$ 10.60$
b. Leading and commonly used Price Earnings Ratio $=\mathrm{P}_{0} / \mathrm{E}_{1}=\$ 10.60 / \$ 3.18=3.33$

Trailing or Historical Price Earnings Ratio $=\mathrm{P}_{0} / \mathrm{E}_{0}=\$ 10.60 / \$ 3.00=3.53$
c. $\quad P V G O=P_{0}-\frac{E_{1}}{k_{e}}=\$ 10.60-\frac{\$ 3.18}{0.16}=-\$ 9.275$

The low P/E ratios and negative PVGO are due to a poor ROE (9\%) that is less than the market capitalization rate ( $16 \%$ ).
d. Now, you revise b to $1 / 3$, $g$ to $1 / 3 \times 9 \%=3 \%$, and $D_{1}$ to:

$$
D_{1}=E_{0}(1+g)(1-b)=E_{0} \times 1.03 \times(2 / 3)=\$ 2.06
$$

Thus:

$$
\mathrm{V}_{0}=\mathrm{D}_{1} /\left(\mathrm{K}_{\mathrm{e}}-\mathrm{g}\right)=\$ 2.06 /(0.16-0.03)=\$ 15.85
$$

$\mathrm{V}_{0}$ increases because the firm pays out more earnings instead of reinvesting a poor ROE. This information is not yet known to the rest of the market.

## B2.

a. $\mathrm{g}=\mathrm{ROE} \times \mathrm{b}=16 \% \times 0.5=8 \%$
$\mathrm{D}_{1}=\$ 2(1-\mathrm{b})=\$ 2(1-0.5)=\$ 1$
$\mathrm{P}_{0}=\mathrm{D}_{1} /\left(\mathrm{k}_{\mathrm{e}}-\mathrm{g}\right)=\$ 1 /(0.12-0.08)=\$ 25$
b. $\quad P_{3}=P_{0}(1+g)^{3}=\$ 25(1.08)^{3}=\$ 31.49$

B3.
a. $\quad P_{0}=\frac{D_{1}}{k-g}=\frac{\$ 8}{0.10-0.05}=\$ 160$
b. The dividend payout ratio is $8 / 12=2 / 3$, so the plowback ratio is $b=1 / 3$. The implied value of ROE on future investments is found by solving:

$$
g=b \times \text { ROE with } g=5 \% \text { and } b=1 / 3 \Rightarrow R O E=15 \%
$$

c. Assuming ROE $=\mathrm{k}$, price is equal to:

$$
P_{0}=\frac{E_{1}}{k}=\frac{\$ 12}{0.10}=\$ 120
$$

Therefore, the market is paying $\$ 40$ per share $(\$ 160-\$ 120)$ for growth opportunities.

## B4.

a. The formula for the Gordon model is:

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\(\mathrm{V}_{0}=\left[\mathrm{D}_{0} \times(1+\mathrm{g})\right] /\left(\mathrm{k}_{\mathrm{e}}-\mathrm{g}\right)\)
where:
\(\mathrm{D}_{0}=\) dividend paid at time of valuation
\(\mathrm{g}=\) annual growth rate of dividends
\(r=\) required rate of return for equity
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In the above formula, $\mathrm{P}_{0}$, the market price of the common stock, substitutes for $\mathrm{V}_{0}$ and g becomes the dividend growth rate implied by the market:
$\mathrm{P}_{0}=\left[\mathrm{D}_{0} \times(1+\mathrm{g})\right] /(\mathrm{r}-\mathrm{g})$
Substituting, we have:
$58.49=[0.80 \times(1+\mathrm{g})] /(0.08-\mathrm{g}) \Rightarrow \mathrm{g}=6.54 \%$
b. Use of the Gordon growth model would be inappropriate to value Dynamic's common stock, for the following reason:
The Gordon growth model assumes a set of relationships about the growth rate for dividends, earnings, and stock values. Specifically, the model assumes that dividends, earnings, and stock
values will grow at the same constant rate. In valuing Dynamic's common stock, the Gordon growth model is inappropriate because management's dividend policy has held dividends constant in dollar amount although earnings have grown, thus reducing the payout ratio. This policy is inconsistent with the Gordon model assumption that the payout ratio is constant.

B5.
a.
$E(R i)=0.058+0.057 \beta i=0.058+0.057 \times 1.24=0.058+0.0706=0.1286$
Or, 12.9\%
b. We add 3 percent to the IBM bond YTM: $\mathbf{6 . 2 3 8 \%} \mathbf{+ 3 \%} \mathbf{= 9 . 2 3 8 \%}$, or $\mathbf{9 . 2 \%}$. Note that the difference between the IBM and T-bond YTM is 0.438 percent, or 44 basis points. This amount plus 3 percent is the total risk premium versus treasury debt.
c. Undervalued means that the value of a security is greater than market price. All else equal, the smaller the discount rate, the higher the estimate of value. The inverse relationship in valuation. If IBM appears to be undervalued using the CAPM cost of equity estimate of 12.9 percent, it will appear to be even more undervalued using a 9.2 percent cost of equity based on the bond yield plus risk premium method.

## B6.

From the data, each component costs can be found and, subsequently, the weighted average cost of capital can be calculated.
The cost of preferred stock is:

$$
r_{P}=\frac{D i v_{P}}{P_{P S}}=\frac{\$ 3.75}{\$ 50.00}=7.75 \%
$$

The cost of common equity capital, which is the cost of equity (rCE), can be calculated in one of three ways:

1) $r_{C E}=r_{D}+R_{E R P}=7.0 \%+2.7 \%=9.7 \% *$
2) $r_{C E}=\frac{D i v_{1}}{P_{C S}}+g_{D i v}=E($ Dividend Yield $)+g_{D i v}=3.7 \%+6.0 \%=9.7 \% *$
3) $r_{C E}=r_{F}+\beta_{C S}\left(r_{M}-r_{F}\right)=6.5 \%+1.07(9.5-6.5)=9.7 \% *$
*Through judicious selection of the data, the cost of equity is the same for all three approaches in this example. In general, all three approaches will not produce the same answer. This only means that there is some uncertainty regarding a firm's cost of common equity capital, which should be viewed as being within a range of values defined by these models.

Therefore, the weighted cost of capital of the firm under these conditions is as follows:

$$
\begin{aligned}
& r_{w}=(1-t) r_{D}\left(\frac{V_{D}}{V_{A}}\right)+r_{P}\left(\frac{V_{P}}{V_{A}}\right)+r_{C E}\left(\frac{V_{C E}}{V_{A}}\right) \\
& =(1-.35)(.07)\left(\frac{106 \text { million }}{1,000 \text { million }}\right)=0.075\left(\frac{52 \text { million }}{1,000 \text { million }}\right)+0.097\left(\frac{842 \text { million }}{1,000 \text { million }}\right)=9.0 \%
\end{aligned}
$$

B7.
a) $V 0=\frac{D_{1}}{r-g}=\frac{\$ 0.83}{0.062-0.037}=\frac{\$ 0.83}{0.025}=\$ 33.20$
b) Because the Gordon growth model estimate of $\$ 33.20$ is $\$ 3.20$ higher than the market price of $\$ 30.00$, ABC appears to be slightly undervalued.
c) Stable dividend growth is a realistic model for ABC for the following reasons:
a. ABC profitability is stable as reflected in its return on equity. This reflects predictable demand and regulated prices for its product, water.
b. Dividends bear an understandable and consistent relationship to earnings as evidenced here by a stable dividend payout ratio.
c. Earnings growth, at 3.7 percent a year, is less than nominal annual GDP growth for the US and is plausibly sustainable long term.
d.
d) The cost of equity as given by the CAPM is
$R_{F}=B_{i}\left[E\left(R_{M}\right)-R_{F}\right]=0.057+(-0.16 \times 0.057)=0.04788$
or 4.8 percent. As noted above, both RF and $\left[E\left(R_{M}\right)-R_{F}\right]$ equal the same rate, here 5.7 percent.
e) The Gordon growth model of ABC using the cost of equity of 4.8 percent is:

$$
V_{0}=\frac{D 1}{r-g}=\frac{\$ 0.83}{0.048-0.037}=\frac{\$ 0.83}{0.011}=\$ 75.45
$$

$\$ 75.45$ is an implausible estimate for the value of ABC judged by a P/E of 24. The $\$ 75.45$ estimated value represents a P/E of 57 on FY2002 earnings, calculated as $\$ 75.45 / \$ 1.33=$ 56.7 or 57. (the number 24 is taken for peer-group comparisons.) In fact, the R-squared for the regression for beta for ABC is about 2 percent, and the CAPM does not do a good job of explaining the returns on this stock.
Note that the question a) used a more plausible cost of equity figure, given as 6.2 percent. ABC does not have publicly traded debt, so the bond yield plus risk premium method was not available. The cost of equity estimate of 0.062 stated in the problem comes from a build-up approximation. As of the year-end 2001, based on the Gordon growth model applied to the S\&P 500, the cost of equity for an average US stock was estimated as 8.2 percent. An average stock has a beta of 1 and should earn the S\&P 500 return, on average. Because ABC has below-average risk (its earnings have above-average stability and its beta is less than 1.0). We subtracted a subjective company-specific risk adjustment of 2 percent. We should note that an APT estimate of the cost of equity is another possibility to consider.
f) Because of the uncertainty in the cost-of-equity estimate, one has less confidence that $A B C$ is undervalued. In particular, the analyst may view $A B C$ as approximately fairly valued.

