## Portfolio Management 2010-2011

1. 

a. Critically discuss the mean-variance approach of portfolio theory
b. According to Markowitz portfolio theory, can we find a single risky optimal portfolio which is suitable to the demands of all investors? Explain why.
c. Briefly define the concept of the yield curve and spot rate
d. True or False? If interest rates in all maturities increase by one percent, the price of shorter maturity bond will fall by a smaller percentage than the fall in price of the longer maturity bond.
e. Briefly define the concept of time value of a call option, and explain why the time value of a call option is always greater than zero?

## 2.

Philip Morris has issued bonds that pay coupons annually with the following characteristics:

| Annual coupon rate | Yield to Maturity <br> per annum | Maturity | Macaulay Duration |
| :---: | :---: | :---: | :---: |
| $4 \%$ | $7 \%$ | 15 years | 10.62 years |

a. Calculate modified duration using the information above.
b. Explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates.
c. Define convexity and explain how modified duration and convexity are used to approximate the bond's percentage change in price, given a change in interest rates.

## 3.

Jane Ludlow's firm requires all its analysts to use a two-stage dividend discount model (DDM) and the Capital Asset Pricing Model (CAPM) to value stocks. Using the CAPM and DDM, Ludlow has valued QuickBrush Company at $\$ 63$ per share. She now must value SmileWhite Corporation.
a. Calculate the required rate of return for SmileWhite by using the information in the following table:

|  | BuickBrush | SmileWhile |
| :--- | :---: | :---: |
| Beta | 1.35 | 1.15 |
| Market price | $\$ 45.00$ | $\$ 30.00$ |
| Intrisic value | $\$ 63.00$ | $?$ |

## Notes:

Risk-free rate 4.50\%
Expected market return $14.50 \%$
b. Ludlow estimates the following EPS and dividend growth rates for SmileWhite:

| First 3 years | $12 \%$ per year |
| :--- | :--- |
| Years thereafter | $9 \%$ per year |

Estimate the intrinsic value of SmileWhite by using the table above, and the twostage DDM. Dividends per share in the most recent year were $\$ 1.72$
c. Recommend QuickBrush or SmileWhite stock for purchase by comparing each company's intrinsic value with its current market price
d. Describe one strength of the two-stage DDM in comparison with the constantgrowth DDM. Describe one weakness inherent in all DDMS.

## 4.

a. An investor purchased 500 half year put options with a strike price equal to $\$ 54$ at a premium of $\$ 2.75$. The investor purchased 500 shares of the stock at $\$ 57.8$. The shortterm risk-free interest rate is $5 \%$ annum.

1) State the name of this strategy
2) Calculate the maximum profit and loss
3) Determine the breakeven point
4) Calculate the premium of one call option with the same maturity as the put option
b. All else equal, will a call option with a high exercise price have a higher or lower hedge ratio than one with a low exercise price? Explain.
c. Present the limitations of the Black-Scholes option-pricing model.

## 5.

a. Identify the fundamental distinction between a futures contract and an option contract and briefly explain the difference in the manner that futures and options modify portfolio risk.
b. Suppose the value of S\&P 500 stock index is currently 1,200 . If the one-year T-bill rate is $6.5 \%$ and the expected dividend yield on the S\&P 500 is $2 \%$, what should the one year maturity futures price be?
c. Consider a stock that pays no dividends on which a futures contract, a call option and a put option trade. The maturity date for all three contracts is $T$, the exercise price of the put and the call are both $X$, and the futures price is $F$. Show that if $X=F$, then the call price equals the put price. Use parity conditions to guide your demonstration.
d. How might a portfolio manager use financial futures to hedge risk when he owns a large position in a relatively illiquid bond that he wants to sell.

## 6.

An analyst wants to evaluate Portfolio W, consisting entirely of U.S. common stocks. The following table provides the average annual rate of return of portfolio W , the market portfolio (as measured by the S\&P 500), and U.S. Treasury bills during the past 5 years. The Beta of portfolio W is 0.60 .

|  | Average Annual <br> Rate of Return (\%) | Standard Deviation <br> of Return (\%) |
| :--- | :---: | :---: |
| Portfolio W | 10 | 18 |
| S\&P 500 | 12 | 13 |
| T-bills | 6 | N/A |

a. Briefly explain whether Portfolio W underperformed or outperformed the S\&P 500 on a risk-adjusted basis using both the Treynor measure and the Sharpe measure.
b. Based on the performance of Portfolio W relative to S\&P 500, briefly explain the reason for the conflicting results when using the Treynor measure versus the Sharpe measure.
c. Which is a better performance index, Sharpe's index or Treynor's index? Explain your answer carefully.

## 7.

Philip Morris has issued bonds that pay annually with the following characteristics:

| Coupon | Yield to Maturity | Maturity | Macaulay Duration |
| :---: | :---: | :---: | :---: |
| $8 \%$ | $8 \%$ | 15 years | 10 years |

a. Calculate modified duration using the information above.
b. Identify the direction of change in modified duration if:
i. The coupon of the bond was $4 \%$, not $8 \%$.
ii. The maturity of the bond was 7 years, not 15 years.

## 8.

The following is a list of prices for zero-coupon bonds of various maturities.

| Maturity (years) | Price of Bond (\$) |
| :--- | :---: |
| 1 | 943.40 |
| 2 | 898.47 |
| 3 | 847.62 |
| 4 | 792.16 |

a. Calculate the yields to maturity of each bond and the implied sequence for forward rates
b. A portfolio manager made the following statement: "To immunize a portfolio in order to satisfy a single liability, all that is necessary is that (1) the market value of the assets is equal to the present value of the liability and (2) the duration of the portfolio is equal to the duration of the liability. There are absolutely no risks except for the risk that any of the bonds in the portfolio will default or decline in value due to credit downgrades."
Explain whether or not you agree with this statement.

## 9.

a. An investor purchases a 3-month put option, which has a strike price of $\$ 30$ and an option market price of $\$ 0.50$, when the stock is initially priced at $\$ 31.50$ per share. If the stock's price is $\$ 27$ when the option expires, what is the expiration-day value of the put?
b. A non-dividend paying stock is selling for $\$ 3.95$ and the risk free interest rate is $1 \%$. A three-month European call option with a strike price of $\$ 4.00$ is selling for $\$ 0.35$. A three month European put option with a strike price of $\$ 4.00$ is trading at $\$ 0.50$. What would an arbitrageur do?
c. An investor is considering stock options with three-month expiration. If she believes stock prices will not change for the next three months, to maximize profit she should:

1. Buy a call option, 2. Buy a call and sell a put option, 3. Sell a put and sell a call option or 4. Buy a put option?

Justify your answer.
d. ABC Corp. stock is currently trading at $\$ 65$. Assuming an investor buys a call option on 100 shares of ABC Corp. stock, which includes a call premium of $\$ 2.5$ per share and a strike price of $\$ 68$, what is the profit or loss on the expiration day of the call if the stock closes at $\$ 72$ ?
e. An investor buys a put option on stock that she owns. Which of the following best describes the total return of her position as compared to the return from the stock alone if the stock price stays the same?

1. Significantly higher return, 2. Slightly higher return, 3. Slightly lower return and, 4. Significantly lower return.
Justify your answer.

## 10.

a. Identify and explain key reasons why, over an extended periods of time, value stock investing might outperform growth stock investing
b. The stock of Nogro Corporation is currently selling for $\$ 10$ per share. Earnings per share in the coming year are expected to be $\$ 2$. the company has a policy of paying out $50 \%$ of its earning each year in dividends. The rest is retained and invested in projects that earn a $20 \%$ rate of return per year. This situation is expected to continue indefinitely.
i. Assuming the current market price of the stock reflects its intrinsic value as computed using the constant-growth DDM, what rate of return do Nogro's investors require?
ii. By how much does its value exceed what it would be if all earnings were paid as dividends and nothing were re-invested?
iii. If Nogro were to cut its dividend payout ratio to $25 \%$, what would happens to its stock price? What if Nogro eliminated the dividend?

## 11.

a. Briefly define the concept of time value of a call option, and explain why the time value of a call option is always greater than zero?
b. Draw a graph showing the value of a call option with its underlying asset (e.g., stock) before expiration.
c. Use an example to show the strategy using call and put options to replicate a long stock position without actually holding the stock.

## 12.

a. Explain why a callable bond's price would be expected to decline less than an otherwise comparable option-free bond when interest rates rise. If necessary use a numeric example.

Use the following arbitrage-free binomial interest rate tree to answer the questions that follow:

b. What is the value of a 3-year Treasury bond with a $9 \%$ coupon rate?
c. What is the value of a 2-year call option on a bond that currently has three years to maturity and coupon rate of $9 \%$ if the strike price is 98 ? Assume in this calculation that the current price of the 3 -year Treasury bond is the value found in part $b$ ).

## 13.

A corporation is issuing a coupon bond with 3 year maturity. The par value of the bond is $£ 1000$. The coupons are paid semi-annually, but in $300,250,200,150,100$, 50 from first coupon payment to the last payment in this order. The yield to maturity on the bond is $5 \%$ semi-annually.
a. Calculate Macaulay duration of the bond.
b. Calculate modified duration
c. If interest rate change $0.1 \%$ to $5.1 \%$, how much the bond value will change?
d. Critically explain the limitation of using duration as a measure of sensitivity of bond price to the change of interest rate change and why

## 14.

You have the following information on a company that you may wish to acquire shares in:

- Current dividend $\$ 1.25$
- Company $\beta 1.235$
- Expected growth over the next 2 years at $0.5 \%$ p.a. above the economy-wide growth rate
- The company will have reached the maturity/decline stage of the industry lifecycle at the end of the next 2 years

You also have the following general information on the macroeconomic environment:

- Growth is expected to average $2.25 \%$ p.a. over the next 20 years
- The risk free rate over the next 20 years is expected to be around $4.15 \%$ pa
- The expected return on the market over the next 20 years is $8.5 \%$ p.a.

What is the "fair" (or intrinsic) value of a share in this company?

## 15.

An investor purchased 1,000 three months put options with a strike price equal to $£ 65$ at a premium of $£ 3.75$. The investor purchased 1,000 shares of the underlying stock at £67.8. The short-term risk-free interest rate is $4 \%$ annum.
a. Calculate the maximum profit and loss
b. Determine the breakeven point
c. Calculate the premium of one call option with the same maturity as the put option

## 16.

Consider a stock, currently trading at $\$ 80$ (assuming no dividend). The interest rate is $1 \%$ per month.
a. What should be the "price" of a 3 month future on this stock?
b. If a 6 month future is trading at $\$ 90$, can you make arbitrage profit? Explain?
c. If a 1 year future is trading at $\$ 90$, can you make arbitrage profit? Explain?

## 17.

Briefly explain why bonds of different maturities have different yields in terms of the expectations and liquidity preference hypotheses. Briefly describe the implications of each hypothesis when the yield curve is
i. Upward sloping
ii. Downward sloping

## 18.

A senior portfolio manager has asked two assistants to consider the acquisition of one of two option-free bond issues knowing that issue 1 has a lower coupon rate and shorter maturity than issue 2 , having both the same credit rating.
One of the assistants argue that issue 1 as greater interest rate risk than issue 2 because of its lower coupon rate, while the other argue that issue 2 has greater interest rate risk because it has a longer maturity than issue 1 .
a. Which assistant portfolio manager is correct with respect their selection to the issue with the greater interest rate risk? Justify your answer.
b. Suppose that you are the senior portfolio manager. How would you suggest the two assistants to determine which issue has the greater interest rate risk? Clearly justify your answer.

## 19.

Use the following arbitrage-free binomial interest rate tree to answer the questions that follow:

a. What is the value of a 3-year Treasury bond with a $9 \%$ coupon rate?
b. What is the value of a 2-year put option on a bond that currently has three years to maturity and coupon rate of $9 \%$ if the strike price is 105 ? Assume in this calculation that the current price of the 3-year Treasury bond is the value found in part a.

## 20.

Critically discuss the following theories regarding the term structure of interest rates:
The Pure Expectations Theory, The Liquidity Theory and the Preferred Habitat Theory.

## Solutions

1. 

a. Mean-variance approach is basic portfolio selection, which assume the mean and variances and covariance between assets in the portfolio are informationally sufficient to identify the asset needed.

The criterion of choosing one asset is mean-variance: for a given mean return, an asset with lower variance is preferred.

But the approach is under some criticisms arguing that the distribution of return is not normal and stress the need for attention paid on shortfall impacts on portfolio selection.
b. We can find a portfolio which suitable for all because different risk attitudes of different investors, that is, tangent portfolio.
c. Yield curve is visual expression of the relation between maturity and interest rates. Spot rates are the YTMs of zero coupon bonds with different maturities.
d. True. The percentage price decline for longer maturity bond will be higher than for the shorter maturity bond, all else being the same.
e. The part of the value of an option that is due to its positive time to expiration.

The time value of a call option is always greater than zero in that it gives the buyer the chance to attain higher return in the future, even though in the case of out-of-the money situation.

## 2.

a. Modified duration $=\frac{\text { Macaulay duration }}{1+Y T M}=\frac{10.62}{1.07}=9.83$ years
b. For option-free coupon bonds, modified duration is a better measure of the bond's sensitivity to changes in interest rates. Maturity considers only the final cash flow, while modified duration includes other factors, such as the size and timing of coupon payments, and the level of interest rates (yield to maturity). Modified duration, unlike maturity, indicates the approximate percentage change in the bond price for a given change in yield to maturity.
c. Convexity measures the curvature of the bond's price-yield curve. Such curvature means that the duration rule for bond price change (which is based only on the slope of the curve at the original yield) is only an approximation. Adding a term to account for the convexity of the bond increases the accuracy of the approximation. That convexity adjustment is the last term in the following equation:

$$
\frac{\Delta \mathrm{P}}{\mathrm{P}}=(-\mathrm{D} * \times \Delta \mathrm{y})+\left[\frac{1}{2} \times \text { Convexity } \times(\Delta \mathrm{y})^{2}\right]
$$

## 3.

a. $\left.k=r_{f}+\left(r_{M}\right)-r_{f}\right]=4.5 \%+1.15(14.5 \%-4.5 \%)=16 \%$
b.

| Year | Dividend |  |
| ---: | :---: | :---: |
| 2003 |  | $\$ 1.72$ |
| 2004 | $\$ 1.72 \times 1.12=$ | $\$ 1.93$ |
| 2005 | $\$ 1.72 \times 1.12^{2}=$ | $\$ 2.16$ |
| 2006 | $\$ 1.72 \times 1.12^{3}=$ | $\$ 2.42$ |
| 2007 | $\$ 1.72 \times 1.12^{3} \times 1.09=$ | $\$ 2.63$ |

Present value of dividends paid in 2004-2006:

| Year | PV of Dividend |  |
| :---: | :---: | :---: |
| 2004 | $\$ 1.93 / 1.16^{1}=$ | $\$ 1.66$ |
| 2005 | $\$ 2.16 / 1.16^{2}=$ | $\$ 1.61$ |
| 2006 | $\$ 2.42 / 1.16^{3}=$ | $\$ 1.55$ |
|  | Total $=$ |  |

Price at year-end 2006 $=\frac{D_{2007}}{k-g}=\frac{\$ 2.63}{0.16-0.09}=\$ 37.57$
PV in 2003 of this stock price $=\frac{\$ 37.57}{1.16^{3}}=\$ 24.07$
Intrinsic value of stock $=\$ 4.82+\$ 24.07=\$ 28.89$
c. The data in the problem indicate that QuickBrush is selling at a price substantially below its intrinsic value, while the calculations above demonstrate that SmileWhite is selling at a price somewhat above the estimate of its intrinsic value. Based on this analysis, Quick Brush offers the potential for considerable abnormal returns, while SmileWhite offers slightly below-market risk-adjusted returns.
d. Strengths of two-stage versus constant growth DDM:

- Two-stage model allows for separate valuation of two distinct periods in a company's future. This can accommodate life cycle effects. It also can avoid the difficulties posed by initial growth that is higher than the discount rate.
- Two-stage model allows for initial period of above-sustainable growth. It allows the analyst to make use of her expectations regarding when growth might shift from off-trend to a more sustainable level.

A weakness of all DDMs is that they are very sensitive to input values. Small changes in k or g can imply large changes in estimated intrinsic value. These inputs are difficult to measure.

## 4.

a)

1) Protective put strategy.
2) The maximum profit is infinite.

The maximum loss $=500((54-57.8)-2.75)=\$ 3,275$
3) The breakeven point $=54+2.75=\$ 56.75$
4) The premium of one call option $=2.75+57.8-54 * \mathrm{e}^{-5 \% * 0.5}=\$ 7.88$
b) Lower. The call option will be less in the money. Both $\mathrm{d}_{1}$ and $\mathrm{N}\left(\mathrm{d}_{1}\right)$ are lower when X is higher.
c) The limitations of the Black-Scholes option-pricing model are as follow:
i. no dividends are paid in the period of option,
ii. applies only to European call options,
iii. risk free rate is constant throughout options life,
iv. standard deviation of returns from the underlying security must be accurately estimated and has to be constant throughout the option's life,
v. no transaction costs or tax effects involved in buying or selling the option or its underlying security

## 5.

a. The important distinction between a futures contract and an options contract is that the futures contract is an obligation. When an investor purchases or sells a futures contract, the investor has an obligation to accept or deliver, respectively, the underlying commodity on the expiration date. In contrast, the buyer of an option contract is not obligated to accept or deliver the underlying commodity but instead has the right, or choice, to accept delivery (for call holders) or make delivery (for put holders) of the underlying commodity anytime during the life of the contract.

Futures and options modify a portfolio's risk in different ways. Buying or selling a futures contract affects a portfolio's upside risk and downside risk by a similar magnitude. This is commonly referred to as symmetrical impact. On the other hand, the addition of a call or put option to a portfolio does not affect a portfolio's upside risk and downside risk to a similar magnitude. Unlike futures contracts, the impact of options on the risk profile of a portfolio is asymmetrical.
b. $\mathrm{F}_{0}=\mathrm{S}_{0}\left(1+\mathrm{r}_{\mathrm{f}}-\mathrm{d}\right)=1,200(1+.065-.02)=1254$
c. The put-call parity relation states that

$$
\mathrm{P}=\mathrm{C}-\mathrm{S}_{0}+\mathrm{X} /\left(1+\mathrm{r}_{\mathrm{f}}\right)^{\mathrm{T}}
$$

If $\mathrm{F}=\mathrm{X}$, then $\mathrm{P}=\mathrm{C}-\mathrm{S}_{0}+\mathrm{F} /\left(1+\mathrm{r}_{\mathrm{f}}\right)^{\mathrm{T}}$
But spot-futures parity tells us that $\mathrm{F}=\mathrm{S}_{0}\left(1+\mathrm{r}_{\mathrm{f}}\right)^{\mathrm{T}}$. Substituting, we find that:
$\mathrm{P}=\mathrm{C}-\mathrm{S}_{0}+\left[\mathrm{S}_{0}\left(1+\mathrm{r}_{\mathrm{f}}\right)^{\mathrm{T}}\right] /\left(1+\mathrm{r}_{\mathrm{f}}\right)^{\mathrm{T}}=\mathrm{C}-\mathrm{S}_{0}+\mathrm{S}_{0}$, which implies that $\mathrm{P}=\mathrm{C}$.
d. Take a short position in T-bond futures, to offset interest rate risk. If rates increase, the loss on the bond will be offset to some extent by gains on the futures.

## 6.

a._Treynor measures

Market: $\quad(12-6) / 1=6.00$
Portfolio W: $\quad(10-6) / .6=6.67$
Sharpe measures
Market: $\quad(12-6) / 13=.462$
Portfolio W: $\quad(10-6) / 18=.222$
Portfolio W outperforms the market based on the Treynor measure, but underperforms based on the Sharpe measure.
b. The two measures of performance are in conflict because they use different measures of risk. Portfolio W has less systematic risk than the market based on its lower beta, but more total risk (volatility) based on its higher standard deviation. Therefore, the portfolio outperforms the market based on the Treynor measure but underperforms based on the Sharpe measure.
c. Which index is better depends on each investor's situation. Sharpe's ratio uses standard deviation as a measure of risk, while Treynor's ratio uses beta to measure risk. Recall that standard deviation measures the total risk of an asset and beta measures the systematic risk. Therefore, Sharpe's index is more relevant for investors who do not hold any other portfolios, whereas Treynor's index is more relevant for investors who hold many other assets apart from the mutual fund.

## 7.

a. Modified duration $=\frac{\text { Macaulay duration }}{1+\text { YTM }}=\frac{10}{1.08}=9.26$ years
b.
i. Modified duration increases as the coupon decreases.
ii. Modified duration decreases as maturity decreases.
8.
a.

| Maturity | Price | YTM | Forward Rate |
| :---: | :---: | :---: | :---: |
| 1 | $\$ 943.40$ | $6.00 \%$ |  |
| 2 | $\$ 898.47$ | $5.50 \%$ | $\left(1.055^{2} / 1.06\right)-1=5.0 \%$ |
| 3 | $\$ 847.62$ | $5.67 \%$ | $\left(1.0567^{3} / 1.055^{2}\right)-1=6.0 \%$ |
| 4 | $\$ 792.16$ | $6.00 \%$ | $\left(1.06^{4} / 1.0567^{3}\right)-1=7.0 \%$ |

b. The statement is incorrect. There are in fact two other risks. First, even if the duration of the portfolio is matched to the duration of the liability, the portfolio is still exposed to a nonparallel shift in the yield curve. This risk is commonly referred to as immunization risk. Moreover, if there are callable securities in the portfolio, the portfolio is exposed to call risk.

## 9.

a. The expiration value of a put option is equal to $\operatorname{Max}(0, \mathrm{X}-\mathrm{S})=\operatorname{Max}(0, \$ 30-\$ 27)$ $=\$ 3.00$
b. Sell the stock and the put, buy the call and a bond that pays $\$ 4.00$ in three months.

Put-Call parity says:
(Protective put) $\mathrm{S}_{0}+\mathrm{P}_{0}=\mathrm{X} /\left(1+\mathrm{r}_{\mathrm{F}}\right)^{\mathrm{T}}+\mathrm{C}_{0}$ (fiduciary call)
Plugging in the values in the problem and calculating
$\mathrm{X} /\left(1+\mathrm{r}_{\mathrm{F}}\right)^{\mathrm{T}}=4 /(1.01)^{0.25}=3.99$, we have
$3.95+0.5=3.99+0.35$
$4.45>4.34$

The protective put is relatively more expensive than the fiduciary call. Selling the protective put requires: sell stock and the put. Buying the fiduciary call requires: buy the call and a bond that pay 4.00 in three months.
c. Choice 3 is the correct. Since she expects the underlying stock price to stay the same, she should write (or sell) both options and keep the premiums.
d. Total cost of the acquiring the option is $100 \times \$ 2.50=\$ 250$

Value of the call at expiration is $\$ 72-\$ 68=\$ 4 \times 100=\$ 400$
Profit/loss equals the value of the call less the option premium paid:
$\$ 400-\$ 250=\$ 150$
e. Choice 3 is the correct. This is a protective put strategy where the investor buys "insurance" to protect from a large decrease in the stock price. If the stock price does not move, the investor loses the put premium, which is small, relative to the price of the stock.
10.
a.

- Investors might extrapolate recent performance of growth stocks too far into the future and thus overestimate the value of growth stocks. The inevitable correction would lead to the result that growth stocks underperform value stocks over extended periods.
- Momentum investors might focus on recently growing firms and bid up their prices. This results in overpricing of these growth stocks.
- Value stocks may not be as extensively researched if they are not as exciting or publicized. The resultant "neglect effect" will lead to higher average returns.
- During market run-ups, investors may underestimate the risk of growth stocks, and not remember that just as these stocks perform well in up markets, they can perform poorly in down markets. Their performance in up markets is not sustainable "long-run" performance.
b.
i. $\quad K=D_{1} / P_{0}+g$
$D_{1}=0.5 \times \$ 2=\$ 1$
$\mathrm{g}=\mathrm{b} \times \mathrm{ROE}=0.5 \times 0.2=0.1$
Therefore: $\mathrm{k}=(\$ 1 / \$ 10)+0.1=0.20=20 \%$
ii. Since $\mathrm{k}=\mathrm{ROE}$, the NPV of future investment opportunities is zero:

$$
\mathrm{PVGO}=\mathrm{P}_{0}-\frac{\mathrm{E}_{1}}{\mathrm{k}}=\$ 10-\$ 10=0
$$

iii. Since $\mathrm{k}=\mathrm{ROE}$, the stock price would be unaffected by cutting the dividend and investing the additional earnings.

## 11.

a. The part of the value of an option that is due to its positive time to expiration. The time value of a call option is always greater than zero in that it gives the buyer the chance to attain higher return in the future, even though in the case of out-of-the money situation.
b.

c. By long call and short put at same strike price, a replicated long stock position can be shown as follow:


## 12.

a. The price of a callable bond can be expressed as follows:

Price of a callable bond = price of option-free bond - price of embedded call option
An increase in interest rates will reduce the price of the option-free bond. However, to partially offset that price decline of the option-free bond, the price of the embedded call option will decrease. This is because as interest rates rise the value of the embedded call option to the issuer is worth less. Since a lower price for the embedded call option is subtracted from the lower price of the option-free bond, the price of the callable bond does not fall as much as that of an option-free bond.
b The value of the 3-year treasury bond is 103.373, as shown below

c. The current price of the bond is 103.373 as found in part i) and the price assumed in the question. The value of the 2 -year call option is $\$ 2.5886$, as shown below:


## 13.

Firstly the student should not be confused by unequal coupons
a. Calculate Macaulay duration

| Time until <br> payment | Cash flow | PV | Weights |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 300 | 285.71 | 0.1710 | 0.1710 |
| 2 | 250 | 226.76 | 0.1357 | 0.2715 |
| 3 | 200 | 172.77 | 0.1034 | 0.3103 |
| 4 | 150 | 123.41 | 0.0739 | 0.2955 |
| 5 | 100 | 78.35 | 0.0469 | 0.2345 |
| 6 | 1050 | 783.53 | 0.4690 | 2.8142 |
|  |  | 1670.52 |  | 2.0485 |

According to above table, Macaulay duration of the bond is 2.0485 years $=4.0970$ half years
b. Modified duration $\mathrm{D}^{*}=\mathrm{D} /(1+\mathrm{y})=4.0970 /(1+5 \%)=3.9019$ half years $=1.9509$ years
c. $0.1 \%$ increase of interest rate will cause

$$
\frac{\Delta P}{P}=-D^{*} \times \Delta y=-3.9019 \times 0.1 \%=-0.39019 \%
$$

That is, the bond price will decrease $0.39019 \%$
d. Because the relation between bond price and interest rate is non-linear, duration only is only a good approximation when the change of interest rate is small, taking convexity into consideration will lead a better measure.

## 14.

Students need to consider what this question is asking, what information they have and what information they need to construct:
Firstly, students can get the required return for the company from its $\beta$, the risk-free rate and the expected return on the market. This requires use of the CAPM.

- Secondly, it is important that students make the assumption that 20 years is, effectively, forever. That way they can simplify their calculations (they would have to make other assumptions if they don't do this, and calculate cash flows for the next 20 years).
- Third, they have $\mathrm{D}_{0}$. They need to calculate $\mathrm{D}_{1}, \mathrm{D}_{2}$ and $\mathrm{D}_{3}$.
- Finally, they must have growth in dividends for the first two years at a rate of $\mathrm{g}_{1}$, then growth at $\mathrm{g}_{2}$ for dividends after this time. They should use a modified version of the specified holding period equity valuation model, where all dividends from time 2 on are used in a Gordon growth model valuation exercise.
The use of the previous assumptions/ideas allows us to get the following information:

$$
\begin{aligned}
& g_{1}=0.0225+0.005=0.0275(2.75 \%)(3 \text { marks }) \\
& g_{2}=0.0225(2.25 \%)(2 \text { marks }) \\
& D_{1}=D_{0} \times\left(1+g_{1}\right)=1.25 \times(1+0.0275)=1.2844(£ 1.28) \\
& D_{2}=D_{1} \times\left(1+g_{1}\right)=1.2844 \times(1+0.0275)=1.3197(£ 1.32) \\
& D_{3}=D_{2} \times\left(1+g_{2}\right)=1.3197 \times(1+0.0225)=1.3494(£ 1.35)(1 \text { mark each for Div. }) \\
& r=r_{f}+\beta \times\left[E\left(r_{M}\right)-r_{f}\right]=0.0415+1.235 \times(0.085-0.0415)=0.0952(9.52 \%)
\end{aligned}
$$

( 3 marks for $\cos t$ of capital and use CAPM)

The solution to the equity valuation problem is then constructed as follows:

$$
\begin{aligned}
& V_{0}=\frac{D_{1}}{(1+r)}+\frac{D_{2}}{(1+r)^{2}}+\frac{D_{3} /\left(r-g_{2}\right)}{(1+r)^{2}}(3 \text { marks for identifica tion }) \\
& =\frac{1.2844}{(1+0.0952)}+\frac{1.3197}{(1+0.0952)^{2}}+\frac{1.3494 /(0.0952-0.0225)}{(1+0.0952)^{2}} \\
& (3 \text { marks for correct use } \text { entry of data }) \\
& =17.7476(£ 17.75)
\end{aligned}
$$

On the basis of the above calculations a 'fair' value for the firm's shares is thus £17.75 each.

## 15.

a. The maximum profit is infinite

The maximum loss $=1,000(65-67.8)-3.75=£ 6,550$
b. Breakeven point: $£ 65+£ 3.75=£ 68.75$
c. Premium of one call option $=3.75+67.8-65 * \mathrm{e}^{-4 \% * 0.25}=£ 7.197$

## 16.

a. The price of the three month future is:

$$
\mathrm{F}_{\mathrm{t}}=\mathrm{S}_{\mathrm{t}}(1+\mathrm{r})^{\mathrm{T}-\mathrm{t}}=80 \times 1.01^{3}=82.42
$$

b. The fair value of the 6 month future should be:

$$
F_{t}=S_{t}(1+r)^{T-t}=80 \times 1.01^{6}=84.92
$$

Given the trading price $\$ 90$, we know that it is overvalued. Hence, to make arbitrage profit, we want to short the future.

At $t=0$, we short one unit of the future, borrow $\$ 80$ to buy one unit of the stock at spot market. The net cash flow is zero.

At $t=1$,we deliver the unit to the buyer, receive payment of $\$ 90$ on the future, and repay the bank loan 84.92 . The net cash flow is $90-84.92=5.08$. This is our arbitrage profit.
c. The fair value of the 1 year future should be:

$$
F_{t}=S_{t}(1+r)^{T-t}=80 \times 1.01^{12}=90.146
$$

Given the trading price $\$ 90$, we know it is undervalued. Hence, to make arbitrage profit, we want to long the future.

At $\mathrm{t}=0$, we long one unit of the future, borrow and short one unit of the stock at spot market, and save the $\$ 80$ proceeds from stock selling in bank. The net cash flow is zero.

At $t=1$, we acquire one unit of the stock from the seller of the future and return the stock to lender. We have to pay $\$ 90$ according to the future contract. On the other hand, we withdraw from the bank $80 \times 1.01^{12}=90.146$. The net cash flow 0.146 is our arbitrage profit.

## 17.

Expectations hypothesis: The yields on long-term bonds are geometric averages of present and expected future short rates. An upward sloping curve is explained by expected future short rates being higher than the current short rate. A downwardsloping yield curve implies expected future short rates are lower than the current short rate. Thus bonds of different maturities have different yields if expectations of future short rates are different from the current short rate.
Liquidity preference hypothesis: Yields on long-term bonds are greater than the expected return from rolling-over short-term bonds in order to compensate investors in long-term bonds for bearing interest rate risk. Thus bonds of different maturities can have different yields even if expected future short rates are all equal to the current short rate. An upward sloping yield curve can be consistent even with expectations of falling short rates if liquidity premiums are high enough. If, however, the yield curve is downward sloping and liquidity premiums are assumed to be positive, then we can conclude that future short rates are expected to be lower than the current short rate.

## 18.

a. While both assistant portfolio managers are correct in that they have identified tow features of an issue that will impact interest rate risk, it is the interaction of the two that will affect an issue's interest rate risk. From the information provided in the question, it cannot be determined which has the greater interest rate risk.
b. You, as the senior portfolio manager, might want to suggest that the two assistant portfolio managers compute the duration of the two issues

## 19.

a. The value of the 3 -year Treasury bond is 103.373 , as shown below

b. The current price of the bond is 103.373 as found in part a) and the price assumed in the question. The value of the put option is $\$ 3.4570$.


## 20.

See slides covered during lectures.

