Derivatives

Black-Scholes Call-Option-Pricing Model

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The Black-Scholes Call-Option-Pricing Model

The Concepts Underlying Black-Scholes Model

The option price and the stock price depend on the same underlying source of uncertainty

We can form a portfolio consisting of the stock and the option which eliminates this source of uncertainty

The portfolio is instantaneously riskless and must instantaneously earn the risk-free rate

This leads to the Black-Scholes differential equation

Assumptions Underlying Black-Scholes

No dividends

Underlying stock returns are normally distributed

No transaction costs

Risk free interest rate for lending and borrowing

Volatility and interest rates are constant up to maturity

Black and Scholes Formulas

$$c_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$d_1 = \frac{\ln(S_0 / X) + [R_F + 0.5Var(R)]T}{\sigma\sqrt{T}}; \; d_2 = d_1 - \sigma\sqrt{T}$$

N(.) is the cumulative distribution function of the standard normal distribution

$N(d_2)$ is the risk adjusted probability that the option will be exercised.

$N(d_1)$ always greater than $N(d_2)$. $N(d_1)$ must not only account for the probability of exercise as given by $N(d_2)$ but must also account for the fact that exercise or rather receipt of stock on exercise is dependent on the conditional future values that the stock price takes on the expiry date.

Example

S_0 = \$60 = market price of the underlying asset (such as the share price of an optioned stock)

X = \$50 = Exercise (strike) price

T = 0.333 = 4 months = (one third of the year) = the time until the option expires and is worthless.

R_F=7% = Risk free rate stated at an annual rate

Var (R) = 0.144 = variance of returns = The riskiness of an investment in the optioned asset.

$$d_1 = \frac{\ln(\$60/\$50) + [0.07 + 0.5 \times 0.144] \times 0.333}{0.3794 \times 0.5773} = 1.0483$$

$$d_2 = 1.0483 - 0.3794\sqrt{0.333} = 0.8293$$

Table of Standard Normal Cumulative Distribution Function Φ(z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7064	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8339
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

$$c_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$c_0 = \$60 \times 0.8531 - \$50 e^{-0.07 \times 0.333} \times 0.7967 = \$12.29$$

$$p_0 = C_0 + \frac{X}{(1+r)^{1/3}} - S_0 \Leftrightarrow p_0 = \$12.29 + \frac{\$50}{(1.07)^{0.333}} - \$60 = \$1.18$$

Black & Scholes in Practice

Forward Use:

Underlying Price
Exercise Price
Time-to-Maturity
Risk-free Interest Rate
Estimated Volatility

Black-Scholes Formula

OPTION PRICE

Backward Use:

Underlying Price
Exercise Price
Time-to-Maturity
Risk-free Interest Rate
Implied Volatility
(forward looking volatility)

Black-Scholes Formula

OPTION PRICE

Example of option valuation using Black-Scholes

What is the value of a European call option with an exercise price of 6.70 and a maturity date of 276 days from now if the current share price is $12.41, standard deviation is 25% p.a. and the risk-free rate is 10.50%. Assume there to be 365 days in a year.

Use the Black-Scholes formula to derive your result.

$$c_0 = P_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$d_1 = \frac{ln(P_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\left[ln\left(\frac{12.41}{6.70}\right) + \left[0.1050 + \frac{1}{2}(0.25)^2\right]\frac{276}{365}\right]}{0.25\sqrt{\frac{276}{365}}}$$

$$d_1 = 3.3093 \qquad d_2 = 3.3093 - 0.25\sqrt{\frac{276}{365}} = 3.0919$$

$$N(d_1) = 0.9995 \qquad N(d_2) = 0.9990$$

$$\mathrm{C} = 12.41 \times 0.9995 - 6.7 \times e^{-0.105\times\left(\frac{276}{365}\right)} \times 0.9990$$

$$C = 12.4038 - 6.1824 = \$6.2214$$

Question

The stock of Cloverdale Food Processors currently sells for \$40. A European Call option on Cloverdale stock has an expiration date six months in the future and a strike price of \$38. The estimate of the annual standard deviation of Cloverdale stock is 45 percent, and the risk-free rate is 6 percent. What is the call worth?

Answer Question

$$d1 = \frac{\left(\frac{40}{38}\right) + \left(0.06 + \frac{0.45^2}{2}\right)\frac{1}{2}}{0.45\sqrt{\frac{1}{2}}} = \frac{0.0513 + 0.0806}{0.3182} = 0.4146$$

$$d2 = d1 - \sigma\sqrt{t} = 0.4146 - 0.45\sqrt{\frac{1}{2}} = 0.0964$$

$$N(0.4146) + 0.6608$$

$$N(0.0964) = 0.5384$$

$$C = 40(0.6608) - 38\left(2.718^{-(0.,06)(0.5)}\right)(0.5384) = \$6.58$$

Question

Stock price S_0 = 100
Exercise Price X = 100 (at the money option)
Maturity T = 1 year
Interest rate (continuous) r = 5%
Volatility σ = 0.15

$\ln(S_0 / X e^{-rT}) = \ln(1.0513) = 0.05$

$d_1 = (0.05)/(0.15) + (0.5)(0.15) = 0.4083$

$N(d_1) = 0.6585$

$d_2 = 0.4083 - 0.15 = 0.2583$

$N(d_2) = 0.6019$

European call :
$100 \times 0.6585 - 100 \times 0.95123 \times 0.6019$
$= 8.60$

$$N(-d_1) = 1 - N(d_1) = 1 - 0.6585 = 0.3415$$

$$N(-d_2) = 1 - N(d_2) = 1 - 0.6019 = 0.3981$$

European put option

$$-100 \times 0.3415 + 95.123 \times 0.3981 = 3.72$$

Questions

A six-month call option with a strike price of $25.00 is selling for $3.50. Assuming the underlying stock price is also $25.00 and the risk-free rate is 6 percent APR, use the following table to determine the volatility (i.e, standard deviation of the return) implied using the option price. (Hint: Price the option using the table to determine which volatility generates a price of $3.50)

Volatility	**N(d1)**	**N(d2)**
40%	0,5799	0,4859
45%	0.6000	0.4742
50%	0.6032	0.4634