

## Bonds

### Question 1

If interest rates in all maturities increase by one percent what will happen to the price of these bonds?

- a. The price of shorter maturity bond and the long maturity bond will fall by the same percentage.
- b. The price of the shorter maturity bond and the longer maturity bond will rise by the same percentage.
- c. The price of shorter maturity bond will fall by a smaller percentage than the fall in price of the longer maturity bond.
- d. The price of the shorter maturity bond will rise by a smaller percentage than the rise in price of the longer maturity bond.

**C is correct. The percent price decline for longer maturity bond will be higher than for the shorter maturity bond, all else being the same.**

### Question 2

Sunn Co.'s bonds, maturing in 7 years, pay 8 percent interest on a \$1,000 face value. However, interest is paid semiannually. If your required rate of return is 10 percent, what is the value of the bond? How would your answer change if the interest were paid annually?

**If interest is paid semiannually:**

$$\text{Value} = \sum_{t=1}^{14} \frac{\$40}{(1.05)^t} + \frac{\$1,000}{(1.05)^{14}}$$

Thus,

$$\$40 (9.899) = \$ 395.96$$

$$\$1,000 (0.505) = \$ 505.00$$

$$\text{Value} = \$ 900.96$$

**If interest is paid annually:**

$$\text{Value} = \sum_{t=1}^7 \frac{\$80}{(1.10)^t} + \frac{\$1,000}{(1.10)^7}$$

$$\$80 (4.868) + \$1,000 (0.513)$$

$$\text{Value} = \$ 902.44$$

### Question 3

Trico Bonds have a coupon rate of 8 percent and a par value of \$1,000, and will mature in 20 years. If you require a return of 7 percent, what price would be willing to pay for the bond? What happens if you pay *more* for the bond? What happens if you pay *less* for the bond?

$$\text{Value} = \sum_{t=1}^{20} \frac{\$80}{(1.07)^t} + \frac{\$1,000}{(1.07)^{20}}$$

Thus,

$$\text{Present value of interest: } \$80 (10.594) = \$ 847.52$$

$$\begin{aligned}\text{Present value of par value: } & \$1,000 (0.258) = \$ 258.00 \\ \text{Value} & = \$1,105.52\end{aligned}$$

If you pay more for the bond, your required rate of return will not be satisfied. In other words, by paying an amount for the bond that exceeds \$1,105.52, the expected rate of return for the bond is less than the required rate of return. If you have the opportunity to pay less for the bond, the expected rate of return exceeds the 7-percent required rate of return.

#### Question 4

A government bond has an 8 percent coupon rate and a \$1,000 face value. Interest is paid semi-annually, and the bond has 10 years to maturity. If investors require a 10 percent yield, what is the bond's value?

Because the bond has an 8 percent coupon yield and investors require a 10 percent return, we know that the bond must sell a discount. Notice that, because the bond pays interest semi-annually, the coupons amount to  $\$80/2 = \$40$  every six months. The required yield is  $10\%/2=5\%$ . Finally, the bond matures in 10 years, so there are a total of 20 six-month periods.

The bond's value is thus equal to the present value of \$40 every six months for the next 20 six-month periods plus the present value of the \$1,000 face amount.

$$\begin{aligned}\text{Bond value} &= \$40 \times \left(1 - 1/1.05^{20}\right) / 0.05 + 1,000 / 1.05^{20} \\ &= \$40 \times 12.462 + 1,000 / 2.6533 \\ &= \mathbf{\$875.38}\end{aligned}$$

#### Question 5

A government bond has a 10 percent coupon rate and a \$1,000 face value. Interest is paid semi-annually, and the bond has 20 years to maturity. If investors require a 12 percent yield, what is the bond's value? What is the effective annual yield on the bond?

Because the bond has a 10 percent coupon yield and investors require a 12 percent return, we know that the bond must sell a discount. Notice that, because the bond pays interest semi-annually, the coupons amount to  $\$100/2 = \$50$  every six months. The required yield is  $12\%/2=6\%$ . Finally, the bond matures in 20 years, so there are a total of 40 six-month periods.

The bond's value is thus equal to the present value of \$50 every six months for the next 40 six-month periods plus the present value of the \$1,000 face amount.

$$\begin{aligned}\text{Bond value} &= \$50 \times \left(1 - 1/1.06^{40}\right) / 0.06 + 1,000 / 1.06^{40} \\ &= \$50 \times 15.04630 + 1,000 / 10.2857 \\ &= \mathbf{\$849.54}\end{aligned}$$

Notice that we discounted the \$1,000 back 40 periods at 6 percent per period, rather than 20 years at 12 percent. The reason is that the effective annual yield on the bond is  $1.06^2 - 1 = 12.36\%$ , not 12%. We thus could have used 12.36 percent per year for 20 years when we calculated the present value of the \$1,000 face amount, and the answer would have been the same.

## Question 6

Describe some of the risks associated with investing in bonds, in particular:

a. Interest-rate risk or market risk,

**Interest-rate risk or market risk:**

As interest rates rise, the price of a bond fall (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price)

b. Call Risk, and,

**Call Risk:**

**Issuer** can retire or “call” all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate)

**Investor** perspective: i) the CF pattern is not known with certainty, ii) exposed to reinvestment risk (issuers will call the bonds when interests have dropped) and iii) capital appreciation of a bond will be reduce

c. Liquidity Risk.

Size and spread btw the bid and ask price. The wider the dealer spread, the more the liquidity risk

## Question 7

A 5 year bond has a face value of \$100 and pays an annual coupon of 4.0%. At the time of issue, the yield to maturity is 3%.

a. What is the bond price at the time of issue?

$$C \left[ \frac{1 - \frac{1}{(1+r)^n}}{r} \right] + \frac{FV}{(1+r)^n} = 4 \left[ \frac{1 - \frac{1}{(1+0.03)^5}}{0.03} \right] + \frac{100}{(1+0.03)^5} = \$104,58$$

b. What is the price of this bond one year later assuming the yield is unchanged at 3%?

$$4 \left[ \frac{1 - \frac{1}{(1+0.03)^4}}{0.03} \right] + \frac{100}{(1+0.03)^4} = \$103,72$$

c. What is the price of this bond one year later if instead of the yield being unchanged the yield increases to 3.5%?

$$4 \left[ \frac{1 - \frac{1}{(1+0.03)^4}}{0.03} \right] + \frac{100}{(1+0.03)^4} = \$103,72$$

d. Explain how the price of a bond changes as the bond approaches its maturity date and compute the change in value that is attributable to the passage of time.

As a bond approaches maturity its price approaches the bond's par value.

To compute the value change attributable to the passage of time, compare the value of the bond at time  $t$  with the bond value in a prior period.

e. Explain how the yield level impacts the interest rate risk of a bond.

The higher the yield at which a bond trades, the lower its price sensitivity for a given basis point change in interest rates will be, all other factors being equal.

### Question 8

You purchased a 4% annual coupon bond one year ago. At the time of the purchase, the yield to maturity of the bond was 3%. Also, over the past year, the yield to maturity of the bond increased and it is now 4%. Which of the following statements is correct?

- a. When you bought the bond one year ago, the price of the bond was higher than its face value.
- b. Over the past year, the bond price increased due to the increase of the general level of interest rates.
- c. The bond is currently trading at discount.
- d. In the past year, the current yield of the bond was smaller than its total rate of return.

Choice "a" is correct. The YTM is lower than the coupon rate when the bond was bought, being therefore at premium (higher than the face value).

**Choice "a", "b" and "d" are incorrect.** The semi-strong form of the EMH suggests that both technical and fundamental analyses are likely to be ineffective in finding stocks that will produce superior risk-adjusted returns.

### Question 9

ABC Bonds have a coupon rate of 5% and a par value of £1,000, and will mature in 10 years. Coupons are paid semi-annually. If you require a return of 6%, what price would be willing to pay for the bond? What happens if you pay *more* for the bond? What happens if you pay *less* for the bond?

$$\text{Bond price} = 25 \left[ \frac{1 - \frac{1}{(1 + 0.03)^{20}}}{0.03} \right] + \frac{1,000}{(1 + 0.03)^{20}} = £925.6126$$

If you pay more for the bond, your required rate of return will not be satisfied. In other words, by paying an amount for the bond that exceeds £925.6126. The expected rate of return for the bond is less than the required rate of return. If you have the opportunity to pay less for the bond, the expected rate of return exceeds the 6% required rate of return.

### Question 10

A government bond has a 6% coupon rate and a £1,000 face value. Interest is paid semi-annually, and the bond has 15 years to maturity. If investors require a 7% yield, what is the bond's value? What is the effective annual yield on the bond?

Because the bond has a 6% coupon yield and investors require a 7% return, we know that the bond must sell at a discount. Notice that, because the bond pays interest semi-annually, the coupons amount to  $£60/2 = £30$  every six months. The required yield is  $7\%/2=3.5\%$ . Finally, the bond matures in 15 years, so there are a total of 30 six-month periods.

The bond's value is thus equal to the present value of £30 every six months for the next 30 six-month periods plus the present value of the £1,000 face amount.

$$\text{Bond Value} = £30 \times ((1 - 1/1.035^{30})/0.035) + 1,000/1.035^{30} = £908,0398$$

Notice that we discounted the \$1,000 back 30 periods at 3.5% per period, rather than 15 years at 7%. The reason is that the effective annual yield on the bond is  $1.035^2 - 1 = 7.1225\%$ , not 7%. We thus could have used 7.1225% per year for 15 years when we calculated the present value of the \$1,000 face amount, and the answer would have been the same.

### Question 11

Which one of the following bonds has the shortest duration?

- a. zero-coupon, 10-year maturity.
- b. zero-coupon, 13-year maturity.
- c. 8% coupon, 10 year maturity.
- d. 8% coupon, 13-year maturity.

**Choice 'c' is correct.** All other things being equal, bonds with higher coupons and shorter maturities have shorter durations than bonds with lower coupons and longer maturities. Lower durations exhibit less price risk. Bond "c" has the combination of higher coupon and shorter maturity.

Choice "a" is incorrect. Since bond "c" has a higher coupon, bond "c" will have lower duration.

Choice "b" is incorrect. Since bond "c" has a higher coupon and a shorter maturity than bond "b", bond "c" will have lower duration.

Choice "d" is incorrect. Since bond "c" has shorter maturity than bond "d" will have lower duration.

### Question 12

Identify the *most accurate* statement concerning duration.

- a. The higher the yield, the greater the duration.
- b. The higher the coupon, the greater the duration.
- c. The difference in duration between two similar coupon-paying bonds maturing in more than 15 years is small.
- d. For coupon bonds, duration is the same as term to maturity.

**Choice "c" is correct.** It is accurate that the difference in durations is small between two bonds maturing in more than 15 years. This is because the present value of principal is less important for bonds with long maturities.

Choice "a" is incorrect. Duration shortens as yields rise, because at higher discount rates, the present values of further-out cash flows become less significant. Therefore, a greater proportion of the bond's value will be attributed to earlier cash flows, which shortens the duration.

Choice "b" is incorrect. When coupons are higher, they become more significant relative to the value of principal to be received at maturity. This shortens duration.

Choice "d" is incorrect. This is accurate only for zero-coupon bonds.

### Question 13

A bond currently sells at \$925 and has a duration of 3.65. Compute the approximate percentage price change of the bond for a 75 basis point decrease in rates.

- a. 2.74%
- b. -2.53%
- c. -2.74%
- d. 2.53%

**Choice "a" is correct.** 75 basis points = 0.75%

$$\% \Delta P = -D(\Delta r) = (-3.65) \times (-0.0075) = 0.0274 = 2.74\%$$

Choice "b" is incorrect. This choice incorrectly states the price to be negative, and uses duration of 3.376 (92.5% of 3.65).

Choice "c" is incorrect. This choice incorrectly assumes duration carries a minus sign.

Choice "d" is incorrect. This choice incorrectly uses duration of 3.376 (92.5% of 3.65).

### Question 14

#### Excel Application

Assume 5% coupon bond maturing in 4 years, with a face value of 1,000 and the YTM is 9% (annualized). Coupons are paid semiannually.

Calculate:

- a. The fair price of the bond.
- b. Macaulay and Modified Duration

### Question 15

Assume two bonds: a 6% coupon bond maturing in 9 years, with a face value of 1,000 and YTM equal to 7% and a 4% coupon bond maturing in 2 years and face value of 100. Both bonds pay coupons semi-annually.

Additionally the following information is provided.

Maturity	0.5	1	1.5	2
Spot Rates	2.00%	2.50%	3.00%	4.00%

The spot rates are all quoted in annual basis. All calculations should be done in semi-annual rates

Calculate:

- a. The fair price for the two bonds.
- b. The Macaulay and Modified Duration for both bonds

### Question 16

Assume a 6.5% coupon bond maturing in 15 years, with a face value of 1,000 and YTM equal to 7.2%. Coupons are paid semi-annually.

- a. The Macaulay Duration, Modified Duration for the 6.5% coupon bond.
- b. Assume that the YTM for the 6.5% bond maturing in 15 years change by  $\pm 1\%$   
Calculate:
  - i. The price approximation using duration
  - ii. Suppose that over the first 10 years of the holding period, interest rates decline, and the yield-to-maturity on the bond falls to 5.5%. What is the price of the bond in 10 years, time?

**See Excel Application**