Exercise Set

1. Find the first-order partial derivatives $\partial y/\partial x$, $\partial y/\partial z$, $\partial y/\partial t$ of:

$$y = 11x^2zt - 5z^3t^2 + 9xzt^3 + 3x^2z^2$$

2. Use appropriate rules to find the first-order partials for each of the following:

(i) $z = (w - x - y)(3w + 2x - 4y)$

(ii) $z = \dfrac{x^2 - y^2}{3x + 2y}$

(iii) $z = (5x^2 - 4y)^2(2x + 7y^3)$

3. Take the first, second, and cross partial derivatives of the following functions:

(i) $z = 7x^3 + 9xy + 2y^5$

(ii) $z = e^{(3x + 2y)}$

(iii) $z = \ln(5x + 9y)$

4. For the following functions, (a) find the critical points, and (b) determine if at these points the function is at a relative maximum, relative minimum, inflection point, or saddle point.

(i) $f(x, y) = 3x^3 + 1.5y^2 - 18xy + 17$

(ii) $z(x, y) = \ln(x^2 - 4x + 3y^2 - 6y)$

5. Find the total differential dz for the function $z = (x - 3y)^3$

6. In monopolistic competition producers must determine the price that will maximize their profit. Assume that a producer offers two different brands of a product, for which the demand functions are:

$$Q_1 = 14 - 0.25P_1$$
$$Q_2 = 24 - 0.5P_2$$

and the joint cost function is:

$$TC = Q_1^2 + 5Q_1Q_2 + Q_2^2$$

Find:

(i) The profit-maximizing level of output for each product

(ii) The profit-maximizing price for each product

(iii) The maximum profit

(Hint: Total Revenue for the firm is $P_1Q_1 + P_2Q_2$)

7. Find the turning point of the following function and determine whether it is a maximum or minimum

$$y = 3x_1^2 - 5x_1 - x_1x_2 + 6x_2^2 - 4x_2 + 2x_2x_3 + 4x_3^2 + 2x_3 - 3x_1x_3$$