TUGAN TUNCAY

STUDENT NUMBER 090019490

FINAL YEAR PROJECT FOR BSC HONOURS

INVESTMENT AND FINANCIAL RISK MANAGEMENT

CREDIT SPREAD ANALYSIS AND CALCULATION

A COMPARATIVE EXAMINATION OF EXISTING MEHTODS FOR THE CALCULATION OF CREDIT SPREAD BASED ON THE EVALUATION OF STATIC RISK FACTORS AND DYNAMIC COMPONENTS.

MARCH 2012

A thesis submitted to the Faculty of Finance at the Sir John Cass Business School, City University London, in fulfilment of the requirements for the degree of Bachelor of Science Honours in 'Investment and Financial Risk Management'.

I certify that I have complied with the guidelines on plagiarism outlined in the Course Handbook in the production of this dissertation and that it is my own, unaided work.

ACKNOWLEDGEMENTS

I wish to thank Professor Mario Levis for the formal supervision of this undergraduate thesis and Dr. Sotiris Staikouras for his continuing assistance. I am particularly and deeply grateful for the support and commitment of Dr. Cesario Mateus who has inspired me though his teaching and his motivation. I am also thankful for my family's support in my wish to pursue studies abroad at Cass Business School London.

ABSTRACT

This paper conducts a thorough comparative analysis into existing calculation methods of credit spread. For this purpose, the dynamic determinants and static risk factors affecting credit spread are systematically examined and evaluated. The calculation models are divided in two distinct categories, i.e. calculation based on individual bonds with matched maturity, duration etc. and calculation based on a spot rate curves. Further, the advantages and disadvantages of specific methods are discussed with regards to appropriate applications. The paper concludes with an assessment of the analysis and points out the limitations of the results.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS 2
ABSTRACT 2
LIST OF ABBREVIATIONS 4
LIST OF TABLES 4
LIST OF SYMBOLS 5
1. Introduction 6
2. Literature Review 7
3. Credit spreads 10
  3.1 Risks of Bonds 10
  3.2 Constituents of credit spreads 14
  3.3 Analysis of appropriate benchmarks 17
4. Credit spread calculation methods 20
  4.1 Calculating credit spread using individual bonds 20
  4.2 Calculating credit spreads using spot rate curves 29
5. Application of credit spread calculation methods 33
6. Conclusion 35
REFERENCES 36

LIST OF ABBREVIATIONS

CDS	Credit Default Swap
EURIBOR	Euro Interbank Offered Rate
E.g.	Exempli gratia ('for example')
FRA	Forward Rate Agreement
LIBOR	London Interbank Offered Rate
i.e.	Id est ('that is')
Mod. Duration	Modified Duration
p. a.	Per annum ('yearly')
REX	'Deutscher Rentenindex' (German Government bond index)
Yield	Yield to Maturity (YTM)
YTM	Yield to Maturity

LIST OF TABLES

TABLE 1 Example for the calculation of credit spread on the basis of two individual bonds with matched maturities. 22

TABLE 2 Example of a credit spread calculation by determining the difference between the par yield of a corporate bond and the swap rate with the same maturity. 28

TABLE 3: Theoretical example for the calculation of spot rates using bootstrapping method 31

TABLE 4: Spot rates calculation with bootstrapping 31

LIST OF SYMBOLS

AI	Accrued interest
β	Regression coefficient beta
c	Coupon rate
C	Coupon
CF_{ti}	Cash flow at time t
Δ	Delta (uppercase)
δ	Delta (lowercase)
D	Duration
D_M	Modified Duration
N	Number of bonds in the dataset
P	'Dirty' price
P^{cl}	'Clean' price
PV	Present value
r	Interest rate
$r_{o,ti}$	Spot rate with maturity of t_i
r_e	Yield to maturity (YTM)
$r_e(t_M, c)$	Yield with regards to maturity and coupon rate
$r_{g,t}$	Spot rate curve of government bond
$r_{c,t}$	Spot rate curve of corporate bond
s	Credit spread
s_t	Credit spread at time t
t	Time
t_i	Time variable
t_M	Term to maturity

1. Introduction

This undergraduate thesis is centred on credit spread. It aims to examine its properties and conduct a comparative analysis into its calculation methods. In financial theory, we define credit spread as the difference between the yield of a corporate bond and the yield of a government bond with similar maturity. Per definitionem, these two bonds should be identical in all aspects except credit risk. However, in practice, the spread includes many additional risks factors. [1] As a result, we attempt to examine and evaluate these static risk factors affecting credit spreads, i.e. liquidity risk, credit risk, spread risk and interest rate risk.[2] Apart from those static risk factors, there are also other dynamic influences on credit spread, for example the term to maturity, stock market prices, volatility and the business cycle. These will need to be examined in regards to their impact on credit spreads. Based on the information from this examination, the paper conducts a comparative analysis into the calculation and estimation methods.

The immense growth of the credit derivative market in recent years and new banking regulation (e.g. Basel III) have increased the need of precise credit spreads, since these spreads are used in banks' assessment of default risk. Increased capital requirements, especially for systemically important financial institutions (SIFIs), by Basel Committee on Banking Supervision, further underline the importance of having reliable data on credit spread in order to calculate overall credit risk by applying them in various structured and reduced-form models.[3]

Structural models (based on the contingent claim approach) such as the Markov chain model described in Jarrow, Lando and Turnbull (1997) or reduced models, e.g. Lando (1998) and Duffie & Singleton (1997), require empirical credit spreads for the identification of implied default probabilities of bonds[4] and will be discussed on broad terms in the following chapters.

There are recent empirical studies on credit spread dynamics.[5] Nevertheless, there is no single, generally accepted method for the calculation of credit spreads; a topic which has been neglected in past academic research and literature[6]. Therefore, the purpose of this thesis is to analyse the properties of credit spreads, offer suitable calculation models and investigate those in detail, especially with respect to their suitability.

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[1] See Brown & Zarnic (2003) p. 3
[2] See Section 3.1
[3] See Van Landschoot (2004) p.7
[4] See Jankowitsch & Pichler (2004) p.49 for reduced and structural models
[5] See Pedrosa & Roll (1998) for examination of the daily time-series properties of credit spreads from October 1995 to March 1997
[6] There has been a study conducted by Jankowitsch & Pichler (2004), which focuses on the parsimonious estimation of credit spreads, however, neglects the actual calculation methods.

2. Literature Review

There are a number of empirical studies and quantitative research conducted on credit risk, which focuses on developing an adequate credit risk measurement framework, however, there is a lack of research into the calculation of credit spread. Nonetheless, we can find some previous academic work on the determinants of credit spread.[7]

A recent Deloitte research report by Annaert & De Ceuster (2009) examines European credit spreads and establishes that the yield spread between corporate and government bonds is not a pure 'credit' spread, adjusted for counterparty and default risk, but includes a premium for liquidity (which may vary over time) and credit migration risk. Further, the paper recommends the unification of market risk and credit risk management structures in corporations and banks since it finds a strong relationship between the spreads and the term structure of risk-free interest rates, i.e. the credit spread decreases if risk-free interest rates increase.

Similarly, Elton et al. (2004) attempt to explain the determinants of corporate rates (credit spreads) between corporate and government bonds. The authors emphasise the influence of default risk on credit spread across different rating classes and find its impact to be rather insignificant. They further elaborate on the influence of state taxes, i.e. the influence of federal tax exemptions on U.S. Treasuries, and conduct a cross-sectional test to prove the presence of a risk premium beyond the credit risk premium in credit spreads. Further, they elaborate on the impact of state taxes on corporate bond yields and consequently the credit spread. Nonetheless, they fail to offer suitable calculation models and their analysis does not identify the impact of other individual risk factors.

A recent research paper published in the 'Financial Review' in 2011 titled 'Credit Spread Changes and Equity Volatility: Evidence from Daily Data' by Hibbert et al. investigates the factors for changes in credit spreads in the U.S. corporate bond market. Their work aims to prove that an increase in stock market volatility is positively correlated with the credit spread beyond the degree that can be determined by standard term structure variables.

As for the application of credit spreads for the valuation of risky debt, one of the earliest and most influential analyses is Merton's (1974) seminal paper 'On the Pricing of Corporate Debt: The Risk Structure of Interest Rates', in which the author uses the idea of Black/Scholes (1973) to price defaultable bonds by assuming the equity of a firm to be the equivalent of a call option on the companies' assets. It the most simplistic case, with the companies' value being the sum of its debt

[7] See della Ratta (2006) p. 5

and equity and a single issue of a zero-coupon bond as debt outstanding, Merton determines the debt value to be equal to the difference between the value of a call option on the company's assets and the company's actual value; with the strike price of the call option equal to the nominal value of debt and term to maturity of the option equal to debt maturity. Ultimately, Merton concludes that the debt value is a combination of short put option on the company's assets and a riskless government bond. Bondholders in effect buy a safe bond and give shareholders the option to sell them the company's assets at the debt value. [8]

The first research on using swap rate curves rather than yield curves [9] comes from Blanco et al. (2003). In their paper, they create a benchmark reference for the calculation of credit spread using swap rates as risk-free rates. Using empirical data, they determine the credit default swap spreads to be similar to the yield spreads of bonds. Moreover, their research shows that highest level of price discovery happens in the credit default swap market since it leads the bond market. Houweling and Vorst (2005) confirm that the swap rate is preferred over the Treasury rate as the risk-free rate in the credit default swap market. [10]

There are various empirical studies that attempt to divide and distinguish the risk components of credit spread, e.g. Elton et al. (2001) or Longstaff et al. (2005). Both papers assume that the relative influence of the individual risks to overall risks is constant, however, the results of their research varies significantly, since there is a lack of a unified model for the calculation of credit spread.

Bernanke and Gertler (1989) analyse the phenomenon of 'flight to quality' in a bust market in order to explain the negative relation between corporate and government rates. Under normal circumstances, high nominal rates would be associated with higher risk premiums for corporate debt, widening the credit spread. Their model suggests that an increase in interest rates, ceteris paribus, would cause agency problems for borrowers, widening the credit spread due to an increase in the difference between internal and external financing costs.

With regards to the credit spread, Jarrow et al. (1997) assume that it is uncorrelated with the risk-free rate, while Das and Tufano (1996) argue that the corporate default recovery rate is negatively correlated with the credit spread. Generally, it is assumed that there is a positive relation between credit spreads and high rates, which causes an increase in the credit risk and thus in the probability of a credit rating downgrade.

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[8] See della Ratta (2006) p. 6
[9] See section 4.2
[10] See Hull et al. (2004) for more information on research on swap rate curves

Morris et al. (1998) find that the work of *"Cornell and Green (1991), Fridson and Kenney (1994), Longstaff and Schwartz (1995), and Duffee (1998) document a significant negative relation between changes in credit spreads and changes in Treasury rates. There are, however, two reasons to question whether these results imply a negative long-run relationship between the levels of Treasury rates and credit spreads. First, the empirical specifications in these studies focus on changes and do not incorporate equilibrium relationships between the variables. This is important because the predictions of the theoretical models are long-run or equilibrium predictions. Since the models do not specify the transition path from one equilibrium to another, it is questionable to draw inference about the equilibrium spread from the short-run dynamics. Second, estimates from these studies on the relation between credit spreads and Treasury rates will be biased and inconsistent if corporate and Treasury rates are cointegrated."*[11]

[11] Excerpt from Morris et al. (1998) pp. 10-11

3. Credit spreads

In theory, credit spread is defined as the yield spread between a default risk-free government bond and a corporate bond, which share all characteristics except credit risk. This definition implies that the credit spread should only compensate the investor for the additional risk of default and thus offer a credit risk premium. In reality, however, it is virtually impossible to find a government bond showing identical characteristics that would be suitable as a risk-free reference bond. Therefore, it becomes obvious that credit spread compensates the investor for all risks that go beyond the risk of a credit risk-free benchmark bond.

Most calculation methods aim to determine the pure risk premium (which investors would demand for investing in corporate rather than government bonds) and therefore attempt to calculate the credit spreads by using riskless reference bonds that have a similar risk profile to a corporate bond. The first part of the analysis will therefore focus on the risks of investing in bonds in general and then try to evaluate different types of risk, with focus on those which affect credit risk in particular. Moreover, certain dynamic factors, i.e. the business cycle, the term structure of interest rates, etc. will influence the credit spreads over time. These factors will be dealt with in the second part of section 3.

3.1 Risks of Bonds

In general terms, bonds are classic instruments of long-term financing for corporations, governments and individual municipalities. Purchasing a bond constitutes a formal contract: the bondholder receives certain, usually regular payments (e.g. annual, semi annual, quarterly, monthly, etc.)[12], known as coupon, for a pre-specified time. In most cases, the holder can redeem the bond upon its maturity date at the face value. Investors in bonds are subject to certain risks when buying a bond, which can impact their wealth. These risks can be subdivided into systematic (market) risk and unsystematic (bond-specific) risk, with some degree of interrelation between them. Liquidity risk and credit risk are among the most common risks that fall into the category of unsystematic risk, whereas spread risk and interest rate risk are considered to be market risks.[13]

3.1.1 Liquidity Risk

Although financial theory offers slightly differing definitions for the concept of liquidity, it is generally accepted as a financial asset's ability to be sold easily with a minimum loss of value at any

[12] See Hull (2009)
[13] See Hull (2009) p. 489

given time and without a significant movement in the price. In a perfectly liquid and frictionless market, changes in price would only occur instantaneously when new information would become available to all investors. Illiquid assets, on the other hand, can only be sold at a discount and usually with some delay. This also implies a lack of feasibility for large transactions. Certain measures are used to quantify liquidity, e.g. the market breadth (spread between ask and bid price) and market depth (tradable volumes of securities). Consequently, liquid securities are characterised by high trading volumes and small spreads. For corporate bonds traded over-the-counter, however, readily available information is limited. Outstanding volumes of OTC transactions and day count figures for a particular corporate bond can be used to approximate liquidity indicators.[14]

A certain degree of concentration of liquidity with respect to a bond's maturity can be observed depending on the geographical location of the market. Additionally, the number of dealers and brokers is used as a liquidity proxy. A debt security, which is quoted by many brokers, offers investors a broader selection of possible counterparties, making it more liquid and therefore reducing its yield.[15]

Moreover, we can use a bond's age as an indicator for its liquidity since recent on-the-run bonds are much more liquid than older off-the-run bonds of the same issuer, which have been traded in the secondary market for some time.[16] With regards to the age of the bond, a shorter term to maturity will cause an increasing percentage of the issued amount to be absorbed in bondholders' buy-and-hold portfolios. Thus, an increase in age will decrease trading activity, which will in turn reduce liquidity. [17] Consequently, once a bond becomes illiquid, it usually stays illiquid until its maturity.

Ultimately, liquidity risk measures the risk of a change of liquidity during the bond's term to maturity, which would lead to a loss in market value and a discount if sold in the secondary market. Thus, the investor demands an additional risk premium in order to compensate him for taking on another type of risk. Since liquidity effects are different for corporate and government bonds, the credit spread must include a certain degree of adjustment for liquidity risk. This causes a change in the structure of credit spread rates.[18]

3.1.2 Credit Risk

[14] See Brown&Zarnic (2003) p. 5-7 for liquidity indicators
[15] See Brown&Zarnic (2003) p. 7 for on-the-run and off-the-run definitions
[16] See Warga (1992) p. 611 for the definition of on-the-run and off-the-run bonds and their use as a liquidity indicator
[17] See Warga (1992) for demonstration of the relationship of bond's age and liquidity
[18] See Hull (2009) for liquidity effects

An investment into a particular bond is subject to the risk of a changing credit rating of the issuing body. In a worst-case scenario, an issuer would no longer be able of repaying the principal or coupon, leading to a default. Even if an issuer can prevent insolvency, a change in credit rating can have a significant impact to the market price of its bonds. As a consequence, an investor in a bond has to account for both the risk of a default (usually corporate or government default), but also for the risk of a change in market price, which would lead to a loss when these bonds were sold in the secondary market upon a change in rating. These risks together are categorized as credit risk.[19]

The default risk of a bond is dependent on the probability of insolvency of a corporation or government. In the case of a corporate default, investors in bonds will receive a certain 'recovery rate', which will depend on the original claim against the corporation, after restructuring and/or bankruptcy proceedings have been finalised. In order to estimate the probability of a default, 3 major credit rating agencies have been established: Standard & Poor's, Moody's Investors Service and Fitch Ratings.[20] These credit rating agencies conduct a series of quantitative and qualitative analyses and apply certain models in order to determine a credit rating. These ratings, however, are not expressed in absolute, but in relative terms to other issuers. As a result, issues are then subdivided into different rating categories with homogenous characteristics and similar probability of default.[21]

The risk of a change in marker price of a debt security as a consequence of a change in credit rating is known as migration risk.[22] If the credit worthiness of an issuer deteriorates, the probability of insolvency will increase. As a consequence, investors are reluctant to pay the full original price for the bond, leading to a decline in market value. At the same time, the issuer is still obliged to make the same regular payments (coupons), causing an increase in the bond's yield and the overall credit spread. An improvement of credit rating will have the inverse effect.

Ultimately, credit risk can affect both corporate and government bonds. However, some government bonds issued by nations with the highest possible credit rating can be regarded as virtually default risk-free and are therefore particularly suitable as a benchmark in the calculation of credit spreads.[23]

[19] See Fabozzi (2005) Counterparty risk can be seen as part of credit risk
[20] A.M. Best has been gaining some importance in ratings for particular securities
[21] See Hull (2009) pp. 489-492 on recovery rates and default probabilities
[22] See Austrian Financial Market Authority for precise definition of migration risk http://www.fma.gv.at/en/special-topics/solvency-ii/glossary.html
[23] See section 3.3

3.1.3 Spread risk

By definition, spread risk will only affect corporate bonds (which have the possibility of default) since it describes a possible change in credit spreads on the assumption that the risk-free benchmark rate does not change. In this case, however, a change in spreads is not caused by a change in credit rating[24], but by changing market tendencies and trends, e.g. increase in number of risk-averse investors.[25] Therefore, spread risk can be categorised into the group of systematic risks that cannot be diversified away under the conditions of the efficient market hypothesis.[26] An increase in credit spread will cause a decline in market value of a bond and consequently constitute a loss for the investor. Spread risks are particularly high in periods of high volatility and during a market crash.

3.1.4 Interest rate risk

In this part, the concept of the 'term structure of interest rates' will be used interchangeably with the 'yield curve'. The yield curve represents a continuous spot rate curve that can be derived by plotting the yields of different governments bonds (usually U.S. Treasuries) across various maturities.

Interest rate risk also falls into the category of systematic risks and describes the possibility of a change in the government bond yield curve, which would have an effect on the market value of all corporate bonds. A rise in interest rates would consequently increase the discount factor, which is used to calculate the present value of future cash flows of a bond, changing overall market prices.[27] Interest rates and the market price of bonds have an inverted relationship, i.e. a rise in interest rates will decrease market price, and vice versa. Interest rate risk can be quantified using the concepts of duration (and convexity), presented in section 4.1.2 of this paper.

3.1.5 Influence on Credit Spreads

As the credit spread represents the difference in yields of a corporate bond to the benchmark bond, credit spread accounts for all risk premiums that go beyond the default risk-free bond. Thus, credit risk, liquidity risk and spread risk are priced into the credit spread.

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[24] See Austrian Financial Market Authority for definition
[25] See Brown & Zarnic (2003) p.10
[26] See Fama & French (1993)
[27] See section 3.1.1

Interest rate risk is not priced into credit spread per se, as both corporate and benchmark bond are affected. However, the term structure of interest rates will have an impact on credit spread; if these spreads are calculated on the base of subtracting the yield to maturity of two individual bonds form another, as discussed in detail in Section 4.1. In comparison, spreads, which are calculated by using spot-rate curves, are not affected by interest rate changes. The change in credit spreads as a result of changes in interest rates will also be categorized as a spread risk.

In addition to the described factors, credit spread may factor in certain covenants and embedded options, such as in callable and putable bonds, etc.[28]. Even tax exemptions can matter in the calculation of credit spreads. For example, interest income from U.S. Treasury bonds is exempt from state and local income taxes, but is subject to U.S. federal income taxes. As a consequence, Treasuries will trade at higher prices and lower yields than other government bonds, so that credit spreads, which are determined on the basis of these bonds, are comparatively higher.

3.2 Constituents of credit spreads

Financial theories in regards to the modelling of default risks can be divided into two separate groups. Supporters of the so-called reduced-form models simulate default as an exogenous, random event. They do not make assumptions about the cause of the failure. Important work in the field of reduced-form models has been conducted by Duffie and Singleton (1997) as well as Elton et al. (2001). The second group of models are known as structural models, which simulates the default event as a function of the capital structure and the value of the assets of the company. This approach was originally developed by Merton (1974)[29] and extended by Longstaff & Schwartz (1995) and Collin-Dufresne et al. (2001) with their own models. In Merton's model, a default occurs when the company's assets are worth less than the value of a single-issue zero-coupon bond. The development of corporate assets is modelled as a stochastic process. The valuation of risky zero-coupon bond can thus be conducted by using the option pricing theory of Black and Scholes (1973).

On the other hand, structural models take into consideration the debt-equity ratio, which reflects the creditworthiness of the company, the risk-free rate as well as the value and the volatility of corporate assets.[30] These factors constitute the basic framework for the theoretical calculation of default risk and credit spread.[31]

[28] See Brown & Zarnic (2003) pp.12-13
[29] See Literature Review in section 1 for definition of Merton model
[30] Please see section 2.1 for an analysis of changing credit rating
[31] See Van Landschoot (2004) pp.10-12

Nevertheless, other dynamic factors need to be examined and taken into consideration (apart from the more static risk factors described in the previous section), e.g. the business cycle, stock market prices and liquidity.[32] These factors will have an effect on credit spreads in the form of compensated risks and higher premia. The credit spread and its constituents are subject to change during the term to maturity of a bond.

3.2.1 Term structure of interest rate

In their structural model, Longstaff & Schwartz (1995) show that there is a negative correlation between the interest rate and the credit spread. The current default risk-free rate represents the growth rate of corporate assets, implying that with larger values of assets, the probability of default will decrease. Consequently, the spread decreases with increasing interest rates. As explained in Section 3.2.4, a strong economy will lead to a fall in credit spreads since more money will be allocated to corporations. At the same time economic expansion will pressure the central banks to lift interest rates. This will add to the negative correlation between interest rates and credit spread, a relationship that has been documented in empirical studies.[33]

Apart from that, the yield curve (term structure of interest rates), which describes the difference between short and long-term government bonds, is often regarded as a vital factor of the credit spread. According to the expectation hypothesis, the slope of the yield curve reflects the expectations of market participants with respect to short-term interest rates. A steep yield curve thus leads to higher short-term interest rates. In addition, a steep yield curve is also associated with good economic prospects. According to the above considerations, this will result in a decline in credit spreads. However, a negative correlation between credit spreads and the slope of a yield curve has not been empirically established in financial literature.[34]

3.2.2 Stock market prices and volatility

The basic idea of the structural model implies that the default risk is negatively correlated with the value of corporate assets. Thus, a negative correlation between the value of corporate assets and the credit spread is expected. Since information on the value of assets from the corporation's balance sheet is only available in large time intervals, the stock price return and the market capitalisation can be used as an approximate figure.[35] Collin-Dufresne et al. (2001) note, that there is

[32] See Van Landschoot (2004) pp. 10-12
[33] See Annaert & De Ceuster (1999) for influence of interest rate changes on credit spread
[34] See Brown & Zarnic (2003) pp. 8-9
[35] See Van Landschoot (2003) p. 13

an even stronger negative relationship between credit spread and equity indices. This may be explained by the fact that the stock market reflects the overall economic situation, which in turn has a direct impact on credit spreads.[36]

On the other hand, there should be a positive correlation between the spread and the volatility of corporate assets (according to the structural models), since increasing volatility increases the probability that the redemption value of the zero-coupon bond will fall short.[37] Numerous empirical studies confirm the relationship described here.[38]

3.2.3 Term to maturity

The credit spread of a corporate bond is affected by its decreasing term to maturity. The relationship, however, is complex and can be demonstrated with two different theoretical approaches. In the investor-oriented approach, investors demand a higher risk premium for longer bond maturities, since increases in the term to maturity of the investment increase credit risk. Therefore, one can state a positive relationship between term to maturity and credit spread. The fundamental approach, on the other hand, focuses on the solvency of the company at maturity. A decreased solvency of the company shortly before the maturity date will cause difficulties in receiving appropriate refinancing, thus increasing the credit spread before the settlement date. This line of argument implies a negative relationship.

Both lines of argument, however, must not contradict each other if one presumes that the effect, which is described in the fundamental approach, is particularly relevant for corporations with an inferior credit rating and overcorrects the effect which is described in the investor-oriented approach. Therefore, credit spread will be the largest shortly before the end of the term to maturity if the issuer has a low credit rating. Conversely, corporations with a high credit rating will experience an increasing credit spread along with an increase in term to maturity. Nonetheless, empirical studies cannot establish this relationship without a certain degree of uncertainty.[39]

3.2.4 Business cycle

The analysis of the impact of the economic climate on credit spreads is intuitive. An economic downturn generally produces higher credit spreads, while an expansion leads to declining credit

[36] See section 3.2.4
[37] See Westphalen (2001) p.8 for structural model volatility and Merton's model in section 2
[38] See Ericsson & Renault (2006) for a summary of those empirical studies
[39] See Brown & Zarnic (2003) p. 9 for in-depth analysis of this relationship

spreads. This can be explained by the fact that in a recessionary phase, the deposits and the surplus of a company decreases and therefore less financial resources are available to make interest and principal payments. For this reason, bondholder's credit risk increases along with the credit spreads. In an expansionary phase, however, credit spreads will decline for the opposite reasons. The counter-cyclical behaviour of credit spreads has been demonstrated in empirical studies, e.g. Duca (1999), etc.

3.3 Analysis of appropriate benchmarks

In order for government bonds to be used as a benchmark in the calculation of credit spread, they must demonstrate certain qualities: Firstly, they have to be default risk free. Secondly, they must be actively traded in a liquid market. Thirdly, they must cover a certain range of maturities. Apart from these fundamental factors, a benchmark should also show very similar characteristics to the corporate bond in order for the credit spread to be precise. In financial theory, government bonds and 'treasuries' of certain countries[40] with the top credit rating are regarded as virtually default risk-free.[41]

However, investors have been increasingly relying on swap rate curves rather than yield curves due to a lack of availability of government bonds, which cover the entire range of different maturities. For this case, interbank loan rates, such as LIBOR or EURIBOR are used to determine short-term risk free rates, forward rate agreements (FRA) or interest rate futures are used to determine mid-term risk-free rates and swap rates are used to determine long term risk-free rates for the new swap curve.[42]

Interbank rates can be used as short-term (less than one year) interest for loans of companies with an AA-rating and are therefore considered to be virtually default-risk free.[43] Since transactions in interest rate futures are also based on the short-term interbank rates, spot rates derived from those can also be considered as approximately default-risk free. A swap rate is defined as the fixed rate that is exchanged for (another rate plus) a variable rate linked to the LIBOR (or EURIBOR, etc.) in an interest rate swap agreement. By multiplying the interest rates with a fixed nominal amount, a fixed and a variable payment stream can be defined. In order to minimize the counterparty risk in a swap agreement, only the difference in interest rate payments will be exchanged at specified (usually semi-annual) intervals. Since the parties engaging in a swap agreement must either demonstrate a

[40] Perceptions of default risk-free countries might change causing downgrades, such as recent downgrades of the US and France, etc.
[41] See Houweling et al. (2001) p.298 for benchmarks
[42] See Hull (2009) pp. 158-159 and Ron (2000)
[43] See Hull (2009) pp. 74-75

high credit rating or have sufficient collateral deposits, swap rates can also be considered as practically free of default risk.[44] Because the swap curve is constructed using various money market interest rates, a large range of maturities can be covered. All inputs of the swap rate are also actively traded on liquid markets. Thus, the swap curve fulfils all theoretical requirements of a default-risk-free benchmark for the calculation of credit spread.

The use of the swap curve is supported by a number of advantages of the swap market.[45] The swap market is very liquid and has narrow bid-ask spreads.[46] In addition, high market efficiency is provided. The swap market covers a wide range of different maturities, with new swaps being issued daily. There is also the aspect of good international comparability, since national authorities (e.g. FSA, SEC, etc.) scarcely regulate the swap market. On the other hand side, tax exemptions, which are granted in some countries when purchasing government bonds, would lead to inconsistent international standards with respect to interest rates. As a consequence, such tax exemptions would also affect credit spreads, which are calculated using government bonds as a benchmark.[47] Finally, there are some studies that demonstrate, using valuation models for credit default swaps, that the swap curves are generally closer to the implicit risk-free rate than government yield curves (term structure of interest rates).[48] The yield curve will usually lie below the swap curve when plotted on a graph since a swap curve must compensate for the lack of a tax exemption by offering a higher yield. Hull et al. (2004) estimate that the market is using a risk-free rate about 10 basis points less than the swap rate.

However, there are some additional benefits in investing in government bonds apart from a possible tax exemption, such as the high degree of liquidity in the bond market, in which market participants facilitate major transactions without a significant change in price. Apart from that, government bonds can be used as a deposit in margin accounts instead of cash in many futures exchanges, e.g. CME Group.[49] Consequently, a premium should be taken into account, when government bonds are used as a benchmark for the calculation of the credit spread.

Hull et al. (2004) regard the traditional practice of calculating the credit spread as problematic. They propose *"credit default swap spreads as an alternative to bond prices in empirical research on credit ratings for two reasons since CDS spread data provided by a broker consists of firm bid and offer*

[44] See Hull (2009) pp.158-159
[45] See Ron (2000) pp. 2-4
[46] See section 3.1.1
[47] See section 3.1.5
[48] Studies confirming this observation are Hull et al. (2004) pp.2971 and Houweling & Vorst (2005) pp.1215-1217
[49] See Hull p. 75 for margin account, etc.

quotes from dealers. Once a quote has been made, the dealer is committed to trading a minimum principal (usually $10 million) at the quoted price. By contrast the bond yield data available to researchers usually consist of indications from dealers. There is no commitment from the dealer to trade at the specified price. The second attraction of CDS spreads is that no adjustment is required: they are already credit spreads. Bond yields require an assumption about the appropriate benchmark risk-free rate before they can be converted into credit spreads."[50]

Furthermore, Hull et al. (2004) identify the main difficulty to be *"choosing the risk-free rate, r. Bond traders tend to regard the Treasury zero curve as the risk-free zero curve and measure a corporate bond yield spread as the spread of the corporate bond yield over the yield on a similar government bond. By contrast, derivatives traders working for large financial institutions tend to use the swap zero curve (sometimes also called the LIBOR zero curve) as the risk-free zero curve in their pricing models because they consider LIBOR/swap rates to correspond closely to their opportunity cost of capital. The choice of the Treasury zero curve as the risk-free zero curve is based on the argument that the yields on bonds reflect their credit risk. A bond issued by a government in its own currency has no credit risk so that its yield should equal the risk-free rate of interest. However, there are many other factors such as liquidity, taxation, and regulation that can affect the yield on a bond. For example, the yields on US Treasury bonds tend to be much lower than the yields on other instruments that have zero or very low credit risk. One reason for this is that Treasury bonds have to be used by financial institutions to fulfil a variety of regulatory requirements. A second reason is that the amount of capital a financial institution is required to hold to support an investment in Treasury bonds is substantially smaller than the capital required to support a similar investment in low risk corporate bonds. For all of these non-credit-risk reasons, the yields on U.S. Treasury bonds tend to be depressed relative to the yields on other low risk bonds."*[51] (pp. 12-13)

[50] Hull et al. (2004)
[51] Excerpt from Hull et al. (2004) (pp. 12-13)

4. Credit spread calculation methods

This section signifies a main focus of this thesis. Possible calculation methods of credit spreads will be conducted, along with a detailed discussion of their respective advantages and disadvantages. In general terms, we can divide existing calculation methods into two broad categories. The first category conducts a calculation of credit spreads using two individual bonds or bond indices. In the second category, credit spread is calculated on the basis of spot-rate curves, which can be modelled and approximated from multiple types of bonds.[52]

4.1 Calculating credit spread using individual bonds

When analysing two individual bonds, the credit spread of a corporate bond is usually the difference in yield to maturity of the corporate bond on the yield to maturity of the risk-free benchmark bond.[53] However, this again raises theoretical and practical questions of which benchmark bond to use. In this section, various different approaches will be evaluated; among those is a method, which matches the maturity and/or duration of two individual bonds (corporate and government bond). Apart from that, the approach of using a bond index as a benchmark shall be presented. In addition, a credit spread calculation based on swap rates is proposed.

4.1.1 Calculating credit spread on the basis of individual bonds with matched maturities

The most common method of calculating credit spread is to subtract the yield to maturity of a government bond from the yield to maturity of a corporate bond, under the condition of both bonds having similar characteristics and the same maturity. Although this is simplest approach, possible distorting factors such as different interest payment dates will affect the precision of the resulting credit spread. The credit spread is described as: $S = r_{c,t} - r_{g,t},$ where $r_{c,t}$ and $r_{g,t}$ are defined as the yield to maturity of a corporate bond and the yield to maturity of a government bond, respectively. These yields can be calculated using the market value of the bonds, i.e. calculating the internal rate of return (IRR).[54]

[52] When comparing individual bonds, the credit spread is usually calculated as the difference in yields to maturity of a corporate and government bond. Similarly, when credit spreads are determined on the basis of spot-rate curves, a spot-rate differential is the result. It is important that these methods are not mixed, and no differences between yields to maturity and spot rates are combined, since that would lead to economically uninterpretable credit spread levels.

[53] This method has been used in Fabozzi et al. (2005) p. 329

[54] In Excel, a simple IRR function can be used for the calculation of yield to maturity (YTM)

All interest rates are effective annual rates, unless otherwise noted. The current market price P of a coupon-bearing bond corresponds to the value of the bond, i.e. the sum of all future cash flows, which are discounted with the spot rates from the yield curve $CF_{ti}, i = 1, \dots, M$: [55]

$$P = P^{cl} + AI = \sum_{i=1}^{M} CF_{ti}\ (1 + r_{o,ti})^{-ti}$$

$r_{o,ti}$ represents the spot rate for the fixed payment dates t_i, i=1,…,M where the coupon payment dates are calculated as the time distance in years from today. Because bonds can be traded between coupon payments dates, t_i does not have to be a whole period. In this case, the accrued interest AI is added to the contract price of a bond transaction on top of the 'clean' price. The formula for accrued interest is:

$$\mathrm{AI} = \mathrm{C}\left(\frac{\text{days from the last coupon payment to the settlement date}}{\text{days in the coupon period}}\right)$$

Bond prices that are quoted by dealers are usually clean prices P^{cl}. Adding the accrued interest AI to that figure will produce the 'dirty price' P. As mentioned before, the yield to maturity of a bond can be derived using the market value. Thus, the yield to maturity r_e is defined as the discount rate, which returns the market price of the bond. It is identical to the required return of investors. Ultimately, yield to maturity is the internal rate of return of an investment in the bond made at the observed price.[56]

$$P = \sum_{i=1}^{M} \frac{CF_{ti}}{(1 + r_e)^{ti}}$$

In order find the value for r_e; one would to solve a non-linear equation. This can be done by using the Newton's method or using an appropriate Excel function.

The table below demonstrates the above mentioned calculation method. We calculate the market price of both the corporate and government bond using appropriate discount factors. In order to determine the true value for spread, we ignore real market data, which would be subject to considerable distortions, as described in the previous sections.[57] Instead, we set the spot rate of the government bond 200 bps (basis points) below the corporate bond. Please note that the spot rates are theoretical predictions and that market forces determine actual yields.

[55] See Fabozzi (2005) pp. 75, 82-86 for explicit information on the calculation method
[56] See Fabozzi (2005) p.75-79 for bond pricing
[57] Theoretical models are more suitable for this analysis compared to real market data since we do not aim to examine empirical data, but to establish accurate calculation methods

Calculation of credit spread using individual bonds with same maturity

Yield curve (annual Spot Rates)												
Year	1	2	3	4	5	6	7	8	9	10		
Corporate bond	5.19%	5.44%	5.69%	5.94%	6.19%	6.44%	6.69%	6.94%	7.19%	7.44%		
Risk free government bond	3.19%	3.44%	3.69%	3.94%	4.19%	4.44%	4.69%	4.94%	5.19%	5.44%		
Corporate bond (Face value £100, Maturity 10 years)												
Cash Flows	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	7.0	107.0		
Discount factor (Spot Rates)	1.05	1.11	1.18	1.26	1.35	1.45	1.57	1.71	1.87	2.05		
Discounted Cash flows (Spot Rates)	6.65	6.30	5.93	5.56	5.18	4.81	4.45	4.09	3.75	52.19	Market value	98.90
Discount factors (YTM)	1.07	1.15	1.23	1.32	1.41	1.51	1.62	1.74	1.86	2.00	Yield	7.16%
Discounted cash flows (YTM)	6.53	6.10	5.69	5.31	4.95	4.62	4.31	4.03	3.76	53.60	Market value	98.90
Government bond (benchmark) (Face value £100 , Maturity 10 years)												
Cash Flows	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	105.0		
Discount factor (Spot Rates)	1.03	1.07	1.11	1.17	1.23	1.30	1.38	1.47	1.58	1.70		
Discounted Cash flows (Spot Rates)	4.85	4.67	4.48	4.28	4.07	3.85	3.63	3.40	3.17	61.80	Market value	98.20
Discount factors (YTM)	1.05	1.11	1.17	1.23	1.29	1.36	1.43	1.50	1.58	1.67	Yield	5.24%
Discounted cash flows (YTM)	4.75	4.51	4.29	4.08	3.87	3.68	3.50	3.32	3.16	63.03	Market value	98.20
											Credit Spread	**1.92%**

TABLE 1 Example for the calculation of credit spread on the basis of two individual bonds with matched maturities.

The great disadvantage of this first method of using two individual bonds with the same maturity is that the spread will depend on the term structure of interest rates (yield curve). Although we can establish a negative correlation between the interest rates and the credit spread[58], this relationship only holds, if the term structure of interest rates is constant. Moreover, the credit spread should theoretically compensate the investor only for credit risk and not for interest rate risk.[59] Therefore, changes in the yield curve should not be factored into the credit spread calculation.[60] If, however, the credit spread is calculated on the basis of two bonds with the same maturity, a change in interest rates will have a different impact on the corporate bond compared to the government bond due to their distinctive sensitivities, despite the credit risk remaining constant.

If the term structure of interest rates changes, the yield curve will cause a change in the market price of a bond. This can be easily demonstrated using the pricing formula of bonds. For simplifying purposes, we therefore assume a flat yield curve, so that the same interest rate r (spot rate) is applied for all discount factors. In this case, the sum of all discounted cash flows is

$$P = \sum_{i=1}^{M} \frac{CF_{ti}}{(1+r)^{ti}}$$

Consequently, the derivation of the market value from the interest rate will be negative:

[58] See section 2 (Literature Review) for structural model
[59] See section 3.1.1
[60] See section 3.1.5

$$\frac{\delta P}{\delta r} = - \sum_{i=1}^{M} t_i \, CF_{ti}(1+r)^{-ti-1} < 0$$

Therefore we can state, that the market value of a bond will decrease with a rise in interest rates. Government bonds usually pay a smaller coupon than corporate bonds and will therefore have a longer duration.[61] In this case, little cash flows are received early. Since duration measures the sensitivity of the price of a bond to a change in interest rates, a rise in interest rates will have a more significant impact on the market price of government bond than on a corporate bond, in relative terms. In this case, a decrease in market prices will occur, even if the credit risk is not affected. This can cause major problems and imprecise inputs, especially when developing a credit spread curve or calculating precise inputs for the valuation of risky debt.

Moreover, there is also an issue affecting the liquidity risk factor in credit spreads. Empirical evidence[62] suggests that the liquidity of government bonds decreases at a slower rate than corporate bonds as they move towards their settlement date. However, it is unusual, especially in European markets, to change the reference benchmark in the calculation of credit spread. This can lead to a distortion of the credit spread curve due to changes in liquidity when credit spreads are calculated on the basis of individual bonds with matched maturities. US markets, however, tend to use the most recent on-the-run Treasuries when constructing credit spread curves.[63] Ultimately, a change in the benchmark bond, which can alter the benchmark's characteristics or include market inefficiencies, will have an impact on credit spread calculation models.

4.1.2 Calculating credit spread on the basis of individual bonds with matched duration

Section 4.1.1 discussed the disadvantages of using the yield of two individual bonds with matched maturities to calculate credit spread. In the following section, we shall attempt to eliminate these weaknesses by using a benchmark bond with the same duration as the corporate bond rather than the same maturity.[64] Our motivation behind this approach is the observation that a portfolio of default-risk-free bonds with matched duration will be interest rate risk free, if held until maturity. Consequently, two individual bonds with matched duration will theoretically experience a parallel shift in interest rates (due to a change in the spot rates in the yield curve), keeping the credit spread insensitive to interest rates. Thus, it seems logical to calculate credit spread on the basis of a

[61] See Papageorgiou & Skinner (2002) pp. 4-5
[62] Bao et al. (2011) pp. 17-19
[63] See Munves (2005) p.426 for U.S. market reference procedures
[64] See Westphalen (2001) and Longstaff et al. (2005) for use of duration-matched calculation methods

corporate bond and a risk free benchmark bond with the same duration. Duration is calculated using the Macaulay formula:[65]

$$D = \frac{1}{P}\sum_{t-1}^{n} t\ x\ PV_t$$

Nonetheless, even this approach shows apparent weaknesses. The yield to maturity, which is the basis for the calculation of credit spread, often needs to be approximated because it is significantly more difficult to find a risk-free benchmark bond that has the exact same duration as the corporate bond than finding a benchmark with the same maturity. Moreover, a credit spread calculated with duration-matched bonds is only theoretically insensitive to changes interest rates. However, this problem, in combination with credit spread calculation, has not been critically evaluated in academic literature.

A fundamental concept of duration is that securities will experience a parallel shift in the yield curve when interest rates change. However, empirical evidence suggests that these changes might not cause parallel shifts, but a steepening or flattening of the yield curve. [66] Nonetheless, the yield to maturity of duration-matched bonds is used to calculate the spread. The resulting inaccuracy can be demonstrated using the following calculation, while accounting for the market price of the two individual bonds $P_1\ and\ P_2$, their yield to maturity $r_{e,1}\ and\ r_{e,2}$ and their modified durations $D_{m,1}\ and\ D_{m,2}$ as well as the combined duration $D_{1,2}$.

$$\frac{\Delta P}{P} \approx -D_m \Delta r \qquad\qquad solve\ for\ \ \Delta r$$

This formula shows the reason why the credit spread is only approximately insensitive to interest rate changes. If the relationship $\Delta r_{e,1}$ = $\Delta r_{e,2}$ did hold, then this calculation method would be insensitive to interest rate changes. In this case the product of the inverted modified duration and the change in market prices for both bonds would be almost identical.

$$\Delta r_{e,1} \approx -\frac{1}{D_{m,1}}\frac{\Delta P_1}{P_1} = -\frac{1+r_{e,1}}{D_{m,1}}\frac{\Delta P_1}{P_1}, \qquad\qquad \Delta r_{e,2} \approx -\frac{1}{D_{m,2}}\frac{\Delta P_2}{P_2} = -\frac{1+r_{e,2}}{D_{m,2}}\frac{\Delta P_2}{P_2}$$

However, strictly parallel shifts in the yield curve are based on a simplistic and unrealistic assumption and thus the proposed method using duration displays an essential flaw. In addition, the equations themselves prove to be approximates.

[65] PV signifies the present value of future cash flows, please refer to List of Symbols
[66] See Fabozzi (2005) pp.210-212 for chapter on convexity

The interest rate sensitivity of the resulting credit spreads and the failure to find an exact duration match for the benchmark make the accuracy of the proposed calculation highly questionable and advantages over the maturity matching method fairly insignificant. Moreover, the duration based approach causes further problems when a credit spread curve needs to be developed. A shift in the term structure of interest rates will in consequence lead to a change in duration values causing a loss of the matched duration property in our calculation.

4.1.2 Calculating credit spread on the basis of an individual bond and a bond index

Another proposed method for the calculation of credit spread on the basis of yield to maturity is the use of an appropriate government bond index as a default-risk-free benchmark.

There are a variety of different bond indices, which are composed of government bonds and created by banks, market intelligence providers (e.g. Bloomberg, Thomson Reuters etc.) and stock exchanges. As described in section 3, a benchmark should have similar features to the observed corporate bond in order to minimize the distortion of the spread. This condition has to be taken into account with respect to choosing an appropriate benchmark index. Apart form the obvious currency issues in choosing the appropriate index, one must also account for corporate bonds with warrants, which are usually excluded from the analysis.

For US corporate bonds, one might consider using the 'Barclays Capital Aggregate Bond Index' (originally constructed by the defunct Lehman Brothers)[67] or the 'Vanguard® U.S. Government Bond Index Fund'[68] as a benchmark in our calculation of credit spread.[69] For UK corporate bonds, the 'FTSE UK Actuaries Gilt Index Series'[70] appears to be the most comprehensive index. For German corporate bonds, we look at the 'Deutsche Rentenindex' (German Bond Index) REX, which is published on a daily basis by Deutsche Börse AG[71]. While all these indices could be potentially used as a benchmark, only the German REX offers publicly available regression coefficients, which will be needed for a the calculation. Thus, we will focus on the 'Deutsche Rentenindex' in the following analysis of the proposed third calculation model.

[67] See Barclays Capital Website for factsheet on https://ecommerce.barcap.com/indices/index.dxml
[68] See Vanguard Website on http://www.vanguardinformation.com/international/pdfs/A4_F9917EN.pdf for explicit information on the index
[69] Other alternatives for US bond indices are the Citigroup US Broad Investment Grade Credit Index, and the Dow Jones Corporate Bond Index.
[70] See FTSE Website on http://www.ftse.com/Indices/FTSE_UK_Gilts_Indices/index.jsp for more information on values, constituents, rules, changes, etc.
[71] See Deutsche Börse (2004) pp.2-5

The German Bond Index REX is calculated from all bonds, debentures and treasury notes of the Federal Republic of Germany, which have a fixed coupon and maturities from 0.5 up to 10.5 years. For this purpose, we first determine the yield to maturity of these fixed-income securities based on their market values on the Frankfurt Stock Exchange. These yields are used for the construction of an appropriate yield curve, which depends on the bond's maturity and the coupon. The yield curve is determined by the following regression:

$$r_e(t_M, c) = \beta_1 + \beta_2 t_M + \beta_3 t_M^2 + \beta_4 t_M^3 + \beta_5 \ln t_M + \beta_6 c + \beta_7 c^2$$

Where $r_e(t_M, c)$ is defined as the yield, t_M as the maturity, c as the coupon in percentages % and β_i, i = 1,…,7 as the regression coefficients. Whereas the calculation method and the specific weights of the German Bond Index REX[72] remains irrelevant in our model, the derived yield curve will be very useful for the calculation of credit spread.

Since Deutsche Börse AG releases the regression coefficients on a daily basis to investors, we can determine the yield to maturity of a synthetic bond, which is then used to calculate the credit spread by subtracting the YTM of the synthetic bond from the YTM of the corporate bond. The great advantage of this third calculation model compared the to the previous two is that it enables us to create a synthetic bond, which not only has a matched maturity, but an approximately matched duration to the corporate bond we are comparing it to. Thus, this method allows us to construct a perfect synthetic bond, which has a common coupon and maturity with the corporate bond. However, the duration match remains an approximate, since a perfect match would require both YTMs to be identical. Nevertheless, if we calculate the yield to maturity of a synthetic bond using the regression equation of the German Bond Index, we can establish an approximately interest rate insensitive relationship for German credit spreads.

Moreover, the calculation of credit spreads of corporate bonds using a benchmark index offers a number of other benefits. If a single government bond is used as the default risk-free reference in the calculation of spread, the reference bond is subject to the specific supply and demand situation of the market. This would directly affect the market values and in consequence, the yield on the benchmark. This limitation is eliminated through the use of a benchmark index, since it incorporates many different bonds with an average trading volume. In addition, the use of a benchmark index rather than an individual benchmark bond reduces possible liquidity-driven distortions in the time

[72] The German Bond Index REX itself is calculated on the basis of synthetic bonds, which have maturities between 0,5 and 10,5 years and offer a coupon of 6%, 7.5% and 9%. Using these 30 bonds and running a regression on them, the REX analysts determine the yield to maturity as well as the market price of those synthetic bonds. The REX represents the weighted average of the bond indices' market prices, in which the weights are based on market share of the individual bonds. See Deutsche Börse [2005] for more detail.

series. Furthermore, it eliminates any distortions of the spread curve that would result from changing the reference benchmark in case of a lack in matching properties.

Moreover, the search for a suitable reference with identical characteristics becomes much easier. If one takes the German Bond Index REX, for example, one can easily determine the yield to maturity of a synthetic government bond, which has a matched duration with the corporate bond. Not every government bond index, however, is based on a yield curve, which is dependent on both maturity and coupon. Consequently, we cannot be absolutely certain that credit spreads, which have been determined using any bond index as a reference, are interest rate risk free. Thus, an absolute advantage of using bond indices as a reference for the calculation of credit spread in any possible scenario cannot be demonstrated per se.[73]

4.1.3 Calculating credit spread on the basis of an individual bond and an interest rate swap

Section 3.3 demonstrated the use of the swap curve, which can be derived from the money markets, as a risk-free benchmark in a calculation of credit spread on the basis of spot rates. Extending that idea, it may also be possible to calculate credit spreads using the yield that is dependent on the swap curve.

A swap rate is the fixed portion of a swap as determined by its particular market, which makes the market value of a given swap at initiation zero. These swap rates can be used as a benchmark in the calculation of credit spreads by constructing a risk-free swap curve which satisfies all theoretical conditions of a benchmark yield.[74] Swap rates that are appropriate for our calculation must have equal values for the fixed and variable cash flows of swap agreement and can be regarded analogously to the par yield of a coupon-bearing bond.[75] *“The par yield of a certain bond maturity is the coupon rate that causes the bond price to equal its par value.”*[76] For bonds that pay an annual coupon, the par rate is identical to yield to maturity. For bonds with coupon payments of less than a year, the par rate differs slightly from the yield to maturity.

The calculation of credit spread could therefore be achieved by measuring the difference between the par yield of a bond and swap rate with the same maturity. For bonds with a regular annual

[73] For example, the FTSE UK Actuaries Gilt Index is only dependent on the maturity, however, not on coupons and are therefore suitable as a benchmark, see FTSE (2008)
[74] See section 3.3
[75] Feldhütter & Lando (2008) calculate the swap spread as the difference between swap rates and par rates of government bonds, which share the same maturity. This paper confirms our approach of comparing par rates and swap rates for the calculation of credit spread.
[76] Definition quoted from Hull (2009) p. 79

coupon, the difference could be determined by subtracting the swap rate from the yield to maturity of a corporate bond.

A possible calculation method for the credit spread of a corporate bond would thus be the calculation of the difference between the par rate of the bond and a swap rate, i.e. the difference between yield to maturity and the swap rate for bonds with annual coupon payments. The following method of calculation is based on a theoretical example:

Calculation of credit spread on the basis of a swap						
Yield structure (Spot Rates)						
Payment dates (In Years)	0.5	1	1.5	2		
Corporate bond	7.06%	7.58%	8.09%	8.60%		
Risk-free swap	5.06%	5.58%	6.09%	6.60%		
Corporate bond (Face value 100 €, Maturity 2 years)						
Cash Flow	5.0	5.0	5.0	105.0		
Discount factor	1.035	1.076	1.124	1.179		
Values of the Cash Flows	4.832	4.648	4.449	89.025	Market price	102.95
Cash-Flows at par yield	4.187	4.187	4.187	104.187	Par Rate	8.37%
Discount factor	1.035	1.076	1.124	1.179		
Values of the Cash Flows	4.047	3.892	3.726	88.335	Face value	100.00
Swap (Face value 100 €, Maturity 2 years)						
Variable Cash Flows	2.5	3.0	3.5	4.0		
Discount factor	1.025	1.056	1.093	1.136		
Values of the variable Cash Flows	2.439	2.842	3.203	3.520	Sum	12.00
Fixed Cash Flows	3.229	3.229	3.229	3.229	Swap Rate	6.46%
Discount factor	1.025	1.056	1.093	1.136		
Values of the fixed Cash Flows	3.150	3.058	2.955	2.841	Sum	12.00
					Credit Spread	**1.92%**

TABLE 2 Example of a credit spread calculation by determining the difference between the par yield of a corporate bond and the swap rate of an interest rate swap agreement with the same maturity.

As in the previous example, we have set the risk-free spot rates to be 200bps below the corporate rate. The swap rate has been derived under the condition of having an equal market value for the variable and fixed cash flows. The pre-determined par yield will make the market value of the bond equal its par value. Thus, the credit spread is determined as the difference between the annual par yield and the annual swap rate.

In order to account for the risk-free swap rate, the investor should always choose the most recent issue of bonds. For corporate bonds with short maturities, an investor may use interbank rates (for example, 3-month, 6-month or 12-month LIBOR) as a benchmark. Since this calculation method has not been extensively examined or evaluated in academic literature, it is advisable to conduct further investigations in order to determine unforeseeable weaknesses and limitations.

The main advantage of this calculation method, compared to the previous approach of using duration- or maturity-matching, is based in the general benefits of the swap market.[77] First of all, the swap market offers swap rates of all ranges of maturities.[78] Second, using swap rates eliminates the need for adjusting the benchmark, which must reflect the most current market-determined risk-free rate. In this context, the swap market tends to offer a higher level of efficiency[79] and produces less distorting factors pertaining the term structure of interest rates, etc.[80] Lastly, an important advantage of the swap rate is its proximity to the 'true' value of a risk-free rate. Nevertheless, using the swap rate will not eliminate interest rate risk, which has been previously discussed in this analysis.[81]

4.2 Calculating credit spreads using spot rate curves

Up to this point, this paper attempted to estimate the credit spread for individual bonds, however, one can also determine the credit spread for certain credit risk categories using spot rate curves.

Spot rates are not directly observable in the capital market, but can be derived directly from the market price of zero-coupon bonds. Since only relatively few bonds are traded as zero-coupon bonds, spot rates must be estimated and approximated in order to cover the entire range of corporate bond maturities.[82] These continuous spot rate curves are constructed from data sets that contain coupon-bearing and, upon availability, zero-coupon bonds. Bonds in such data sets are subdivided into groups of similar features such as issuer, credit rating, sector and currency, and are therefore practically homogeneous with respect to their credit risk.[83] Thus, an individual spot rate curve can be determined for each risk class of corporate bonds. In order to determine the default risk-free spot rate curve, one can use the same approach with government bonds. Ultimately, this enables the investor to derive a credit spread curve by calculating the difference between a default-risk free spot rate curve and a corporate spot rate curve. The credit spread s_t at time t will then be:

$$s_t = r_{c,0,t} - r_{g,0,t}$$

Where $r_{c,0,t}$ and $r_{g,0,t}$ represent the spot rates curves of the corporate bonds and government bonds, respectively.

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77 See section 3.3
78 See Hull et al. (2004) in section 3.3 for thorough analysis of advantages of using swap rates
79 Please refer to section 3.3 for advantages of swap rates over term structure of interest rates
80 See section 4.1.1 and 4.1.2
81 See section 3.1.4 and 4.1.1 and 4.1.2
82 The method of approximating synthetic zero-coupon bonds is known as stripping or bootstrapping
83 See Brown & Zarnic (2003) p. 13

In financial theory, spot rate curves are derived using the bootstrapping method or the curve-fitting model of Nelson-Siegel[84] (or Svenson[85]), etc. Moreover, some requirements are imposed on the procedure of constructing a precise yield curve using forward substitution: On the one hand, the method should offer a high degree of adaptability and flexibility, so that the implied bond prices deviate only slightly from the observed prices and complicated structures are adequately modelled. On the other hand, the result should show sufficiently smooth curves, which can be easily interpreted. Finally, the method should be easy to implement.[86] The first two requirements are conflicting objectives, so that a balanced decision, considering all necessary factors, will be taken into account when choosing the appropriate method. For purposes of simplicity, this thesis will focus on the bootstrapping method.

The calculation of credit spreads based on spot rates offers several advantages. Firstly, the investor can construct credit spread curves for arbitrary and constant maturities by repeating the spot-rate curve estimation on each trading day. Secondly, spreads, which have been calculated using spot rate curves, are insensitive to changes in interest rates because the credit spread itself has been determined by the yield curve. Finally, the calculation of credit spreads based on spot rates is preferable for arbitrage reasons, especially if these are used for further rating purposes, since cash flows can only be evaluated under the arbitrage-free condition.[87]

4.2.1 Estimation of spot rate curves using the bootstrapping method

Bootstrapping is a fairly simple method, which is used to determine the discrete spot rates from the market prices of bonds. In a modified form, many large banks and corporations, such as Bloomberg, use the bootstrapping method. In order to construct a fixed-income yield curve, prices of a set of coupon-bearing and zero coupon bonds are required. In most cases, these bonds have different maturities and credit risk ratings. In the first step of the bootstrapping method, the spot rates of bonds with the shortest maturities are determined. In the second step, spot rates with successively longer maturities are calculated using the spot rates which had already been determined.[88] The following theoretical example demonstrates the bootstrapping method, based on five hypothetical coupon-bearing bonds:

[84] See Nelson & Siegel (1987) for model
[85] See Svenson (1994) for estimation of forward interest rates
[86] See Choudry (2005) p.965
[87] See Elton et al. (2001) pp.251-252
[88] See Hull (2009) pp. 80-82

Data for the bootstrapping method			
Face value	Coupon	Maturity	Market value
100	3.0%	1	100.98
100	3.5%	2	100.99
100	4.0%	3	100.82
100	4.5%	4	101.16
100	5.0%	5	102.58

TABLE 3: Theoretical example for the calculation of spot rates using bootstrapping method

For reasons of simplicity, the bonds shown in the table make annual coupon payments. The bond with only 1 year until maturity, offers investors a redemption value of £103, which consists of the final coupon payment and the face value of the bond. Using the equation below, an investor could derive the one-year spot rate $r_{0,1}$ which will equal the yield to maturity due to single outstanding payment:

$$\sum_{i=1}^{M} CF_{ti}\ (1+r_{o,ti})^{-ti} \qquad £100{,}98 = \frac{£103}{1+r_{0,1}} \Leftrightarrow r_{0,1} = 2\%$$

The bond with the term to maturity of two years offers two payments in the amount of £ 3.5 and £ 103.5. Therefore, an investor may use a derivation of the above-mentioned formula to calculate the yield.

$$£100{,}98 = \frac{£3{,}5}{1+r_{0,1}} + \frac{£103{,}5}{(1+r_{0,2})^2}$$ with $r_{0,2}$ as the two-year spot rate.

Since the first variable $r_{0,1}$, has been already determined, an investor may solve the equation for $r_{0,2}$ which is 3%. Yields of all subsequent bonds can be determined analogously, see below:

Bootstrapping Result					
Spot Rate	$r_{0,1}$	$r_{0,2}$	$r_{0,3}$	$r_{0,4}$	$r_{0,5}$
Yield[in % p.a.]	2.00	3.00	3.75	4.25	4.50

TABLE 4: Spot rates calculation using bootstrapping

From a theoretical perspective, the bootstrapping method is sophisticated, but there are some practical problems, which further complicate its use. Considering the above example, it is clear that the only reason why spot rates could be determined easily is because all the bonds had exactly the same coupon payment dates and had consecutive maturities. If a full spot rate curve needs to be estimated, i.e. a curve which covers the spectrum of maturities of up to 30-years, it is difficult to find a suitable data set that would make the bootstrapping method applicable.[89] In order to construct an extensive data set, analysts most commonly use interpolation.[90]

Moreover, in practice, the spot rate curves calculated with the bootstrapping method are strongly dependent on the market prices of bonds. These market prices, however, can be distorted due to transaction costs and market inefficiencies.[91] As a result, the method often leads to irregular and uneven spot rate curves. Finally, the bootstrapping method only allows for discrete points of the spot rate curve to be estimated. Consequently, there are no available spot rates of maturities of less than one year for the example above. However, it is vital to derive a continuous spot-rate curve for any maturity, especially when these yield curves are needed for the credit spread calculation. Analyst may use sophisticated interpolation techniques to approximate continuous spot rate curves.

[89] See Choudry (2005) p. 955
[90] See Hull (2009) p. 82 for interpolation
[91] See McCulloch (1971) p. 21 for distortion factors

5. Application of credit spread calculation methods

The previous section presented various methods for the calculation of credit spreads. The analysis has demonstrated that there is no single universal method, but that there are certain advantages and disadvantages to every model. Instead, the appropriate calculation method will depend on the specific application of the credit spread, when valuing risky debt in structured or reduced-form models, etc. The sections below examine and evaluate three different applications of credit spread with regards to appropriate calculation methods. Ultimately, a recommendation for each of the existing calculation methods, depending on the application, will be offered.

5.1 Calculation of credit spread of a corporate bond at a specified instant of time:

In order to account for the specific credit risk of the corporate bond, this application requires the credit spread to be determined by calculating the difference in yields to maturity of two individual bonds. In this case, a simple calculation based on two individual bonds with matched maturities seems appropriate. The advantage of the calculation based on duration-matched bonds, which aims to offer near-insensitivity to interest rate changes, is insignificant if the estimation is at a specified instant of time.

Moreover, using bonds with matched maturities rather matched duration appears to be more feasible due to the lack of suitable duration-matched reference benchmarks. Alternatively, a calculation based on an individual bond and an adjusted bond index as a reference benchmark can be conducted. This approach offers a specific advantage; the YTM of the risk-free benchmark would be based on a number of individual government bonds (rather than a single issue) and thus be independent from specific supply and demand effects. Taking into consideration that certain government bonds have special characteristics[92] (e.g. tax exemptions, etc.), which would cause distortions in the previous method, an analyst may revert to calculating credit spreads on the basis of a corporate bond and an interest rate swap with matched maturities. This calculation would offer all advantages of using a bond index, however, neglect any distortions due to tax exemptions.[93]

5.2 Estimation of a credit spread time series for an individual corporate bond:

For this application, a calculation again based on comparing individual bonds will be appropriate in order to compensate the investor for the specific default risk of a bond. In addition, the possibility of

[92] See section 3
[93] See section 4.2

distortions of the credit spread time series due to changes in liquidity or any change in the benchmark should be minimized. Therefore, an analyst shall conduct either a calculation based on an individual corporate and an adjusted government bond index, or a calculation on the basis of a swap rate benchmark. Nonetheless, the use of a swap rate will not eliminate the need of changing the benchmark per se, however, only minor distortions will occur under normal circumstances due to the high efficiency of the swap market.

5.3 Calculation of a credit spread curve:

Investors require accurate credit spread curves for the pricing of credit default swaps (CDS), using the probability theory. These credit spread curves can be calculated by determining the difference between a corporate yield curve and a risk-free yield curve. While bootstrapping a spot rate curve was the proposed method in this analysis, there are other models (such as bootstrapping a swap rate curve, spline interpolation, etc.) to determine appropriate yield curves. However, measures of comparability and possible distortion factors have to be taken into account in order to provide accurate credit spreads as an input for further sophisticated pricings and calculations.

6. Conclusion

This bachelor thesis conducts a thorough comparative analysis into the field of credit spread estimation. Thus, the study addresses an issue of increasing relevance and importance. The paper examines both static risk factors and dynamic components; and evaluates various calculation methods of credit spread. Furthermore, an investigation into the advantages and disadvantages of the various methods is conducted. Ultimately, the paper offers suggestions for various applications of credit spread.

We conclude that there is no single universal model, but that every method offers certain advantages and disadvantages. We must therefore find an appropriate estimation process and account for suitability and feasibility. This paper aims to establish the benefits and drawbacks of each individual calculation of credit spread and therefore provide a guideline for the practical implementation.

Numerical examples, tables and formulae in this thesis aim to help professionals and academics to implement the results of this analysis in real-world tasks, especially in the valuation of risky debt, default probabilities and pricing of credit default swaps (see literature review).

While the thesis recognises that there is still lack of research on the topic of calculating credit spreads, it attempts to offer financial academics a good starting point for further research into this relevant topic.s

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