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# FINA 1082 Financial Management

Dr Cesario MATEUS

Senior Lecturer in Finance and Banking

Room QA257 – Department of Accounting and Finance

[c.mateus@greenwich.ac.uk](mailto:c.mateus@greenwich.ac.uk)

[www.cesariomateus.com](http://www.cesariomateus.com)

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# Lecture 1

Introduction to Business Finance

Introduction to Financial Economics

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# Subject Objectives

- Solve basic problems in financial mathematics
- Discuss the basic theories underlying the pricing of risky assets
- Comprehend the concepts of portfolio formation and risk diversification
- Explain the fundamentals of capital budgeting, including the use of alternative criteria, allowing for inflation and the treatment of risk
- Analyze the issues facing managers in capital structure and dividend policy decisions
- Use the features of financial derivatives to achieve specific financial outcomes.

## Recommended Text

Brealey R., Myers S., and Allen F., Principles of Corporate Finance, 8<sup>th</sup> Edition, McGraw-Hill/Irwin, 2006

# Introduction to Business Finance

- Overview of the Finance discipline
- Compare simple interest to compounded interest
- Compute the future value of a single cash flow
- Compute the present value of a single cash flow
- Compute an unknown interest rate and time period

# What is Finance?

**Finance** is the study of how individuals, businesses and institutions acquire, spend and manage financial resources

- **Major areas of Finance**

- Investment analysis and management
- Corporate Finance
- Capital markets and Financial Institutions
- International Finance
- Personal finance
- Real Estate Finance

**This subject provides an introduction to Investment Analysis and Corporate Finance**

# Overview of Business Finance

- The study of Finance is related to the corporate objective of maximizing shareholder wealth
- Our focus is on financial decision making...
  - **Individuals/Investors**
    - Financial Security Valuation: Earnings and Dividend models
    - Portfolios and Risk Diversification: Portfolio Analysis
    - Determination of Security Prices and Rates of Return: Capital Asset Pricing Model and Arbitrage Pricing Model
    - Using Financial Derivatives: Futures, Forwards and Options
  - **Financial Managers**
    - Investment Decisions: Capital Budgeting Analysis
    - Financing Decisions: Capital Structure and Dividend Policies
  - ....and the interaction among these decisions

# This Subject

**Investment Analysis** is mainly concerned with **where** and **how** to invest

- Valuation of stocks, bonds and derivatives
- Portfolio Diversification
- Asset Pricing and Market efficiency
- These topics (except derivatives) are covered in the first half of this subject ,

**Corporate Finance** is mainly concerned with the **decisions of managers**

- Capital Budgeting – **What** investments to make
- Capital Structure – **How** to finance these investments
- Dividend Policy – **What** to payout to Shareholders
- These topics (along with derivatives) are covered in the second half of this subject.



# Why Study Finance

- To make informed economic decisions
- To better manage existing financial resources and accumulate wealth over time
- To be successful in the business world you need to have an understanding of finance.

# The Finance Function

- The main goal of managers is to maximize the market value of the firm

Value of the firm = Present value of Future expected cash flows

- This maximizes the wealth of shareholders

Shareholder wealth = Present Value of shareholder's future expected cash flows

- Do managers always maximize firm value?
  - What about these?
    - HIH Insurance and OneTel collapses (Australia)
    - Enron and Worldcom collapses (USA)
  - A qualified “YES”?

# The Market Value of the Firm

Firm Value is the Present Value of the Future expected cash flows

$$\text{Firm Value} = \sum_{t=1}^n \frac{E(CF_t)}{(1+k)^t}$$

- $E(CF_t)$  = Expected cash flows received at the end of period  $t$
- $n$  = Number of periods over which cash flows are received
- $k$  = required rate of returns by investors

## Main factors to consider when valuing a firm:

- Magnitude of expected cash flows –  $E(CF_t)$
- Timing of cash flows –  $n$
- Risk of Expected Cash Flows –  $k$
- Efficiency of capital markets

# Introduction to Financial Mathematics 1

## Simple Interest

The value of a cash flow is calculated without including any accrued interest to the principal

**Example:** If you invest \$1,000 at 8% p.a. earning simple interest for 5 years what amount will you have in your account at the end of that time period?

Interest earned in each of the five years  $= 1000 \times 0.08 = \$80$

Interest earned in over five years  $= 1000 \times (5 \times 0.08) = \$400$

Future value at the end of year 5  $= 1000 + 400 = \$1,400$

Future value at the end of year 5  $= 1000 \times (1 + 5 \times 0.08) = \$1,400$

Future value (Simple interest):  $S_n = P_0 \times (1 + n \times i)$

Present value (Simple interest):  $P_0 = S_n / (1 + n \times i)$

# Simple Versus Compounded Interest

## Compound Interest

Interest accrued is added to the principal

The value of a cash flow is calculated based on the principal and interest accrued

**Example:** If you invest \$1,000 at 8% p.a. earning compounded interest for 5 years what amount will you have in your account at the end of that time period?

Future value at the end of year 1  $= 1000 \times (1.08) = \$1,080.00$

Future value at the end of year 2  $= 1080 \times (1.08) = \$1,166.40$

Future value at the end of year 5  $= 1000 \times (1.08)^5 = \$1,469.33$

The difference of **\$69.33** ( $= 1469.33 - 1400.00$ ) is due to the compounding of interest

# Simple Versus Compounded Interest

Amount Invested \$1,000

Interest Rate 8%

End of year	Simple Interest	Compounded Interest	Difference
1	\$1,080.00	\$1,080.00	\$0.00
2	\$1,160.00	\$1,166.40	\$6.40
3	\$1,240.00	\$1,259.71	\$19.71
4	\$1,320.00	\$1,360.49	\$40.49
5	\$1,400.00	\$1,469.33	\$69.33
20	\$2,600.00	\$4,660.96	\$2,060.96
50	\$5,000.00	\$46,901.61	\$41,901.61
100	\$9,000.00	\$2,199,761.26	\$2,190,761.26

## Future Value of a Single Cash Flow

The future value (or sum) at  $i\%$  p.a. of  $\$P_0$  today is the dollar value to which it grows at the end of time period  $n$

$$S_n = P_0 \times (1 + i)^n$$



*Cash flows occur at the end of the period*

## Future Value of a Single Cash Flow

The future value at  $r\%$  p.a. of  $\$P_0$  today is the dollar value to which it grows at the end of time  $n$

$$FVIF_{r,n} = \$1(1 + r)^n$$

FVIF is short for Future Value Interest Factor



*Cash flows occur at the end of the period*



# Future Value of a Single Cash Flow

**Example:** You decide to invest \$1,000 for different time periods. What is the future value of this \$1,000 in 5, 20 and 100 years at an interest rate of (a) 4% and (b) 6%?

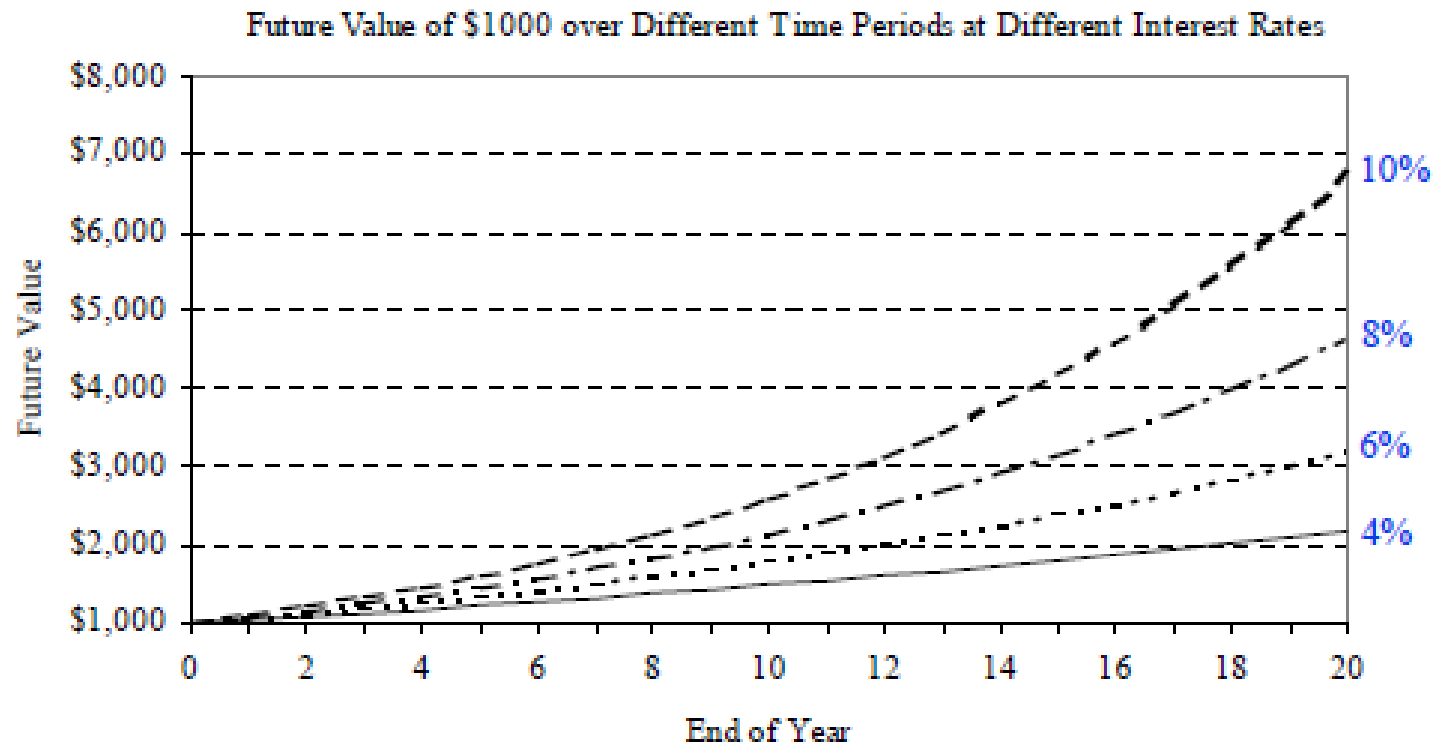
At  $i = 4\%$  p.a.

$$\begin{aligned} S_5 &= 1000 \times (1.04)^5 &&= \$1,217 \\ S_{20} &= 1000 \times (1.04)^{20} &&= \$2,191 \\ S_{100} &= 1000 \times (1.04)^{100} &&= \$50,505 \end{aligned}$$

At  $i = 6\%$  p.a.

$$\begin{aligned} S_5 &= 1000 \times (1.06)^5 &&= \$1,338 \\ S_{20} &= 1000 \times (1.06)^{20} &&= \$3,207 \\ S_{100} &= 1000 \times (1.06)^{100} &&= \$339,302 \end{aligned}$$

# Future Value of a Single Cash Flow



# Future Value of a Single Cash Flow

The future value of a cash flow depends on the following factors:

- The time period,  $n$ 
  - Future value increases as  $n$  increases
- The interest rate,  $i$ 
  - Future value increases as  $i$  increases
- The method of calculating interest
  - Future value increases as the compounding interval increases (more on this later).

## Present Value of a Single Cash Flow

The present value ( $P_0$ ) at  $i\%$  p.a. of  $\$S_n$  at the end of time  $n$  is the amount which invested today would grow to  $\$S_n$  in time  $n$

$$P_0 = S_n / (1 + i)^n = S_n \times (1 + i)^{-n}$$



*Cash flows occur at the end of the period*

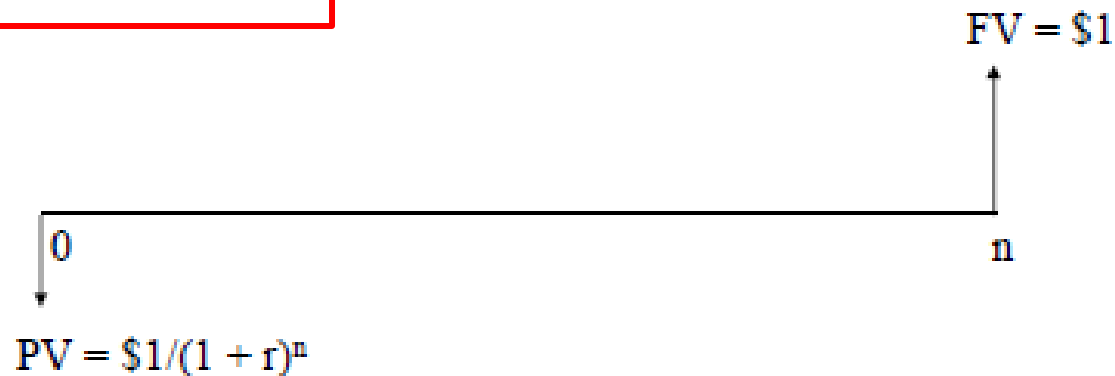
# Present Value of a Single Cash Flow

The present value (PV) at  $r\%$  p.a. of \$1 at the end of time  $n$  is the amount which invested now would grow to \$1 in time  $n$

$$PVIF_{r,n} = \$1/(1 + r)^n = \$1(1 + r)^{-n}$$

PVIF is the short for Present Value Interest Factor

Note:  $PVIF_{r,n} = 1/FVIF_{r,n}$



*Cash flows occur at the end of the period*

# Present Value of a Single Cash Flow

**Example:** If you needed \$10,000 in (a) five years, (b) ten and (c) twenty years how much would you need to save and invest today if the interest rates were (a) 4% and (b) 6%?

**The present value of \$10,000 in five years**

At 4% p.a.,  $P_0 = 10000/(1.04)^5 = \$8,219.27$

At 6% p.a.,  $P_0 = 10000/(1.06)^5 = \$7,472.58$

**The present value of \$10,000 in ten years**

At 4% p.a.,  $P_0 = 10000/(1.04)^{10} = \$6,755.64$

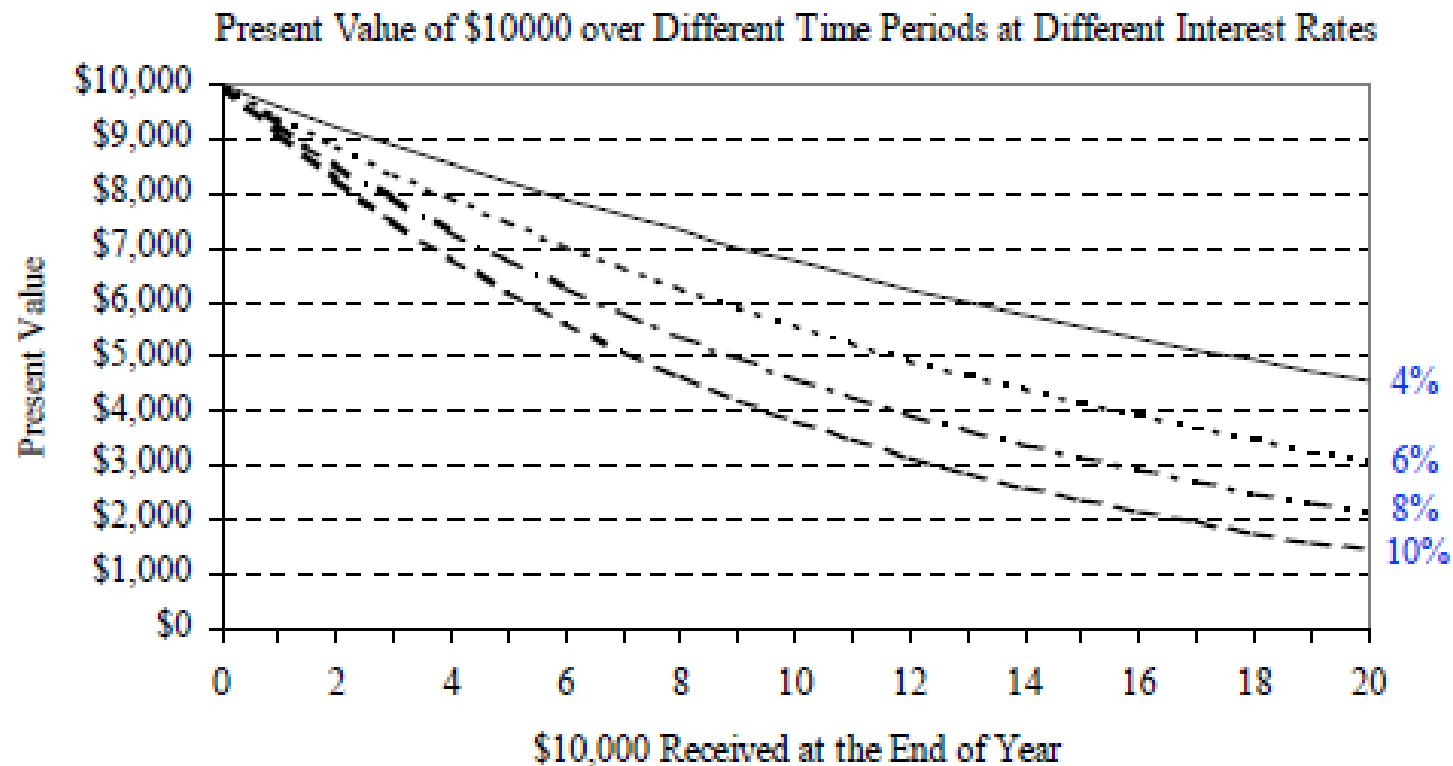
At 6% p.a.,  $P_0 = 10000/(1.06)^{10} = \$5,583.95$

**The present value of \$10,000 in twenty years**

At 4% p.a.,  $P_0 = 10000/(1.04)^{20} = \$4,563.87$

At 6% p.a.,  $P_0 = 10000/(1.06)^{20} = \$3,118.05$

# Present Value of a Single Cash Flow



# Factors Influencing Present and Future Values

The present and future values of a cash flow depend on the following factors

## The time period, $n$

- Future value increases as  $n$  increases
- Present value decreases as  $n$  increases

## The interest rate, $i$

- Future value increases as  $i$  increases
- Present value decreases as  $i$  increases

## The method of calculating interest

- Future value increases as the compounding interval increases
- Present value decreases as the compounding interval increases



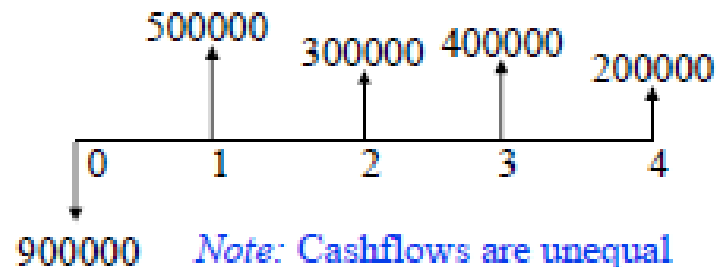
# Net Present Value

Net Present Value (NPV) is defined as the present value (PV) of cash inflows **minus** the present value of cash outflows

$$\text{NPV} = \text{PV}(\text{Cash inflows}) - \text{PV}(\text{Cash outflows})$$

**Class Exercise 1:** Your company is considering investing \$900,000 in a project that is expected to yield the following net cash flows over its four-year life. Assume that the company uses an interest rate of 12% p.a. to evaluate its investments. Should your company make this investment?

<i>Year 1</i>	<i>Year 2</i>	<i>Year 3</i>	<i>Year 4</i>
\$500,000	\$300,000	\$400,000	\$200,000



# Answer to Class Exercise 1

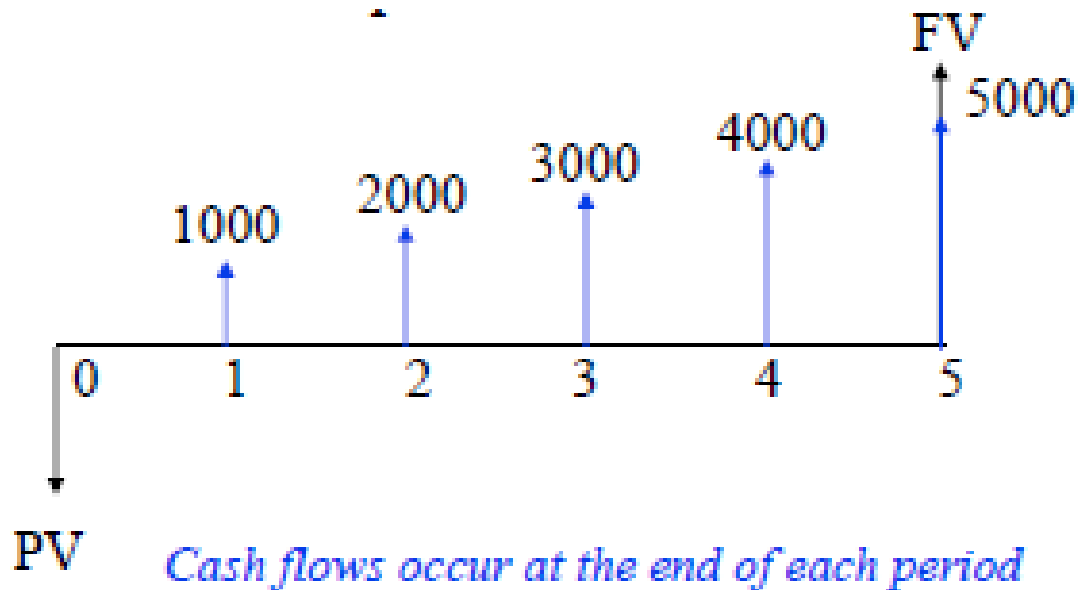
Need to look at whether

$$PV(\text{Cash Inflows}) > PV(\text{Cash Outflows})$$

- Net Present Value =  $PV(\text{Cash Inflows}) - PV(\text{Cash Outflows})$ 
  - Accept project if  $NPV > 0$
  - Reject project if  $NPV < 0$
  - Point of indifference where  $NPV = 0$
- $PV(\text{Cash Inflows}) = 500000/1.12 + 300000/1.12^2 + 400000/1.12^3 + 200000/1.12^4 = \$1,097,402$
- $PV(\text{Cash Outflows}) = \$900,000$
- Net Present Value,  $NPV = \$1,097,402 - \$900,000 = \$197,402 > 0$

## Valuing Unequal Cash Flows

**Class Exercise 2:** You decide to invest \$1,000 at the end of year 1 and then an additional \$1,000 at the end of every year for five years. What is the future value of these cash flows at the end of five years? What equivalent lump-sum amount could you invest today to get this future amount? Assume an interest rate of 10% p.a.



## Answer to Class Exercise 2

To get the total future (present) value of different cash flows occurring at different time periods compute their individual future (present) values and then add across

- Future value of cash flows at the end of five years
  - $FV_5 = 1000 \times (1.10)^4 + 2000 \times (1.10)^3 + 3000 \times (1.10)^2 + 4000 \times (1.10) + 5000$
  - $FV_5 = \$17,156.10$
- Equivalent single amount that could be invested today to get this future amount

- $PV_0 = 1000/(1.10) + 2000/(1.10)^2 + 3000/(1.10)^3 + 4000/(1.10)^4 + 5000/(1.10)^5 = \$10,652.59$

--or equivalently--

- $PV_0 = FV_5 / (1.10)^5 = 17156.10 / (1.10)^5 = \$10,652.59$

# Key Concepts

- Managers should aim to **maximize the value of the firm** by taking on profitable investments. This results in maximum shareholder wealth.
- Money has time value because of **compounded interest**
- In simple interest, the value of a cash flow is calculated **without including any accrued interest to the principal**
- In compounded interest, the value of a cash flow is calculated **based on the principal and interest accrued**
- The present and future values of a cash flow depend on the **time period, interest rate, and method of calculating interest**

# Key Relationships

Future value (Simple interest)

$$S_n = P_0 \times (1 + n \times i)$$

Present value (Simple interest)

$$P_0 = S_n / (1 + n \times i)$$

Future value (or sum) of  $\$P_0$

$$S_n = P_0 \times (1 + i)^n$$

Present value of  $\$S_n$

$$P_0 = S_n / (1 + i)^n = S_n(1 + i)^{-n}$$

# Introduction to Financial Economics

## Objectives

- Compute an unknown interest rate and time period
- Define and compute effective interest rates
- Compute the present value perpetuities
- Compute the present and future values of ordinary annuities
- Compute the present and future values of annuities due
- Applications using financial mathematics

## Unknown Interest Rate or Time Period

**Example:** You invest \$10,000 for a five year period. What interest rate do you need to earn for the funds to double in that time period? If you invest \$10,000 at an interest rate of 10% p.a. how long will it take for these funds to double in value?

In the first case we have an unknown interest rate,  $i$

$$P_0 = \$10,000, S_5 = \$20,000, n = 5, i = ?$$

$$10,000 \times (1 + i)^5 = 20,000$$

$$\text{So, } (1 + i)^5 = 20,000/10,000 = 2$$

$$i = 2^{1/5} - 1 = 14.9\%$$

**Rule of 72:** The approximate interest rate required to double your funds is given as:  $i \approx 72/n$

$$i \approx 72/5 = 14.4\%$$



## Unknown Interest Rate or Time Period

In the second case we have an unknown time period,  $n$

$$P_0 = \$10,000, S_n = \$20,000, i = 10\%, n = ?$$

**Rule of 72:** The approximate time it will take for the funds to double is given as:  $n \approx 72/i$

$$n \approx 72/10 = 7.2 \text{ years (Note: use the interest rate in percentages)}$$

$$\text{Alternatively, } 10,000 \times (1.10)^n = 20,000$$

$$\text{So, } (1.1)^n = 20,000/10,000 = 2$$

$$\text{Taking natural logs we have, } n \times \ln(1.1) = \ln(2)$$

$$\text{So, } n = 0.6931/0.0953 = 7.3 \text{ years}$$

# The Effective Interest Rate

Interest may not always be earned or paid on an annual basis

**Example:** Bank A pays an interest rate of 5% p.a. while Bank B pays an interest rate of 4.9% p.a. but with interest compounded monthly. Which bank's return is better?

The **effective interest rate** (ie) is the annualized rate that takes account of compounding within the year

$$ie = (1 + i/m)^m - 1$$

where  $i/m$  is the per period interest rate

As the compounding becomes more frequent,  $m$  approaches infinity, and in the limit  $(1 + i/m)^m$  approaches  $e^i$

Effective annual rate with continuous compounding:  $ie = e^i - 1$   
(where  $e = 2.71828...$ )

# The Effective Interest Rate

**Example:** Assume the stated interest rate is 5% p.a. What is the effective annual interest rate if interest is paid: (a) semiannually, (b) quarterly, (c) monthly, (d) daily, and (e) continuously? In some markets, (eg. the US) daily compounding is based on a 360 day year. What is the effective interest rate in this case?

Effective interest rate,  $ie = (1 + i/m)^m - 1$

Note:  $i = ie$  only when the compounding interval is one year ( $m = 1$ ), otherwise  $ie$  will always exceed  $i$

The effective annual interest rate **always rises** with the **compounding interval**

# The Effective Interest Rate

## Effective annual interest rates for different compounding intervals

Semi-Annual:  $ie = (1 + 0.05/2)^2 - 1 = 0.0506 = 5.0625\%$

Quarterly:  $ie = (1 + 0.05/4)^4 - 1 = 0.0509 = 5.0945\%$

Monthly:  $ie = (1 + 0.05/12)^{12} - 1 = 0.0512 = 5.1162\%$

Daily:  $ie = (1 + 0.05/365)^{365} - 1 = 0.05127 = 5.1267\%$

Continuous:  $ie = e^{0.05} - 1 = 0.05127 = 5.1271\%$

Daily (360 day basis):  $ie = (1 + 0.05/360)^{365} - 1$   
 $ie = 1.0001389365 - 1 = 5.2\%$

# The Effective Interest Rate

If the interest rate is  $i\%$  p.a. but interest is paid  $m$  times a year, after  $n$  years  $\$P_0$  will have the following future value:

$$S_n = P_0 \times (1 + i/m)^{m \times n}$$

$i/m$  = Per period interest rate

$m \times n$  = Total periods over which interest is compounded

**Example:** Suppose your ancestor saved \$1,000 one hundred years ago with interest compounded monthly. What would its (future) value be today at an interest rate of (a) 5% and (b) 7%?

Here,  $m = 12$  and  $m \times n = 1200$

At 5% p.a.,  $FV = 1000 (1 + 0.05/12)^{1200} = \$146,880$

At 7% p.a.,  $FV = 1000 (1 + 0.07/12)^{1200} = \$1,074,555!$

# Continuous Compounding

The relationship between the present and future values when interest is compounded continuously is:

$$FV = (PV)e^{r n} \text{ and } PV = (FV)e^{-r n}$$

$$\ln(FV) = \ln(PV) + rn$$

$$r = (1/n) \ln (FV/PV)$$

$\ln (FV/PV)$  is the log price relative

**Example:** If FV in 1 year = \$110, PV = \$105, the interest rate is  
 $r = \ln(110/105) = \ln(1.0476) = 0.0465$  or **4.65%**

# Continuous Compounding

**Class Exercise 1:** Your great-grandmother invested \$1,000, one hundred years ago earning continuously compounded interest

- a) How much is this amount worth today if the interest rate is 5% and 7%?
- b) What are the effective interest rates for the above stated rates when interest is continuously compounded?

## Answer to Class Exercise 1

a) The (future) value of the \$1,000 with continuous compounding is:

At 5% p.a.,  $FV = 1000(e^{0.05})^{100} = \$148,413$

At 7% p.a.,  $FV = 1000(e^{0.07})^{100} = \$1,096,633$

Difference between monthly and continuous compounding at 7% is  
 $\$10,966.33 - \$10,745.55 = \$22,078$

b) The effective interest rates are

At 5%,  $re = e^{0.05} - 1 = 0.05127 = 5.127\% \text{ p.a.}$

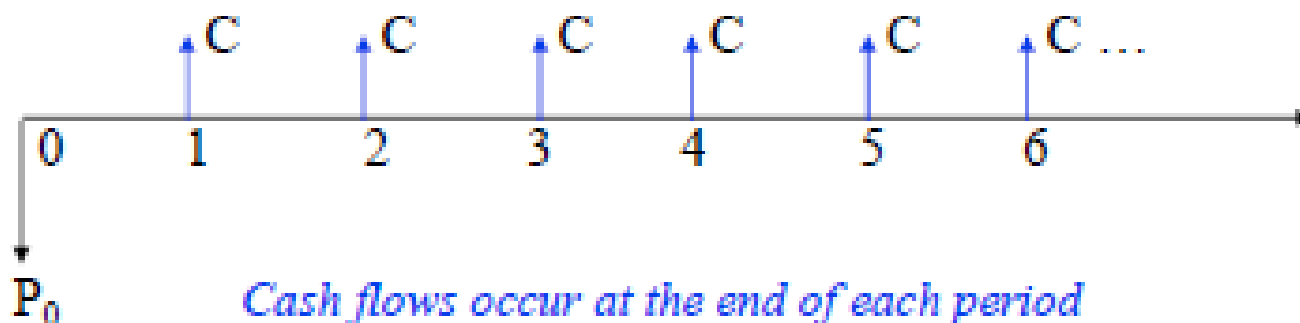
At 7%,  $re = e^{0.07} - 1 = 0.07251 = 7.251\% \text{ p.a.}$

**Note:** The difference in the effective interest rate between daily and continuous compounding is very small



# Valuing Perpetuities and Annuities

A **perpetuity** is a equal, periodic cash flow that goes on forever



The present value of a perpetuity is

$$P_0 = C / (1+i) + C / (1+i)^2 + \dots + C / (1+i)^n + C / (1+i)^{n+1} + \dots$$

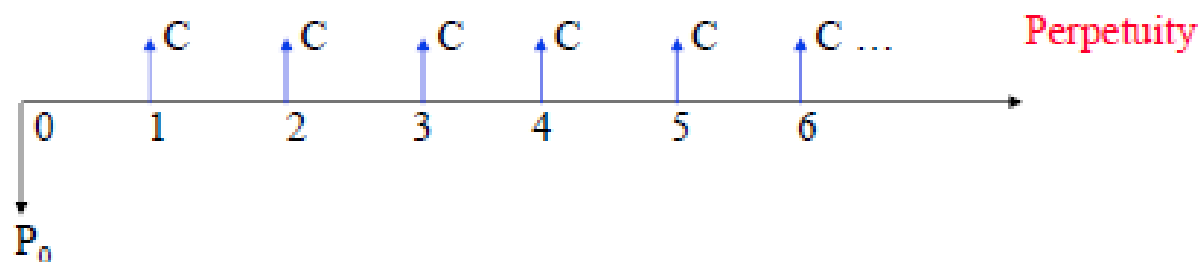
$$P_0 = C [1/(1+i) + 1/(1+i)^2 + \dots + 1/(1+i)^n + 1/(1+i)^{n+1} + \dots]$$

As  $n$  approaches  $\infty$ ,  $[1/(1+i) + 1/(1+i)^2 + \dots + 1/(1+i)^n + \dots]$  approaches  $1/i$

So, the present value of a perpetuity,  $P_0 = C / i$

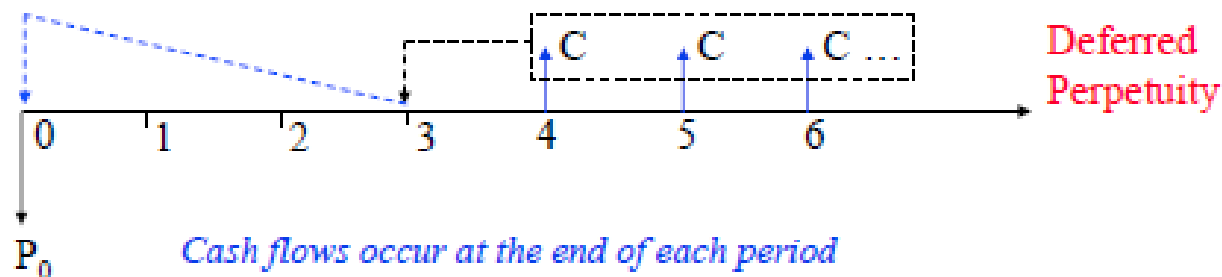
# Valuing Perpetuities and Annuities

The present value of a perpetuity,  $P_0 = C / i$



A **deferred perpetuity** is a perpetuity that starts at some future date and then goes on forever

The present value of a deferred perpetuity,  $P_0 = [C / i] / (1 + i)^n$



# Valuing Perpetuities and Annuities

**Example:** A prize guarantees you \$1,000 per year forever with the first payment to be made at the end of year 1. How much would you sell the prize for if the interest rate were 10% p.a.?

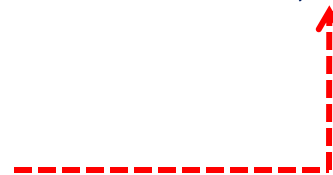
What would the perpetuity's value be if were deferred to year 4 (i.e., the first cash flow occurred at the end of year 4 and not year 1)? (See the time line in the previous slide)

Present value of perpetuity,  $P_0 = 1000 / 0.10 = \$10,000$

Present value of deferred perpetuity

$$P_3 = 1000 / 0.10 = \$10,000$$

$$P_0 = 10000 [1/(1.1)^3] = \$7,513.15$$

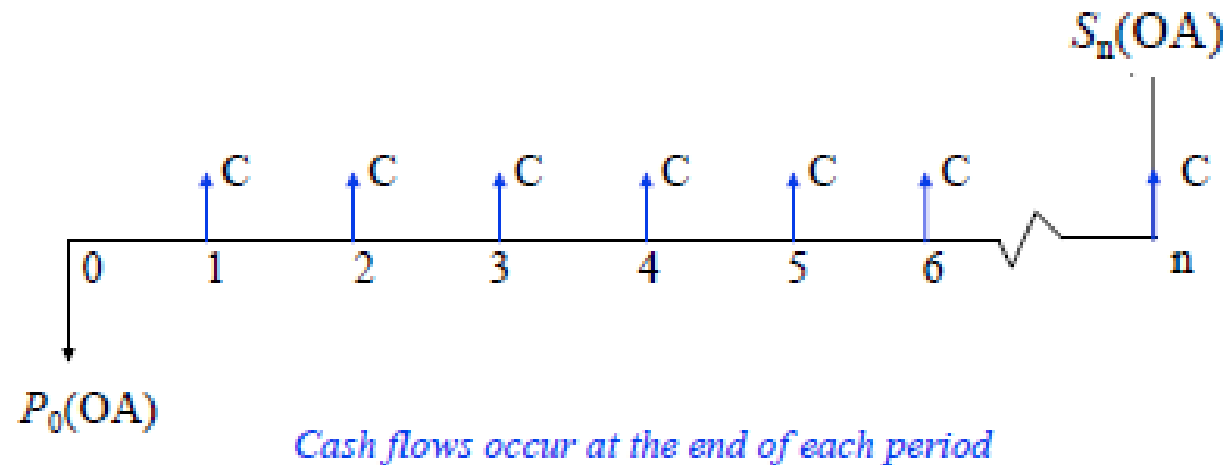


# Valuing Ordinary Annuities

An **ordinary annuity** is a series of equal, periodic cash flows occurring at the end of each period and lasting for  $n$  periods

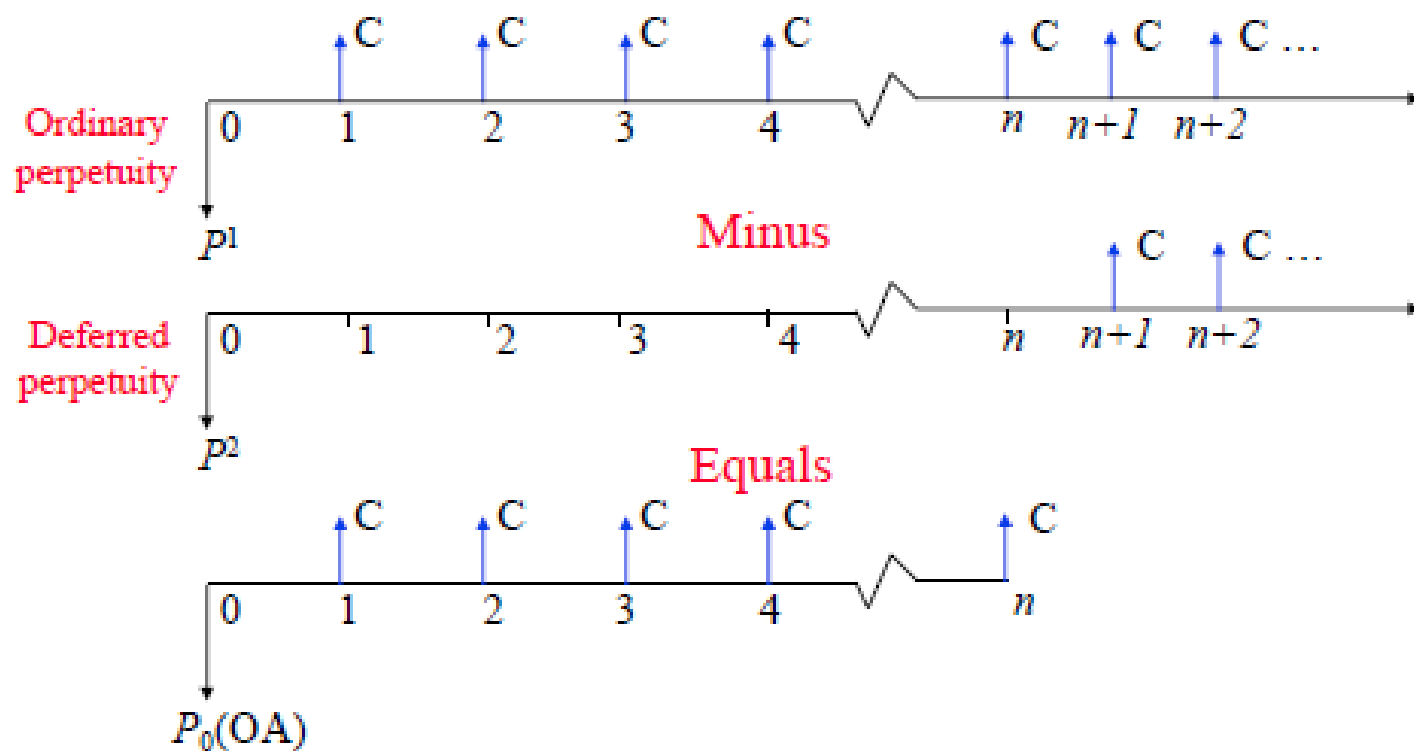
**Note:** The first cash flow occurs at the end of period 1 and the last cash flow occurs at the end of period  $n$

**Ordinary annuities** can be valued as the difference between two perpetuities



# Valuing Ordinary Annuities

An  $n$  year annuity can be valued as the difference between two perpetuities,  $P_0(\text{OA}) = P^1 - P^2$



## Valuing Ordinary Annuities

The **present value** of a \$C annuity can be obtained as the difference between an **ordinary perpetuity** and a **deferred perpetuity**

$$P_0(\text{OA}) = P^1 - P^2$$

$$P_0(\text{OA}) = C / i - [C / i][1/(1 + i)^n]$$

Simplifying the above equation, we get

$$P_0(\text{OA}) = [C / i][1 - 1/(1 + i)^n] = [C / i][1 - (1 + i)^{-n}]$$

The **future value** of a \$C annuity can be obtained by taking the future value of the above present value to time period n

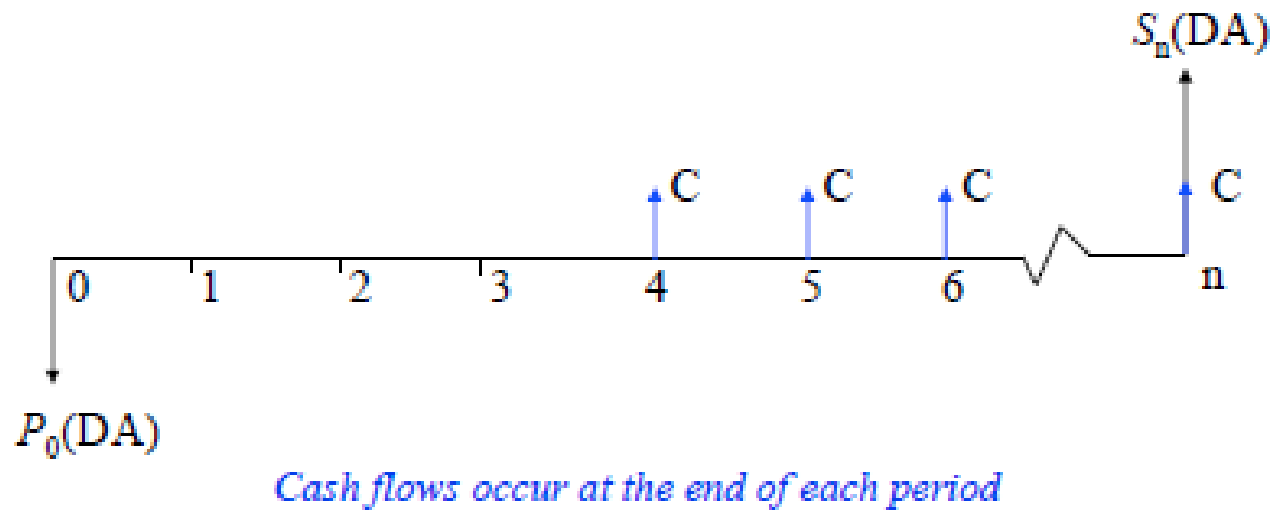
$$S_n(\text{OA}) = [C / i][1 - 1/(1 + i)^n](1 + i)^n$$

Simplifying the above equation, we get

$$S_n(\text{OA}) = [C / i][(1 + i)^n - 1]$$

# Valuing Deferred Ordinary Annuities

A **deferred ordinary annuity** is a series of equal, periodic cash flows occurring at the end of each period where the first cash flow occurs at a future date.



## Valuing Deferred Ordinary Annuities

**Class Exercise 2:** Suppose you invest \$1,000 every year for (i) 10 years and (ii) 50 years earning an annual return of 10%.

- a) What is each investment's value at the point where you stop investing?
- b) What is the present worth of your investments in part (a)?
- c) What is the relationship between the future and present values calculated in parts (a) and (b)?
- d) What are the present and future values at the end of year 10 assuming that you only invest funds in years 6 - 10? (That is, no funds are invested during years 1 - 5)



## Answer to Class Exercise 2

a) Future values at the end of 10 and 50 years

In 10 years:  $S_{10} = [1000/0.1][1.1^{10} - 1] = \$15,937$

In 50 years:  $S_{50} = [1000/0.1][1.1^{50} - 1] = \$1,163,909$

b) Present values of the above investments

Over 10 years:  $P_0 = [1000/0.1][1 - 1.1^{-10}] = \$6,145$

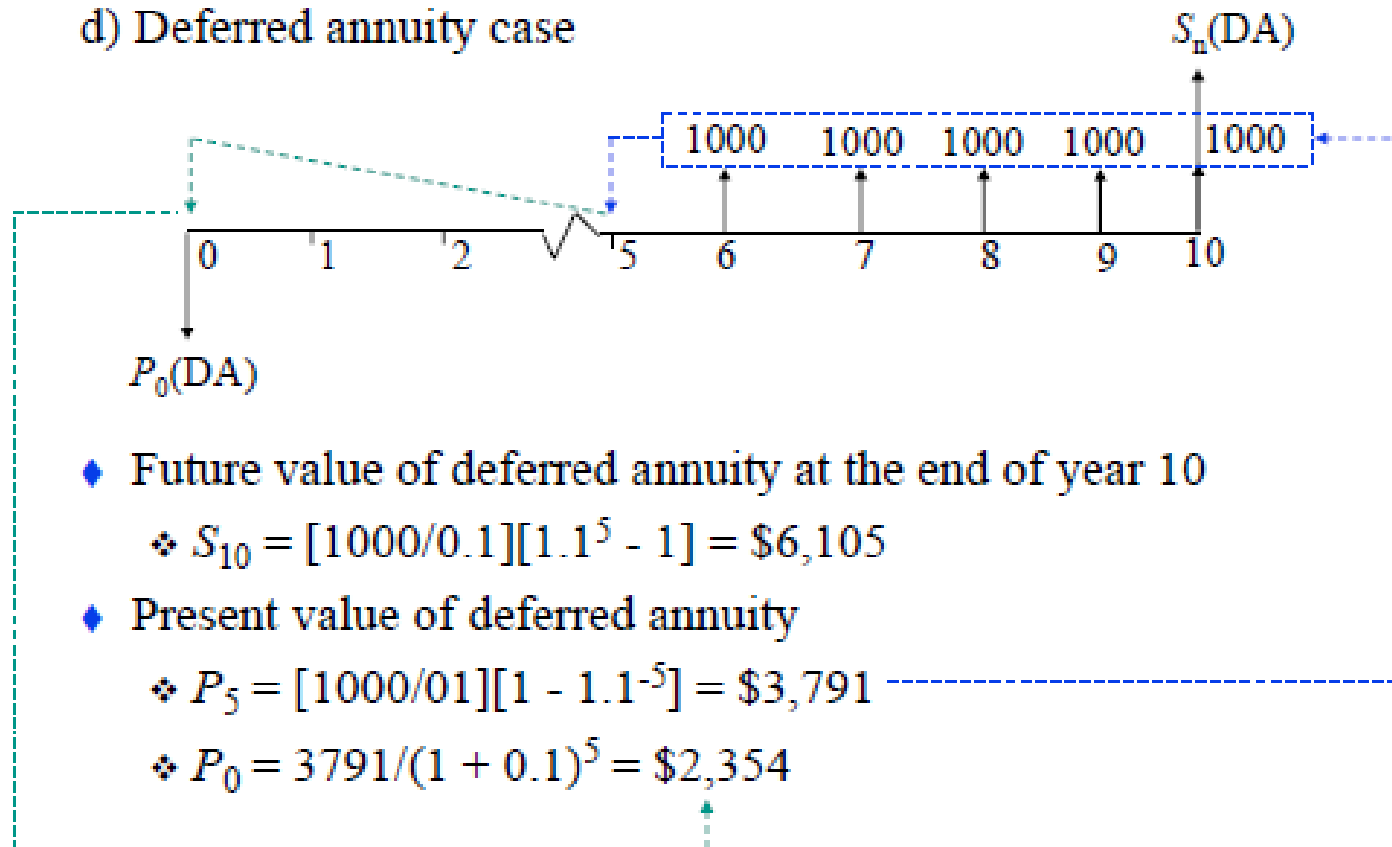
Over 50 years:  $P_0 = [1000/0.1][1 - 1.1^{-50}] = \$9,915$

c) If you had invested \$9,915 for a 50 year period, it would be worth \$1,163,909 at a 10% return p.a.

$9915(1.1)^{50} = \$1,163,930$  (rounding error)

## Answer to Class Exercise 2

d) Deferred annuity case



- ◆ Future value of deferred annuity at the end of year 10

$$\diamond S_{10} = [1000/0.1][1.1^5 - 1] = \$6,105$$

- ◆ Present value of deferred annuity

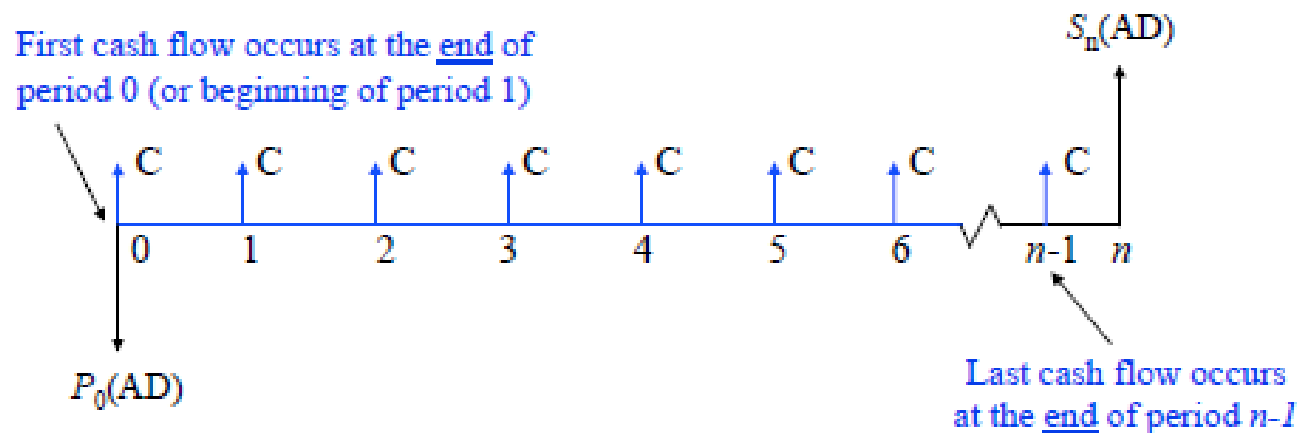
$$\diamond P_5 = [1000/0.1][1 - 1.1^{-5}] = \$3,791$$

$$\diamond P_0 = 3791/(1 + 0.1)^5 = \$2,354$$

# Valuing Annuities Due

An **annuity due** is a series of **equal, periodic cash flows** occurring at the **beginning of each period**

Note: The beginning of period  $t$  is the same as the end of period  $t-1$ . The overall effect is to move the annuity back one period on our standard (end of period) time line



*Cash flows occur at the end of each period*

## Valuing Annuities Due

The present value of a \$C annuity due at i% p.a. is equivalent to the present value of an ordinary annuity compounded one additional period

$$P_0(AD) = [C / i][(1 - (1 + i)^{-n})[1 + i]]$$

The future value of a \$C annuity due at i% p.a. is equivalent to compounding by one additional period the future value of an ordinary annuity

$$S_n(AD) = [C / i][(1 + i)^n - 1][1 + i]$$

**Class Exercise 3:** In Class Exercise 2, suppose the \$1,000 was invested every year at the beginning of each year. What are the future and present values of these investment earning a 10% p.a. return over (a) 10 years and (b) 50 years?

## Answer to Class Exercise 3

Future values at the end of 10 and 50 years are now

In 10 years:  $S_{10} = [1000/0.1][1.1^{10} - 1][1.1] = \$17,530$

In 50 years:  $S_{50} = [1000/0.1][1.1^{50} - 1][1.1] = \$1,280,300$

Present values of the above investments

Over 10 years:  $P_0 = [1000/0.1][1 - 1.1^{-10}][1.1] = \$6,759$

Over 50 years:  $P_0 = [1000/0.1][1 - 1.1^{-50}][1.1] = \$10,906$

# Valuing Perpetuities

A perpetuity is an annuity that goes on forever and ever and...

$$PV = C/r$$

A perpetuity growing at  $g\%$  p.a., compounded at  $r\%$  p.a. has a present value of

$$PV = C / (r - g)$$

Notes: The first cash flow occurs at the end of year 1

The above relationship requires that  $r > g$

**Class Exercise 4:** A prize guarantees \$1,000 per year forever with the first payment to be made at the end of year 1.

a) What would you sell the prize to your lecturer for if the interest rate were 10% p.a.?

b) If a prize guaranteed \$1,000 per year forever (first cash flow to be paid in year 1) growing at 5% p.a., what would its worth be today if the interest rate were 10% p.a.?

## Answer to Class Exercise 4

a) Present value =  $1000 / 0.10 = \$10,000$

If the prize paid \$1,000 per year for 60 years its present value would be

$$PV = 1000(PVIFA_{10, 60}) = \$9,967$$

b) Present value now =  $1000 / (0.10 - 0.05) = \$20,000$

Note: The difference of \$10,000 in parts (a) and (b) is referred to as the present value of growth opportunities (more on this later)

## Financial Mathematics Application

**Application:** You have borrowed \$20,000 from your bank with the loan to be repaid in equal annual installments. Your bank charges an annual interest rate of 10% p.a. with interest compounded annually

- a) What annual payment would you be making on this loan?
- b) Develop a loan amortization schedule for this loan. Then using this table obtain the following information
  - (i) The principal balance outstanding at the end of year 1
  - (ii) The total interest paid in year 2
  - (iii) The total principal repaid in year 3



## Financial Mathematics Application

a) What annual payments would you be making during the next four years?

Loan amount at any point = PV(Remaining payments)

$$20000 = \text{Payment} \times [1 - 1.1^{-4}]/0.1 = \text{Payment} \times (3.1699)$$

$$\text{Payment} = 20000/3.1699 = \$6,309 \text{ (rounded)}$$

b) The loan amortization schedule shows the interest paid, principal repaid and principal remaining over the loan's duration. It uses the following relationships:

Interest paid = (Previous period's principal)  $\times$  (Interest rate)

Principal repaid = Loan Payment - Interest paid

Principal remaining = Previous period's principal – Principal repaid

# Financial Mathematics Application

## b) Loan amortization schedule

Year	Annual Payment	Interest Paid <sup>1</sup>	Principal Repaid <sup>2</sup>	Principal Remaining <sup>3</sup>
0				\$20,000
1	\$6,309	\$2,000	\$4,309	\$15,691 (i)
2	\$6,309	\$1,569 (ii)	\$4,740	\$10,951
3	\$6,309	\$1,095	\$5,214 (iii)	\$5,732
4	\$6,309	\$574	\$5,735	\$0
Totals	\$25,238	\$5,238	\$20,000	

<sup>1</sup> Interest paid = Previous period's principal  $\times$  Interest rate

<sup>2</sup> Principal repaid = Loan Payment - Interest paid

<sup>3</sup> Principal remaining = Previous period's principal - Principal repaid

*Note:* The principal outstanding at the end of year 1 can also be computed as the present value of the *remaining* payments,  $P_1 = \text{Payment} \times [1 - 1.1^{-3}]/0.1$

## Key Concepts

- The effective interest rate is the annualized rate that takes account of compounding within the year
- Ordinary annuities are periodic, end-of-the-period cash flows  
Deferred ordinary annuities are periodic, end-of-the-period cash flows that start at a future date
- Annuities due are periodic, beginning-of-the-period cash flows
- The future value of an annuity is the sum of the future values of each cash flow compounded at the relevant interest rate
- The present value of an annuity is the sum of the present values of each cash flow discounted at the relevant interest rate

# Key Concepts

- The total future (present) value of different cash flows occurring at different time periods equals the sum of their individual future (present) values
- The effective interest rate is the annualized rate that takes account of compounding within the year
- Ordinary annuities are periodic, end-of-the-period cash flows
- Deferred ordinary annuities are periodic, end-of-the-period cash flows that start at a future date
- Annuities due are periodic, beginning-of-the-period cash flows
- The future value of an annuity is the sum of the future values of each cash flow compounded at the relevant interest rate
- The present value of an annuity is the sum of the present values of each cash flow discounted at the relevant interest rate

## Key Relationships

- Future value (or sum) of \$P<sub>0</sub> today:  $S_n = P_0 \times (1 + i)^n$
- Present value of \$S<sub>n</sub> at time n:  $P_0 = S_n / (1 + i)^n = S_n(1 + i)^{-n}$
- Rule of 72:  $i \approx 72/n$
- Effective interest rate:  $i_e = (1 + i/m)^m - 1$
- Present value of a perpetuity:  $C/i$
- Present value of a deferred perpetuity:  $[C/i] / (1 + i)^n$
- Present value of ordinary annuity:  $P_0(OA) = [C/i][1 - 1/(1 + i)^n]$
- Future value of ordinary annuity:  $S_n(OA) = [C/i][(1 + i)^n - 1]$
- Present value of an annuity due:  $P_0(AD) = [C/i][(1 - (1 + i)^{-n})[1 + i]]$
- Future value of an annuity due:  $S_n(AD) = [C/i][(1 + i)^n - 1][1 + i]$