
FINA 1082 Financial Management

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Lecture 11

Capital Budgeting/Project Evaluation II

December, 7, 2011

Comparing the IRR and NPV Methods

- **Independent** projects are projects that can be evaluated on their own and independently of each other
 - The decision to accept a project does not affect the decision to accept or reject other projects
 - Assumes that there are enough funds for all potential projects being considered
 - **Examples:** A development of two separate pieces of land, or expanding the Melbourne and London offices
- **Decision rule for independent projects**
 - Invest in **all** positive NPV projects
- **Does the IRR rule work the same way here?**

Comparing the IRR and NPV Methods

Example: Consider two projects with the following pattern of net cash flows and a required rate of return of 10%. What decision would the firm make if the projects are independent?

Year	Project A	Project B
0	-\$120,000	-\$120,000
1	\$100,000	\$10,000
2	\$50,000	\$60,000
3	\$15,000	\$120,000
<i>NPV</i>	\$23,501	\$28,835
<i>IRR</i>	24.8%	19.8%

Comparing the IRR and NPV Methods

- **Mutually exclusive projects** are projects where the acceptance of one project **rules out** the acceptance of other (competing) projects
 - **Example:** A piece of land is used to build a factory, which rules out an alternate project of building a warehouse on the same land
- **Decision rule for mutually exclusive projects**
 - Assuming the projects being considered are worth undertaking (that is, they are positive NPV projects).
 - Invest in the **highest** NPV project

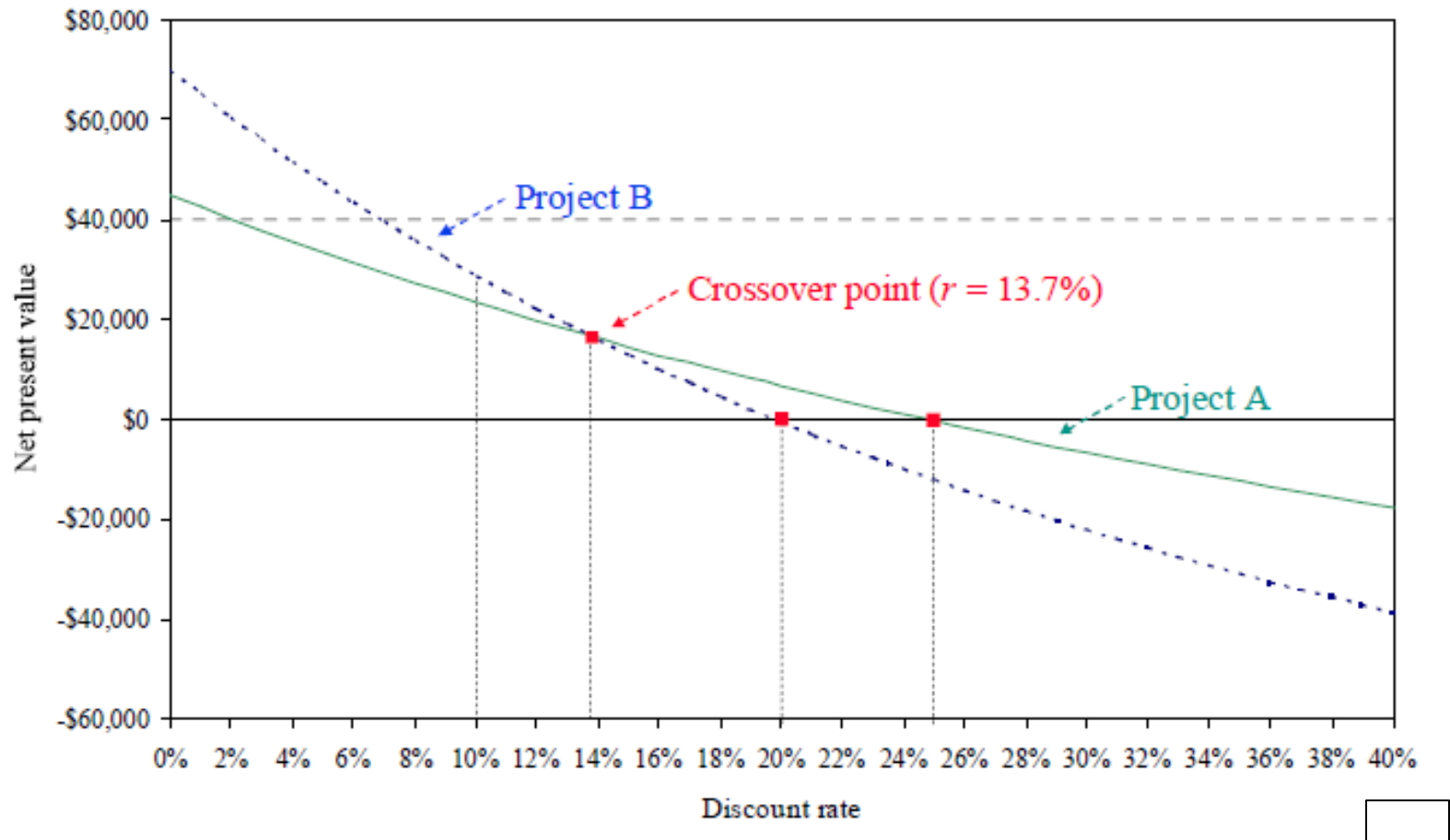
Does the IRR rule work the same way here?

Comparing the IRR and NPV Methods

Example: Consider two projects with the following pattern of net cash flows and a required rate of return of 10%. What decision would the firm make if the projects are mutually exclusive?

Year	Project A	Project B
0	-\$120,000	-\$120,000
1	\$100,000	\$10,000
2	\$50,000	\$60,000
3	\$15,000	\$120,000
<i>NPV at 10%</i>	\$23,501	\$28,835
<i>IRR</i>	24.8%	19.8%

Comparing the IRR and NPV Methods



Comparing the IRR and NPV Methods

- For mutually exclusive projects the IRR and NPV methods can be made consistent by considering the incremental “projects” A - B or B – A.
- Look at the difference in net cash flows of the lower IRR project and the net cash flows of the higher IRR project

Year	Project A	Project B	“Project” B - A
0	-\$120,000	-\$120,000	\$0
1	\$100,000	\$10,000	-\$90,000
2	\$50,000	\$60,000	\$10,000
3	\$15,000	\$120,000	\$105,000
<i>NPV</i>	\$23,501	\$28,835	–
<i>IRR</i>	24.8%	19.8%	13.7%

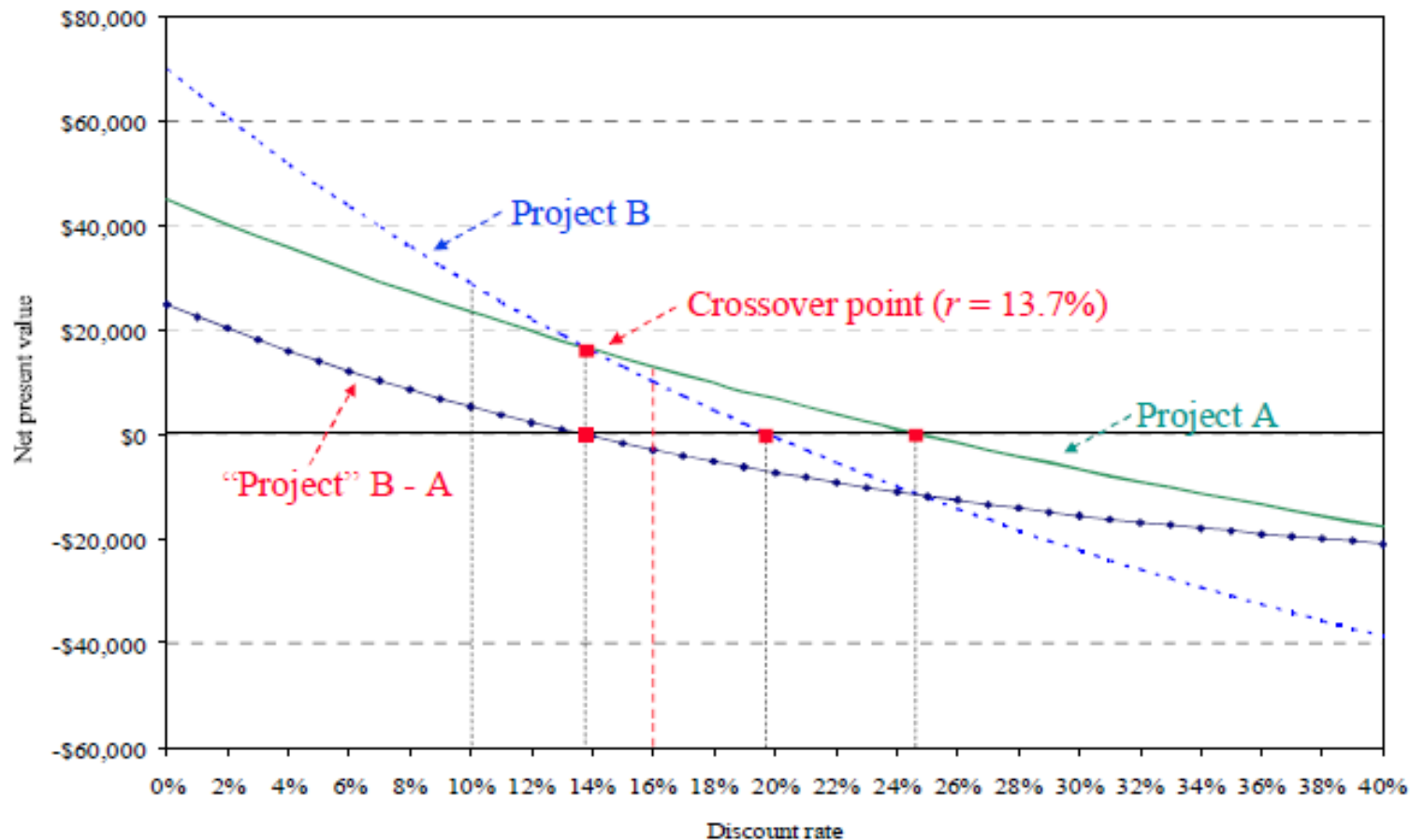
Comparing the IRR and NPV Methods

Is it worth investing in the lower IRR project B in preference to the higher IRR project A?

Year	Project A	Project B	“Project” B - A
0	-\$120,000	-\$120,000	\$0
1	\$100,000	\$10,000	-\$90,000
2	\$50,000	\$60,000	\$10,000
3	\$15,000	\$120,000	\$105,000
<i>NPV</i>	\$23,501	\$28,835	–
<i>IRR</i>	24.8%	19.8%	13.7%

- ◆ $NPV_{B-A} \equiv 0 = 10000/(1 + r_{B-A}) + 105000/(1 + r_{B-A})^2 - 90000$
- ◆ So, $r_{B-A} = 13.7\% > 10.0\%$
- ◆ Which method is preferable - *NPV* or incremental *IRR*?

Comparing the IRR and NPV Methods



What would the decision be if the discount rate was 16%?

Payback Period

- A project's payback period is the time it takes for the initial cash outlay on a project to be recovered from the net after-tax cash flows
 - Note that in computing the payback period we assume that the cash flows are distributed *evenly over the year (rather than at the end of each year)*
- *Decision rule*
 - A project is acceptable if its payback period is less than a prespecified maximum payback period
 - For mutually exclusive projects, the project with the shortest payback period is preferred (assuming they all meet the maximum payback period threshold)

Payback Period

Example: A firm is considering three mutually exclusive projects that require an initial outlay of \$100,000 and that generate the following pattern of cash flows. The firm typically accepts projects with a payback period less than 2 years

Project	Year 1	Year 2	Year 3	Year 4	Payback
C	\$100,000	-	-	\$10,000	1 year
D	\$50,000	\$50,000	\$50,000	\$50,000	2 years
E	\$50,000	\$30,000	\$30,000	\$90,000	2.7 years
F	\$50,000	-\$30,000	\$60,000	\$40,000	3.5 years

- Payback for project E = $2 + 20/30 = 2.7$ years
- Payback for project F = $3 + 20/40 = 3.5$ years

Decision?

Problems With Payback Period

- Fails to take account of the cash flows that occur after the payback period cutoff date
- Biased against projects that have longer development periods
 - **Examples:** Mining and exploration projects
- Ignores the time value of money
- Is there any use for the payback period and accounting rate of return methods?
- What method(s) should a company use?

Key Concepts

- The NPV method is recommended for investment evaluation
- NPV is consistent with maximization of shareholder wealth
- NPV is also simple to use and gives rise to fewer problems than the IRR method
- In practice, other valuation methods such as the accounting rate of return and payback period are used in conjunction with NPV, despite their inferiority

Capital Budgeting II

- Examine the issues related to the estimation of incremental cash flows
- Examine the role of inflation in the capital budgeting process
- Analyze mutually exclusive projects with different lives
- The weighted average cost of capital
- Estimate the weighted average cost of capital
- Use the weighted average cost of capital in capital budgeting
- Examine the limitations of the weighted average cost of capital

Issues in Cash Flow Estimation

- Timing of cash flows

- The exact timing of project cash flows can affect the valuation of a project
- The simplifying assumption used is that the net cash flows are received at the end of a period

- Financing charges

- Cash outflows relating to how the project is to be financed are not included in the analysis
- The value of a project is independent of how it will be financed
- The discount rate used represents the rate of return required by equityholders, debtholders and other securityholders
- Financing costs are not used in the cash flows because that results in their being double counted!

Issues in Cash Flow Estimation

- **Incremental cash flows**
 - Only cash flows that change if the project is accepted are relevant in evaluating a project
 - Need to be careful with sunk costs and allocated costs
- **Sunk costs**
 - These costs are not included as they have been incurred in the past and will not be affected by the project's acceptance or rejection
- **Allocated costs**
 - Overhead costs allocated by management to firm's divisions
 - These costs do not vary with the decision and are usually ignored
 - Examples are administrative costs incurred by head office and allocated to divisions

Issues in Cash Flow Estimation

- Taxes and tax effects

- Taxes need to be included where they have an effect on the net cash flows generated by a project

- Taxes have *three* main effects on cash flows

- Corporate taxes
- Depreciation tax shield
- Taxes on disposal of assets

- Corporate income tax

- Corporate taxes should be included as a cash outflow
- After tax cash flow = Before tax cash flow $\times (1 - t_c)$
- t_c = The effective corporate tax rate

Issues in Cash Flow Estimation

- Depreciation tax savings or depreciation tax shield
 - Depreciation itself is **not** an operating expense and is excluded from the net cash flows
 - However, depreciation affects net cash flows as it decreases the taxes payable due to the depreciation tax shield
 - Depreciation tax savings (or shield) = $t_c \times \text{Depreciation expense}$

Issues in Cash Flow Estimation

- Disposal or salvage value of assets
 - The disposal or salvage value of assets needs to be taken into account **after taxes**
 - Taxes are payable when an asset is sold for **more** than its book value
 - There is a tax saving when an asset is sold for **less** than its book value as the loss can be offset against taxable income
- Book value = Acquisition cost - Accumulated depreciation
- Gain (or loss) = Disposal value - Book value
- Taxes payable of gain = $t_c \times \text{Gain on sale}$
- Tax saving on loss = $t_c \times \text{Loss on sale}$

Issues in Cash Flow Estimation

The incremental net after-tax cash flows are...

$$C_t = (R_t - OC_t - D_t)(1 - t_c) + D_t$$

Alternatively, the incremental net after-tax cash flows are...

$$C_t = (R_t - OC_t)(1 - t_c) + t_c D_t$$

Separating the after-tax residual or salvage value (SV), we get...

$$NPV = \sum_{t=1}^N \left[\frac{(R_t - OC_t)(1 - t_c) + t_c D_t}{(1 + k)^t} \right] + \frac{SV_N}{(1 + k)^N} - I_0$$

Note: SV_N = After-tax residual (or salvage) value

Issues in Cash Flow Estimation

Example: Suppose the initial cost of a machine is \$12,000. It has a useful life of 6 years and will be depreciated on a straight line basis over 6 years. The machine will generate a before-tax cash flows of \$6,000 per year over its useful life and the effective corporate tax rate is 30%. If at the end of year 4 the company sells the machine for \$5,000 what is the net cash flow in year 4 of the machine's life?

The net after tax cash inflows are...

$$C_4 (R_4 - OC_4)(1 - t_c) = 6000(1 - 0.3) = \$4,200$$

Issues in Cash Flow Estimation

The depreciation tax saving in year 4 is...

$$t_c D_4 = 0.3 \times 2000 = \$600$$

The book value of assets in year 4 is...

End of Year	Depreciation	Accumulated Depreciation	Book Value
0			\$12,000
1	\$2,000	\$2,000	\$10,000
2	\$2,000	\$4,000	\$8,000
3	\$2,000	\$6,000	\$6,000
4	\$2,000	\$8,000	\$4,000

Issues in Cash Flow Estimation

- Before tax proceeds from the sale of the machine = \$5,000
- Taxes on proceeds from sale of the machine are...
 - Total gain = Disposal value . Book value
 - Total gain = 5000 – 4000 = \$1,000
 - Taxes payable = $0.3 \times 1000 = \$300$
- The net, after-tax salvage value is.
 - $SV_4 = 5000 - 300 = \$4,700$

Issues in Cash Flow Estimation

The total after tax net cash flow in year 4 is...

$$C_4 = (R_4 - OC_4)(1 - t_c) + t_c D_4 + SV_4$$

After tax net cash inflow	\$4,200	$(R_4 - OC_4)(1 - t_c)$
Depreciation tax saving	\$600	$t_c D_4$
Proceeds from sale	\$5,000	} SV_4
Taxes payable on gains	-\$300	
Total after tax net cash flow	\$9,500	

Inflation and Capital Budgeting

- It is important to be consistent in your treatment of **inflation**
 - For **nominal** cash flows use the **nominal** discount rate
 - For **real** cash flows use the **real** discount rate
- From the Fisher relationship we have...

$$\diamond (1 + r) = (1 + r^*)(1 + i) \quad \text{or}$$

$$\diamond (1 + r^*) = (1 + r)/(1 + i)$$

- $\diamond r$ = Nominal rate of return (or nominal interest rate) per annum
- $\diamond r^*$ = Real rate of return (or real interest rate) per annum
- $\diamond i$ = Expected inflation rate per annum

Inflation and Capital Budgeting

Example: *A one-year project is expected to generate net after tax cash flows of \$1,000,000 in real terms one year from now.*

The nominal discount rate is 10% p.a. and the expected inflation rate is 2% p.a. If the project costs \$900,000 what is the net present value of the project?

- Two methods can be used.
 - Use **real** net cash flows and discount with the **real** discount rate
 - Use **nominal** net cash flows and discount with the **nominal** discount rate
- Don't mix these up!
 - Real cash flows discounted using the nominal discount rate
 - Nominal cash flows discounted using the real discount rate

Inflation and Capital Budgeting

Method 1: Use real discount rate with real cash flows

- ❖ Real rate, $r^* = (1 + r)/(1 + i) - 1 = 1.10/1.02 - 1 = 7.84\%$
- ❖ Present value of cash inflows = $1000/1.0784 = \$927.27$
- ❖ Net present value = $927.27 - 900 = \$27.27$

Method 2: Use nominal discount rate with nominal cash flows

- ❖ The nominal cash flow = $1000(1.02) = \$1020$
- ❖ Present value of cash inflows = $1020/1.10 = \$927.27$
- ❖ Net present value = $927.27 - 900 = \$27.27$

Projects With Different Lives

Example: Consider the following two mutually exclusive projects and assume a 10% p.a. discount rate. Which machine should the firm choose at this discount rate?

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000
3	\$40,000	
<i>NPV (at 10%)</i>	\$25,920	\$11,157

We **cannot** compare the NPVs of the projects because project B lasts only 2 years

Projects With Different Lives

- We assume that both projects are repeated with identical projects until they achieve a common duration (or life)
 - The constant chain of replacement assumption
- The constant chain of replacement assumption can be applied using two different methods...
 - The lowest common multiple method
 - The perpetuity method
 - Both methods will give the same decision

Projects With Different Lives

- Constant chain of replacement using the lowest common multiple method
 - Replicate the projects until they achieve the same “lives”
 - A 2-year versus a 3-year project is replicated 3 and 2 times...
 - A 2-year versus a 4-year project is replicated 2 and 1 time(s)...
 - A 4-year versus a 5-year project is replicated 5 and 4 times...

Projects With Different Lives

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000 – \$50,000
3	\$40,000 – \$100,000	\$40,000
4	\$60,000	\$30,000 – \$50,000
5	\$50,000	\$40,000
6	\$40,000	\$30,000

NPV for repeated project A = \$45,390

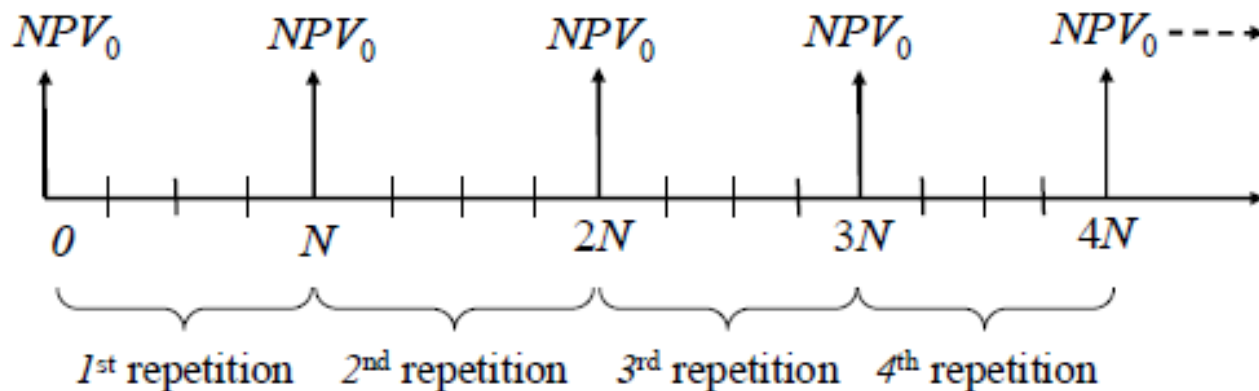
NPV for repeated project B = \$28,000

Projects With Different Lives

- The lowest common multiple method can be cumbersome
 - **Example:** Two projects with lives of 11 and 13 years require present value calculations for 143 ($= 11 \times 13$) years!
- **Constant chain of replacement in perpetuity method**
 - Assumes that both projects are replicated **forever**
 - The chains of cash flows are of “**equal length**” because they are both of infinite length
 - Method is generally easier to use

Projects With Different Lives

- Consider a project with a life of N years and a required rate of return of $k\%$ p.a. Its net present value is NPV_0
- The project repeated forever will have the following profile of NPVs



Projects With Different Lives

The net present value of the infinite chain (NPV_{∞}) is...

$$NPV_{\infty} = NPV_0 + \frac{NPV_0}{(1+k)^N} + \frac{NPV_0}{(1+k)^{2N}} + \dots + \frac{NPV_0}{(1+k)^{\infty}}$$

$$NPV_{\infty} = NPV_0 \left[1 + \frac{1}{(1+k)^N} + \frac{1}{(1+k)^{2N}} + \dots + \frac{1}{(1+k)^{\infty}} \right]$$

$$NPV_{\infty} = NPV_0 \left[\frac{1}{1 - \frac{1}{(1+k)^N}} \right] = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1}$$

Projects With Different Lives

Example (continued): Consider the previous two mutually exclusive projects and assume the same 10% p.a. discount rate. Reevaluate the two machines using the constant chain of replacement method

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000
3	\$40,000	
<i>NPV (at 10%)</i>	<i>\$25,920</i>	<i>\$11,157</i>

Projects With Different Lives

- The net present values of the machines are given as.
 - NPV for project A = \$25,920
 - NPV for project B = \$11,157
- The net present value of the infinite chain (NPV_{∞}) is...

$$NPV_{\infty} = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1}$$

$$NPV_{\infty}^A = 25920 \frac{(1.10)^3}{(1.10)^3 - 1} = \$104,228$$

$$NPV_{\infty}^B = 11157 \frac{(1.10)^2}{(1.10)^2 - 1} = \$64,286$$

Projects With Different Lives

A variant to the constant chain of replacement in perpetuity method is the **equivalent annuity value method** which involves the following steps.

Step 1: Obtain the NPVs of the projects

NPV for project A = \$25,920

NPV for project B = \$11,157

Step 2: Convert these NPVs to an equivalent annual value (EAV) series by dividing the NPVs by the present value of an ordinary annuity factor, that is, $[1 - (1 + k)^{-n}]/k$

- EAV for project A = $25920 / \{[1 - (1 + 0.10)^{-3}] / 0.10\} = \$10,423$
- EAV for project B = $11157 / \{[1 - (1 + 0.10)^{-2}] / 0.10\} = \$6,429$

Projects With Different Lives

Step 3: Because the EAVs obtained in Step 2 are assumed to be perpetuities, their present values are obtained by dividing by the discount rate as follows

$$\text{NPV of project A} = \text{EAV}/k = 10423/0.10 = \$104,228$$

$$\text{NPV of project B} = \text{EAV}/k = 6429/0.10 = \$64,286$$

- Note that...
 - $\text{NPV}_{\infty} = \text{EAV}/k$

Key Concepts

- The NPV method is recommended for investment evaluation because it is consistent with the maximization of shareholder wealth
- The NPV method requires a careful analysis of which incremental cash flows are included in the analysis
- Inflation can distort NPV analysis and it is important to evaluate real (nominal) net cash flows using the real (nominal) discount rate
- The constant chain of replacement assumption is used to evaluate and compare projects of differing lives

Key Relationships/Formula Sheet

◆ Fisher relationship: $(1 + r) = (1 + r^*)(1 + i)$

◆ Net present value

$$NPV = \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \dots + \frac{C_N}{(1+k)^N} - I_0$$

$$NPV = \sum_{t=1}^N \frac{C_t}{(1+k)^t} - I_0$$

$$NPV = \sum_{t=1}^N \left[\frac{(R_t - OC_t)(1 - t_c) + t_c D_t}{(1+k)^t} \right] + \frac{SV_N}{(1+k)^N} - I_0$$

◆ Net present value (constant chain replacement method in perpetuity and the equivalent annuity value method)

$$NPV_{\infty} = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1} = EAV / k$$

The Weighted Average Cost of Capital

- the weighted average cost of capital
- Estimate the weighted average cost of capital
- Use the weighted average cost of capital in capital budgeting
- Examine the limitations of the weighted average cost of capital

The Weighted Average Cost of Capital

- The weighted average cost of capital (WACC or k_0) is the benchmark required rate of return used by a firm to evaluate its investment opportunities
 - The discount rate used to evaluate projects of **similar risk to the firm**
- It takes into account **how** a firm finances its investments
 - How much debt versus equity does the firm employ?
- The WACC depends on...
 - Qualitative factors
 - The market values of the alternative sources of funds
 - The market costs associated with these sources of funds

Estimating the WACC

- The main steps involved in the estimation of the WACC are...
 - Identify the financing components
 - Estimate the current (or market) values of the financing components
 - Estimate the cost of each financing component
 - Estimate the WACC
- We will consider each step for typical financing components

Identify the Financing Components

- Debt

- Identify all externally supplied debt items
- Do not include creditors and accruals as these costs are already included in net cash flows

- Ordinary shares

- Obtain number of issued shares from the balance sheet
- Do not include reserves and retained earnings

- Preference shares

- Obtain number of issued shares from the balance sheet

Valuing the Financing Components

- Use market values and **not** book values
- Value coupon paying debt using the following pricing relation (see Lecture 3)

$$P_0 = \frac{C_1}{(1+k_d)} + \frac{C_2}{(1+k_d)^2} + \dots + \frac{C_n}{(1+k_d)^n} + \frac{F_n}{(1+k_d)^n}$$

$$P_0 = \sum_{t=1}^n \frac{C_t}{(1+k_d)^t} + \frac{F_n}{(1+k_d)^n}$$

where P_0 = Market price of the debt security
 C_t = Periodic interest payment on debt in period t
 k_d = Required rate of return on debt

Valuing Long Term Debt

Example: BLD Ltd has 10,000 bonds outstanding and each bond has a face value of \$1,000 with two years remaining to maturity. The bonds pay coupons (or interest) at a rate of 10% p.a. every six months. If the market interest rate appropriate for the bond is 15% p.a., what is the current price of each bond? What is the total market value of debt in BLD Ltd's capital structure?

Valuing Long Term Debt

- Coupon (or interest) payments are made every six months
- Number of payments, $n = 4$, semi-annual payments
- Annual interest payments = $0.10(1000) = \$100.00$
 - So, semi-annual interest payments = $\$50.00$
- Repayment of principal at the end of year 2 = $\$1000.00$
- Required return on debt, $k_d = 15\%$ p.a.
- So, semi-annual required return on debt, $k_d = 7.5\%$

Valuing Long Term Debt

The price of the bond is...

$$P_0 = \frac{50}{(1.075)^1} + \frac{50}{(1.075)^2} + \frac{50}{(1.075)^3} + \frac{1050}{(1.075)^4}$$

$$P_0 = \$916.27$$

- So, total value of debt = $10000(916.27) = \$9,162,700$
- **Note:** As the coupon rate is lower than the market rate, the price is less than the face value, that is, the bond is selling at a **discount** to face value
 - If the coupon rate is greater than the market rate, the price would be at a **premium** to face value

Valuing Ordinary Shares

- **Example:** ABC Ltd has 300,000 shares on issue which each have a par value of \$1.00. If the shares are currently trading at \$3.50 each what is the total market value of ABC's ordinary shares?
- There are 300,000 shares on issue with a market value of \$3.50 per share
- Market value of equity = $300000 \times 3.50 = \$1,050,000$
 - The par (or book) value of shares is **not** relevant here

Valuing Preference Shares

- Preference shares pay a fixed dividend at regular intervals
- If the shares are non-redeemable, then the cash flows represent a perpetuity and the market value can be computed as...
- $P_0 = D_p / k_p$

Where

P_0 = The current market price

D_p = Value of the periodic dividend

k_p = Required return on preference shares

Valuing Preference Shares

- Example: Assume the preference shares of XYZ Ltd pay a dividend of \$0.40 p.a. and the cost of preference shares is 10% p.a. What is the price of the preference shares? If XYZ Ltd has 500,000 preference shares outstanding, what is the market value of these shares?
- The cash flows from the preference shares are...
 - $D_p = \$0.40$ per share
 - So, $P_0 = 0.40/0.10 = \$4.00$
 - Market value of shares = $500000 \times 4.00 = \$2,000,000$

Estimating the Costs of Capital

- The costs of a firm's financing instruments can be obtained as follows...
 - Use observable market rates - may need to be estimated
 - Use effective annual rates
 - For the cost of debt use the market yield
- Focus here is on the costs of debt, ordinary shares and preference shares
 - **Note:** We ignore the complications of flotation costs and franking credits associated with dividends (sections 15.5.3 and 15.5.5 of the text)

Cost of Debt

- **Example:** The bonds of ABD Ltd have a face value of \$1,000 with one year remaining to maturity. The bonds pay coupons at the rate of 10 percent p.a. If the current market price of the bonds is \$1,018.50, what is the firm's cost of debt?
 - The annual interest (coupon) paid on the debt is...
 - $1000 \times 0.10 = \$100$
 - So, $1018.50 = (1000 + 100)/(1 + k_d)$
 - $k_d = (1100/1018.50) - 1 = 8.0\%$

Cost of Ordinary Shares

Example: Assume that the risk free rate is 6 percent, the expected market risk premium is 8 percent and the equity beta of XYW Ltd's equity is 1.2. What is the firm's cost of equity capital?

Using the CAPM, we have...

$$\begin{aligned} \diamond k_e &= r_f + [E(r_m) - r_f]\beta_e \\ \diamond k_e &= 0.06 + 0.08 \times 1.2 = 15.6\% \end{aligned}$$

Note: Can also use the dividend discount models covered in Lecture 4 (but not commonly used by managers...)

$$\begin{aligned} \diamond P_0 &= D_1 / (k_e - g) \\ \diamond \text{So, } k_e &= D_1 / P_0 + g \end{aligned}$$

Cost of Preference Shares

- Recall that, $P_0 = D_p/k_p$
- Thus, $k_p = D_p/P_0$
- **Example:** The preference shares of DBB Ltd pay a dividend of \$0.50 p.a. If the preference shares are currently selling for \$4.00 per share, what is the cost of these shares to the firm?
- The cost of preference shares is given as...
$$k_p = D_p/P_0$$

So, $k_p = 0.50/4.00 = 12.5\%$

Weighted Average Cost of Capital

The weighted average cost of capital (k_o) uses the cost of each component of the firm's capital structure and weights these according to their relative market values

Assuming that only debt and equity are used, we have...

$$k_o = k_d(D/V) + k_e(E/V)$$

where k_d = Cost of debt

k_e = Cost of equity

D = Market value of debt

E = Market value of equity

$V = D + E$

Weighted Average Cost of Capital

Assuming that preference shares are used as well as debt and equity...

$$k_o = k_d(D/V) + k_e(E/V) + k_p(P/V)$$

where P = Market value of preference shares

k_p = Cost of preference shares

$$V = D + E + P$$

- Be careful of rounding errors in initial calculations
- Be careful to work in consistent terms
 - Calculations in percentages versus decimals
- Check your answers with some common sense logic...

$$\diamond k_e > k_p > k_d > k_d(1 - t_c) \text{ (Why?)}$$

Taxes and the WACC

- Under the classical tax system...
 - Interest on debt is tax deductible
 - Dividends have no tax effect for the firm
- The after-tax cost of debt, $k'_d = (1 - t_c) k_d$
where t_c corporate tax rate
- The cost of equity (k_e) is unaffected
- The after-tax WACC is defined as...

$$k_o = k_d(1 - t_c)(D/V) + k_e(E/V) \quad \text{and}$$
$$k_o = k_d(1 - t_c)(D/V) + k_e(E/V) + k_p(P/V)$$

Calculating and Using the WACC

Example: You are given the following information for BCA Ltd. Note that book values are obtained from the firm's balance sheet while market values are based on market data.

The firm's marginal tax rate is 30%. Estimate the firm's before-tax and after-tax weighted average costs of capital

	Book values	Market values	Market costs
Bonds	\$30,000,000	\$50,000,000	8.0%
Preference shares	\$10,000,000	\$20,000,000	10.0%
Ordinary shares	\$60,000,000	\$80,000,000	14.0%
Total	\$100,000,000	\$150,000,000	

Calculating and Using the WACC

- Before-tax weighted average cost of capital
 - WACC weights are based on market values so book values are not relevant

$$k_o = k_d(D/V) + k_e(E/V) + k_p(P/V)$$
$$V = D + E + P$$

	Market values	Weights	Market costs	Weights×Costs
Bonds	\$50,000,000	0.333	8.0%	2.67%
Preference shares	\$20,000,000	0.133	10.0%	1.33%
Ordinary shares	\$80,000,000	0.533	14.0%	7.47%
Total	\$150,000,000	1.000		11.47%

Note: Weight in bonds, $D/V = 50/150 = 0.333$, and so on

- Before-tax cost of capital = **11.47%**

Calculating and Using the WACC

The after-tax cost of capital requires the after tax cost of debt

$$k'_d = k_d (1 - t_c)$$

$$k'_d = 0.08(1 - 0.30) = 5.6\%$$

	Market values	Weights	After tax market costs	Weights×Costs
Bonds	\$50,000,000	0.333	5.6%	1.87%
Preference shares	\$20,000,000	0.133	10.0%	1.33%
Ordinary shares	\$80,000,000	0.533	14.0%	7.47%
Total	\$150,000,000	1.000		10.67%

- **Note:** Weight in bonds, $D/V = 50/150 = 0.333$, and so on
 - After-tax cost of capital = 10.67%

Calculating and Using the WACC

Example: Assume that a firm is financed by 60 percent equity, 10 percent preference shares and the remainder by debt. The corporate tax rate is 30 percent. The costs of capital for debt, preference and equity capital are 10 percent, 12 percent and 15 percent, respectively. What is the firm's after-tax weighted average cost of capital? If the firm is considering three independent projects with IRRs of 10%, 12% and 14% which of these projects should it accept?

Calculating and Using the WACC

$$k_o = k_d(1 - t_c)(D/V) + k_p(P/V) + k_e(E/V)$$

The debt ratio is...

$$D/V = 1 - 0.60 - 0.10 = 0.30$$

$$k_o = [0.10 \times (1 - 0.30) \times 0.30] + (0.12 \times 0.10) + (0.15 \times 0.60)$$

$$k_o = 12.3\%$$

- The firm should accept all projects with an IRR greater than the cost of capital (why?)
 - Accept the project with an IRR of 14%
 - Reject the projects with IRRs of 10% and 12%

Calculating and Using the WACC

- **Example:** ASL Ltd has a debt-to-equity ratio of 25%. The cost of debt is 8 percent and the corporate tax rate is 30 percent. If the after-tax weighted average cost of capital is 20 percent, what is the firm's cost of equity?
- The cost of equity can be obtained using the weighted average cost of capital relationship...

$$k_o = k_d(1 - t_c)(D/V) + k_e(E/V)$$

- Note that we're given a D/E ratio of 0.25
 - We need the $D/V = D/(D + E)$ ratio

Calculating and Using the WACC

$D/E = 0.25$ implies...

$$D = 0.25(E)$$

$$\text{So, } D/(D + E) = 0.25(E)/[0.25(E) + E] = 0.25(E)/1.25(E)$$

$$D/(D + E) = 0.20 \text{ and } E/(D + E) = 1 - 0.20 = 0.80$$

The weighted average cost of capital is...

$$k_0 = 0.20 = 0.08(1 - 0.30)(0.20) + k_e(0.80)$$

$$\text{So, } k_e = [0.20 - 0.08(1 - 0.30)(0.20)]/(0.80)$$

$$k_e = 23.6\%$$

Limitations on Using the WACC

- **Recall:** The weighted average cost of capital is the discount rate that is used to evaluate projects of similar risk to the firm
- The WACC cannot be used in the following situations...
 - If the project alters the operational (or business) risk of the firm
 - If the project alters the financial risk of the firm by dramatically altering its capital structure
- Examples of risk altering projects?
- What should the firm do if the WACC cannot be used?

Key Concepts

- The weighted average cost of capital is the discount rate that is used to evaluate projects of similar risk to the firm
- There are four main steps involved in the estimation of the weighted average cost of capital
 - Identify the financing instruments
 - Estimate the current (or market) values of the financing components
 - Estimate the cost of each financing component
 - Estimate the weighted average cost of capital
- The WACC cannot be used to evaluate projects that alter the business or financial risks of the firm

Key Relationships/Formula Sheet

Price of debt:
$$P_0 = \sum_{t=1}^n \frac{C_t}{(1 + k_d)^t} + \frac{F_n}{(1 + k_d)^n}$$

Price of preference shares:
$$P_0 = D_p / k_p$$

Cost of equity:
$$k_e = r_f + [E(r_m) - r_f] \beta_e$$

Cost of preference shares:
$$k_p = D_p / P_0$$

Before-tax weighted average cost of capital

❖
$$k_o = k_d (D/V) + k_e (E/V)$$

❖
$$k_o = k_d (D/V) + k_e (E/V) + k_p (P/V)$$

After-tax weighted average cost of capital

❖
$$k_o = k_d (1 - t_c) (D/V) + k_e (E/V)$$

❖
$$k_o = k_d (1 - t_c) (D/V) + k_e (E/V) + k_p (P/V)$$