



## 333-201 Business Finance

Dr Cesario MATEUS

PhD in Finance

Senior Lecturer in Finance and Banking

Room 219 A – Economics & Commerce Building

8344 – 8061

[c.mateus@greenwich.ac.uk](mailto:c.mateus@greenwich.ac.uk)



## 333-201 Business Finance

### Lecture 16:

### Capital Budgeting / Project Evaluation 3

# Capital Budgeting III

- Examine the issues related to the estimation of incremental cash flows
- Examine the role of inflation in the capital budgeting process
- Analyze mutually exclusive projects with different lives

# Issues in Cash Flow Estimation

- Timing of cash flows

- The exact timing of project cash flows can affect the valuation of a project
- The simplifying assumption used is that the net cash flows are received at the end of a period

- Financing charges

- Cash outflows relating to how the project is to be financed are not included in the analysis
- The value of a project is independent of how it will be financed
- The discount rate used represents the rate of return required by equityholders, debtholders and other securityholders
- Financing costs are not used in the cash flows because that results in their being double counted!

# Issues in Cash Flow Estimation

- Incremental cash flows

- Only cash flows that change if the project is accepted are relevant in evaluating a project
- Need to be careful with sunk costs and allocated costs

- Sunk costs

- These costs are not included as they have been incurred in the past and will not be affected by the project's acceptance or rejection

- Allocated costs

- Overhead costs allocated by management to firm's divisions
- These costs do not vary with the decision and are usually ignored
- Examples are administrative costs incurred by head office and allocated to divisions



# Issues in Cash Flow Estimation

- Taxes and tax effects

- Taxes need to be included where they have an effect on the net cash flows generated by a project

- Taxes have *three* main effects on cash flows

- Corporate taxes
- Depreciation tax shield
- Taxes on disposal of assets

- Corporate income tax

- Corporate taxes should be included as a cash outflow
- After tax cash flow = Before tax cash flow  $\times (1 - t_c)$
- $t_c$  = The effective corporate tax rate

# Issues in Cash Flow Estimation

- Depreciation tax savings or depreciation tax shield
  - Depreciation itself is **not** an operating expense and is excluded from the net cash flows
  - However, depreciation affects net cash flows as it decreases the taxes payable due to the depreciation tax shield
  - Depreciation tax savings (or shield) =  $t_c \times \text{Depreciation expense}$

# Issues in Cash Flow Estimation

- Disposal or salvage value of assets
  - The disposal or salvage value of assets needs to be taken into account **after taxes**
  - Taxes are payable when an asset is sold for **more** than its book value
  - There is a tax saving when an asset is sold for **less** than its book value as the loss can be offset against taxable income
- Book value = Acquisition cost - Accumulated depreciation
- Gain (or loss) = Disposal value - Book value
- Taxes payable of gain =  $t_c \times \text{Gain on sale}$
- Tax saving on loss =  $t_c \times \text{Loss on sale}$



# Issues in Cash Flow Estimation

The incremental net after-tax cash flows are...

$$C_t = (R_t - OC_t - D_t)(1 - t_c) + D_t$$

Alternatively, the incremental net after-tax cash flows are...

$$C_t = (R_t - OC_t)(1 - t_c) + t_c D_t$$

Separating the after-tax residual or salvage value (SV), we get...

$$NPV = \sum_{t=1}^N \left[ \frac{(R_t - OC_t)(1 - t_c) + t_c D_t}{(1 + k)^t} \right] + \frac{SV_N}{(1 + k)^N} - I_0$$

Note:  $SV_N$  = After-tax residual (or salvage) value

## Issues in Cash Flow Estimation

**Example:** Suppose the initial cost of a machine is \$12,000. It has a useful life of 6 years and will be depreciated on a straight line basis over 6 years. The machine will generate a before-tax cash flows of \$6,000 per year over its useful life and the effective corporate tax rate is 30%. If at the end of year 4 the company sells the machine for \$5,000 what is the net cash flow in year 4 of the machine's life?

The net after tax cash inflows are...

$$C_4 (R_4 - OC_4)(1 - t_c) = 6000(1 - 0.3) = \$4,200$$

# Issues in Cash Flow Estimation

The depreciation tax saving in year 4 is...

$$t_c D_4 = 0.3 \times 2000 = \$600$$

The book value of assets in year 4 is...

End of Year	Depreciation	Accumulated Depreciation	Book Value
0			\$12,000
1	\$2,000	\$2,000	\$10,000
2	\$2,000	\$4,000	\$8,000
3	\$2,000	\$6,000	\$6,000
4	\$2,000	\$8,000	\$4,000

# Issues in Cash Flow Estimation

- Before tax proceeds from the sale of the machine = \$5,000
- Taxes on proceeds from sale of the machine are...
  - Total gain = Disposal value . Book value
  - Total gain =  $5000 - 4000 = \$1,000$
  - Taxes payable =  $0.3 \times 1000 = \$300$
- The net, after-tax salvage value is.
  - $SV_4 = 5000 - 300 = \$4,700$

## Issues in Cash Flow Estimation

The total after tax net cash flow in year 4 is...

$$C_4 = (R_4 - OC_4)(1 - t_c) + t_c D_4 + SV_4$$

After tax net cash inflow	\$4,200	$(R_4 - OC_4)(1 - t_c)$
Depreciation tax saving	\$600	$t_c D_4$
Proceeds from sale	\$5,000	} $SV_4$
Taxes payable on gains	-\$300	
Total after tax net cash flow	\$9,500	

# Inflation and Capital Budgeting

- It is important to be consistent in your treatment of **inflation**
  - For **nominal** cash flows use the **nominal** discount rate
  - For **real** cash flows use the **real** discount rate
- From the Fisher relationship we have...

$$\diamond (1 + r) = (1 + r^*)(1 + i) \quad \text{or}$$

$$\diamond (1 + r^*) = (1 + r)/(1 + i)$$

- ❖  $r$  = Nominal rate of return (or nominal interest rate) per annum
- ❖  $r^*$  = Real rate of return (or real interest rate) per annum
- ❖  $i$  = Expected inflation rate per annum

# Inflation and Capital Budgeting

**Example:** *A one-year project is expected to generate net after tax cash flows of \$1,000,000 in real terms one year from now.*

The nominal discount rate is 10% p.a. and the expected inflation rate is 2% p.a. If the project costs \$900,000 what is the net present value of the project?

- Two methods can be used.
  - Use **real** net cash flows and discount with the **real** discount rate
  - Use **nominal** net cash flows and discount with the **nominal** discount rate
- Don't mix these up!
  - Real cash flows discounted using the nominal discount rate
  - Nominal cash flows discounted using the real discount rate

# Inflation and Capital Budgeting

Method 1: Use real discount rate with real cash flows

- ❖ Real rate,  $r^* = (1 + r)/(1 + i) - 1 = 1.10/1.02 - 1 = 7.84\%$
- ❖ Present value of cash inflows =  $1000/1.0784 = \$927.27$
- ❖ Net present value =  $927.27 - 900 = \$27.27$

Method 2: Use nominal discount rate with nominal cash flows

- ❖ The nominal cash flow =  $1000(1.02) = \$1020$
- ❖ Present value of cash inflows =  $1020/1.10 = \$927.27$
- ❖ Net present value =  $927.27 - 900 = \$27.27$



## Projects With Different Lives

**Example:** Consider the following two mutually exclusive projects and assume a 10% p.a. discount rate. Which machine should the firm choose at this discount rate?

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000
3	\$40,000	
<i>NPV (at 10%)</i>	\$25,920	\$11,157

We **cannot** compare the NPVs of the projects because project B lasts only 2 years

# Projects With Different Lives

- We assume that both projects are repeated with identical projects until they achieve a common duration (or life)
  - The constant chain of replacement assumption
- The constant chain of replacement assumption can be applied using two different methods...
  - The lowest common multiple method
  - The perpetuity method
  - Both methods will give the same decision

# Projects With Different Lives

- Constant chain of replacement using the lowest common multiple method
  - Replicate the projects until they achieve the same “lives”
  - A 2-year versus a 3-year project is replicated 3 and 2 times...
  - A 2-year versus a 4-year project is replicated 2 and 1 time(s)...
  - A 4-year versus a 5-year project is replicated 5 and 4 times...

## Projects With Different Lives

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000 – \$50,000
3	\$40,000 – \$100,000	\$40,000
4	\$60,000	\$30,000 – \$50,000
5	\$50,000	\$40,000
6	\$40,000	\$30,000

NPV for repeated project A = \$45,390

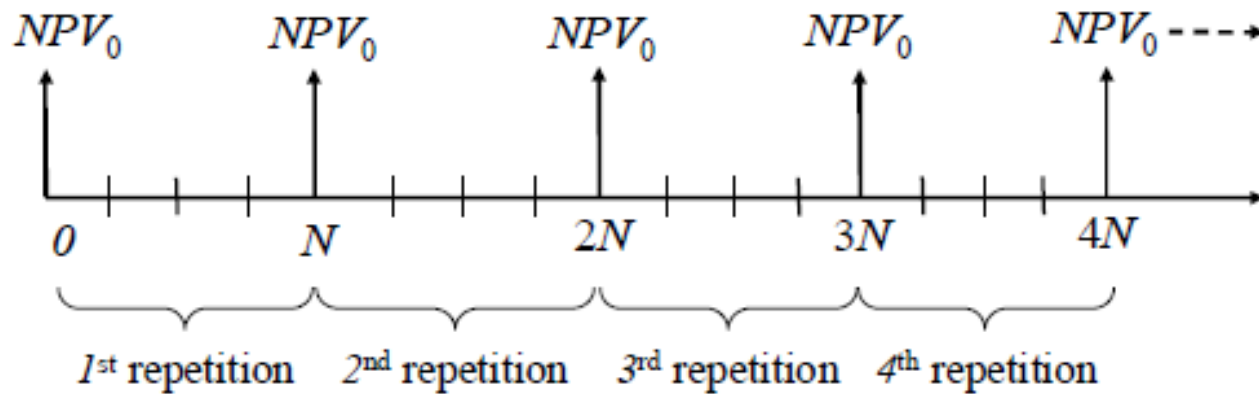
NPV for repeated project B = \$28,000

# Projects With Different Lives

- The lowest common multiple method can be cumbersome
  - **Example:** Two projects with lives of 11 and 13 years require present value calculations for 143 ( $= 11 \times 13$ ) years!
- **Constant chain of replacement in perpetuity method**
  - Assumes that both projects are replicated **forever**
  - The chains of cash flows are of “**equal length**” because they are both of infinite length
  - Method is generally easier to use

## Projects With Different Lives

- Consider a project with a life of  $N$  years and a required rate of return of  $k\%$  p.a. Its net present value is  $NPV_0$
- The project repeated forever will have the following profile of NPVs



## Projects With Different Lives

The net present value of the infinite chain ( $NPV_{\infty}$ ) is...

$$NPV_{\infty} = NPV_0 + \frac{NPV_0}{(1+k)^N} + \frac{NPV_0}{(1+k)^{2N}} + \dots + \frac{NPV_0}{(1+k)^{\infty}}$$

$$NPV_{\infty} = NPV_0 \left[ 1 + \frac{1}{(1+k)^N} + \frac{1}{(1+k)^{2N}} + \dots + \frac{1}{(1+k)^{\infty}} \right]$$

$$NPV_{\infty} = NPV_0 \left[ \frac{1}{1 - \frac{1}{(1+k)^N}} \right] = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1}$$

## Projects With Different Lives

**Example (continued):** Consider the previous two mutually exclusive projects and assume the same 10% p.a. discount rate. Reevaluate the two machines using the constant chain of replacement method

End of Year	Project A	Project B
0	-\$100,000	-\$50,000
1	\$60,000	\$40,000
2	\$50,000	\$30,000
3	\$40,000	
<i>NPV</i> (at 10%)	\$25,920	\$11,157



## Projects With Different Lives

- The net present values of the machines are given as.
  - NPV for project A = \$25,920
  - NPV for project B = \$11,157
- The net present value of the infinite chain ( $NPV_{\infty}$ ) is...

$$NPV_{\infty} = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1}$$

$$NPV_{\infty}^A = 25920 \frac{(1.10)^3}{(1.10)^3 - 1} = \$104,228$$

$$NPV_{\infty}^B = 11157 \frac{(1.10)^2}{(1.10)^2 - 1} = \$64,286$$

## Projects With Different Lives

A variant to the constant chain of replacement in perpetuity method is the **equivalent annuity value method** which involves the following steps.

**Step 1:** Obtain the NPVs of the projects

NPV for project A = \$25,920

NPV for project B = \$11,157

**Step 2:** Convert these NPVs to an equivalent annual value (EAV) series by dividing the NPVs by the present value of an ordinary annuity factor, that is,  $[1 - (1 + k)^{-n}]/k$

- EAV for project A =  $25920 / \{[1 - (1 + 0.10)^{-3}] / 0.10\} = \$10,423$
- EAV for project B =  $11157 / \{[1 - (1 + 0.10)^{-2}] / 0.10\} = \$6,429$

## Projects With Different Lives

**Step 3:** Because the EAVs obtained in Step 2 are assumed to be perpetuities, their present values are obtained by dividing by the discount rate as follows

$$\text{NPV of project A} = \text{EAV}/k = 10423/0.10 = \$104,228$$

$$\text{NPV of project B} = \text{EAV}/k = 6429/0.10 = \$64,286$$

- Note that...
  - $\text{NPV}_{\infty} = \text{EAV}/k$

# Key Concepts

- The NPV method is recommended for investment evaluation because it is consistent with the maximization of shareholder wealth
- The NPV method requires a careful analysis of which incremental cash flows are included in the analysis
- Inflation can distort NPV analysis and it is important to evaluate real (nominal) net cash flows using the real (nominal) discount rate
- The constant chain of replacement assumption is used to evaluate and compare projects of differing lives

# Key Relationships/Formula Sheet

- ◆ Fisher relationship:  $(1 + r) = (1 + r^*)(1 + i)$
- ◆ Net present value

$$NPV = \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \dots + \frac{C_N}{(1+k)^N} - I_0$$

$$NPV = \sum_{t=1}^N \frac{C_t}{(1+k)^t} - I_0$$

$$NPV = \sum_{t=1}^N \left[ \frac{(R_t - OC_t)(1 - t_c) + t_c D_t}{(1+k)^t} \right] + \frac{SV_N}{(1+k)^N} - I_0$$

- ◆ Net present value (constant chain replacement method in perpetuity and the equivalent annuity value method)

$$NPV_{\infty} = NPV_0 \frac{(1+k)^N}{(1+k)^N - 1} = EAV / k$$