FINA 1082 Financial Management

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Lecture 2
Valuation of Debt Securities I

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Objectives

- Analyze the purpose of the capital market and their participants
- Examine the characteristics of debt securities
- Outline the basic valuation principle
- Examine the pricing of discount debt securities
- Examine the pricing of coupon paying debt securities
- Relate the coupon rate to the yield to maturity
- Analyze the sensitivity of prices to changes in interest rates

Purpose of the Capital Market

Firms and individuals use the capital markets for long-term investments

Primary reason that individuals and firms choose to borrow long-term is to reduce the risk that interest rates will rise before they pay off their debt**.**

Capital Market Participants

Primary users: federal and local governments and corporations

Largest purchasers: households. Frequently, individuals and households deposit funds in financial institutions that use the funds to purchase capital market instruments such as bonds or stock

Capital Market Trading

Occurs in either primary market or the secondary market

Primary market: where new issues of stocks and bonds are introduced. Investment funds, corporations, and individual investors can all purchase securities offered in the primary market. (*IPO – initial public offering*).

Secondary market: where the sale of previously issued securities takes place, and it is important because most investors plan to sell long-term bonds before they reach maturity.

Types of Bonds

- Bonds are Securities that represent a debt owned by the issuer to the investor.

- Bonds obligate the issuer to pay a specified amount at a given date, generally with periodic payments

- The par, face, or maturity value of the bond is the amount that the issuer must pay at maturity

- The coupon rate is the rate of interest that the issuer must pay

• This rate is usually fixed for the duration of the bond and does not fluctuate with market interest rates,

• If the repayment terms of a bond are not met, the holder of a bond has a claim on assets of the issuer

Par or Face Value - The amount of money that is paid to the bondholders at maturity. For most bonds this amount is $1,000. It also generally represents the amount of money borrowed by the bond issuer.

Coupon Rate -The coupon rate, which is generally fixed, determines the periodic coupon or interest payments. It is expressed as a percentage of the bond's face value. It also represents the interest cost of the bond to the issuer.

Coupon Payments - The coupon payments represent the periodic interest payments from the bond issuer to the bondholder. The annual coupon payment is calculated by multiplying the coupon rate by the bond's face value. Since most bonds pay interest semiannually, generally one half of the annual coupon is paid to the bondholders every six months.

Maturity Date - The maturity date represents the date on which the bond matures, *i.e.,* the date on which the face value is repaid. The last coupon payment is also paid on the maturity date.

Original Maturity -The time from when the bond was issued until its maturity date.

Remaining Maturity -The time currently remaining until the maturity date.

Call Date - For bonds which are callable, *i.e.,* bonds which can be redeemed by the issuer prior to maturity, the call date represents the earliest date at which the bond can be called.

Call Price - The amount of money the issuer has to pay to call a callable bond (there is a premium for calling the bond early). When a bond first becomes callable, *i.e.,* on the call date, the call price is often set to equal the face value plus one year's interest.

Required Return - The rate of return that investors currently require on a bond.

Yield to Maturity - The rate of return that an investor would earn if he bought the bond at its current market price and held it until maturity. Alternatively, it represents the discount rate which equates the discounted value of a bond's future cash flows to its current market price.

Yield to Call - The rate of return that an investor would earn if he bought a callable bond at its current market price and held it until the call date given that the bond was called on the call date.

Current Yield: yield of the bond at the current moment. It is equal to the annual interest payment divided by the bond’s price. It does not reflect the total return over the life of the bond. In particular, it takes no account of reinvestment risk (the uncertainty about the rate at which future cashflows can be reinvested) or the fact that bonds usually mature at par value, which can be an important component of a bond's return.

At discount: YTM > current yield > coupon yield

At Premium: coupon yield > current yield > YTM

At Par: YTM = current yield = coupon yield.

Treasury Securities

Type	Maturity
Treasury Bill	Less than 1 year
Treasury Note	1 to 10 years
Treasury Bond	10 to 20 years

The Valuation Principle

The price of a security today is the present value of all future expected cash flows discounted at the "appropriate" required rate of return (or discount rate)

The valuation variables are

1. Current price
2. Future expected cash flows - Face value and/or coupons
3. Yield or required rate of return

The valuation problem is to

1. Estimate the price; given the future cash flows and required rate of return, or
2. Estimate the required rate of return; given the future cash flows and price

Valuing Discount Securities

The face value (P_n) is promised at a pre-specified date – no other payment promised

Interest earned is "implicit" in the difference between the face value and current market price, P_0

Examples: Treasury Bills, Commercial Paper, Bank Bills. Face value is typically $100,000 or its multiple

The price is computed as the present value at a specified yield

The interest factor for maturity less than 1 year is:

Interest rate factor = (Time to maturity/365) × r = (n/365) × r

Example: Consider two Treasury bills, each with a face value of $100,000 and maturing in 180 and 90 days, respectively. Assume the yield is 6% p.a. What are their prices today? What happens to T-bill prices as they approach maturity?

Valuing Discount Securities

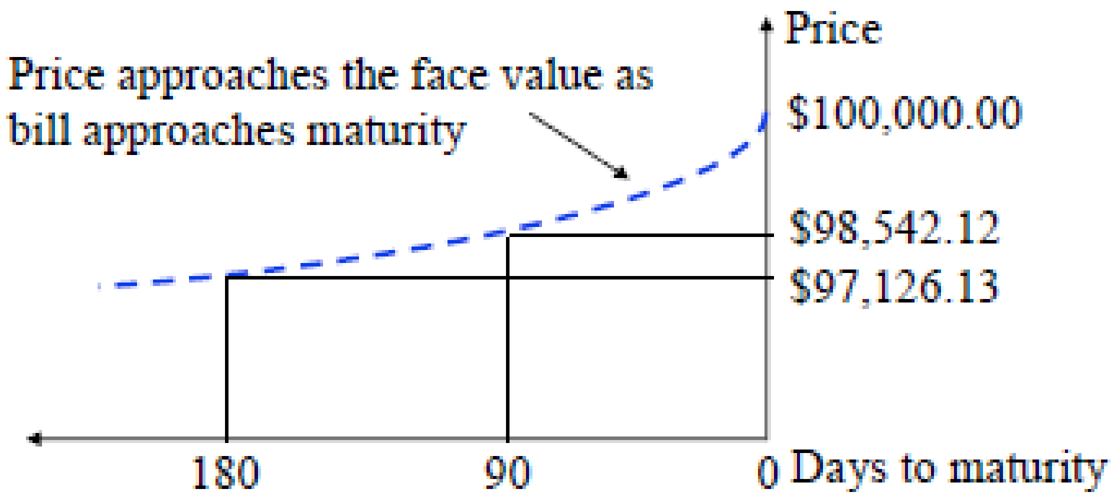
Pricing the 180 day T-bill

Interest rate factor = (n/365)×r = (180/365)×0.06 = 2.959%

Price = 100,000/[1 + 0.02959] = $97,126.13

Pricing the 90 day T-bill

Price = 100,000/[1 + (90/365)×0.06] = $98,542.12



Valuing Discount Securities

Effective annual return (re) on the 180 day T-bill

The effective annual return computation assumes that you invest in a 180 day T-bill and then roll over your investment for another 180 days

$$re = (1 + (180/365)0.06)365/180 - 1 = 6.09\%$$

Effective annual return on the 90 day T-bill

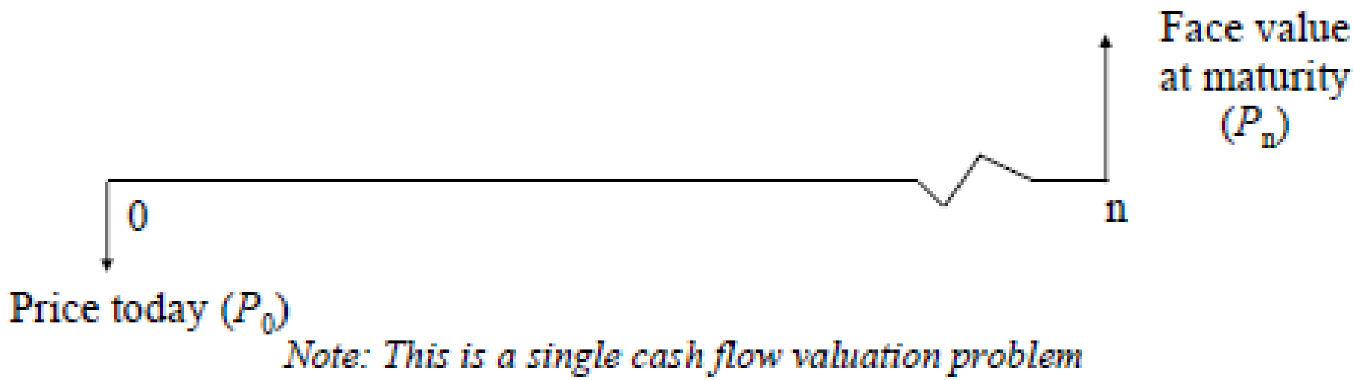
The effective annual return computation assumes that you invest in a 90 day T-bill and then roll over your investment for three more 90 day periods

$$re = (1 + (90/365)0.06)365/90 - 1 = 6.14\%$$

Zero Coupon Securities

Zero coupon bonds are long-term securities paying the face value at maturity

- No coupon or interest payment made
- Issued at deep discount to face value
- Return earned is based on the appreciation in bond's value (price) over time



Pricing Zero Coupon Securities

Example: Consider a zero coupon bond which matures in 5 years with a face value of $1,000

a) If the bond has a yield to maturity of 8% what price should it be selling for today?

b) Suppose interest rates change suddenly and the price of these bonds rises to $700. What has happened to the yield to maturity of the bonds and why?

Given: P_n = $1,000, n = 5 years, and k_d = YTM = 8%

a) P_0 = 1000/1.085 = $680.58

b) The price has risen so you'd expect the YTM to be lower

New price, $P_0 = 700 = 1000/(1 + k_d^*)^5$

k_d^* = (1000/700)1/5 - 1 = 7.39% or 7.4%

Note: Prices and yields are inversely related

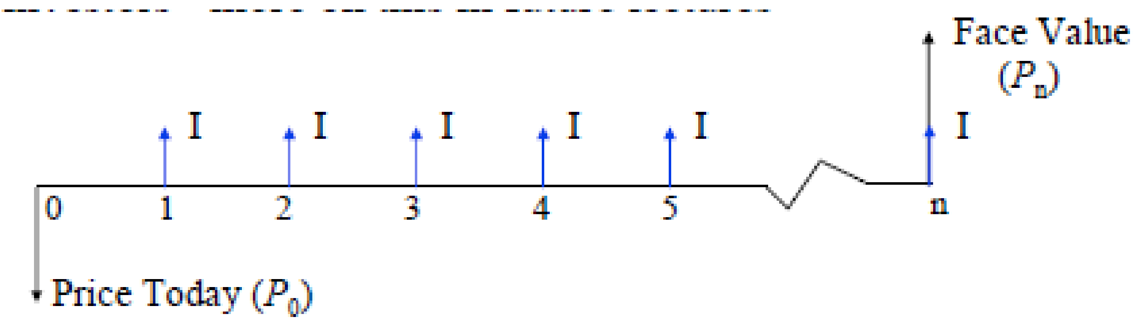
Coupon Paying Securities

- Fixed coupon payment, typically every six months
  - Non coupon paying bonds called zero coupon bonds

- Repayment of face value at maturity

- Typically issued at face value
  - Examples: Treasury bonds, corporate bonds

- Market price depends on the rate of return required by investors - more on this in future lectures



Note: This is a single cash flow plus annuity valuation problem

Pricing a Bond

Equal to the present value of the expected cash flows from the financial instrument. Determining the price requires:

- An estimate of the expected cash flows
- An estimate of the appropriate required yield

The price of the bond is the present value of the cash flows, it is determined by adding these two present values:

i) The present value of the semi-annual coupon payments

ii) The present value of the par or maturity value at the maturity date

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}$$

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

P = Price

n =number of periods (nr of years times 2, if semi-annual)

C = semi-annual coupon payment

r = periodic interest rate (required annual yield divided by 2, if semi-annual)

t = time period when payment is to be received

Because the semi-annual coupon payments are equivalent to an ordinary annuity, applying the equation for the present value of an ordinary annuity gives the present value of the coupon payments:

$$C\left[\frac{1-\frac{1}{(1+r)^{n}}}{r}\right]$$

Consider a 20 year 10% coupon bond with a par value of $1,000. The required yield on this bound is 11%.

$$\$50\left[\frac{1-\frac{1}{(1+0.055)^{40}}}{0.055}\right]=\$802.31$$

The PV of the par or maturity value of $1,000 received 40 six-month periods from now discounted at 5.5%, is $117.46, as follows:

$$\frac{\$1,000}{(1.055)^{40}}=\frac{\$1,000}{8.51332}=\$117.46$$

Price = PV coupon payments + PV of par (maturity value)

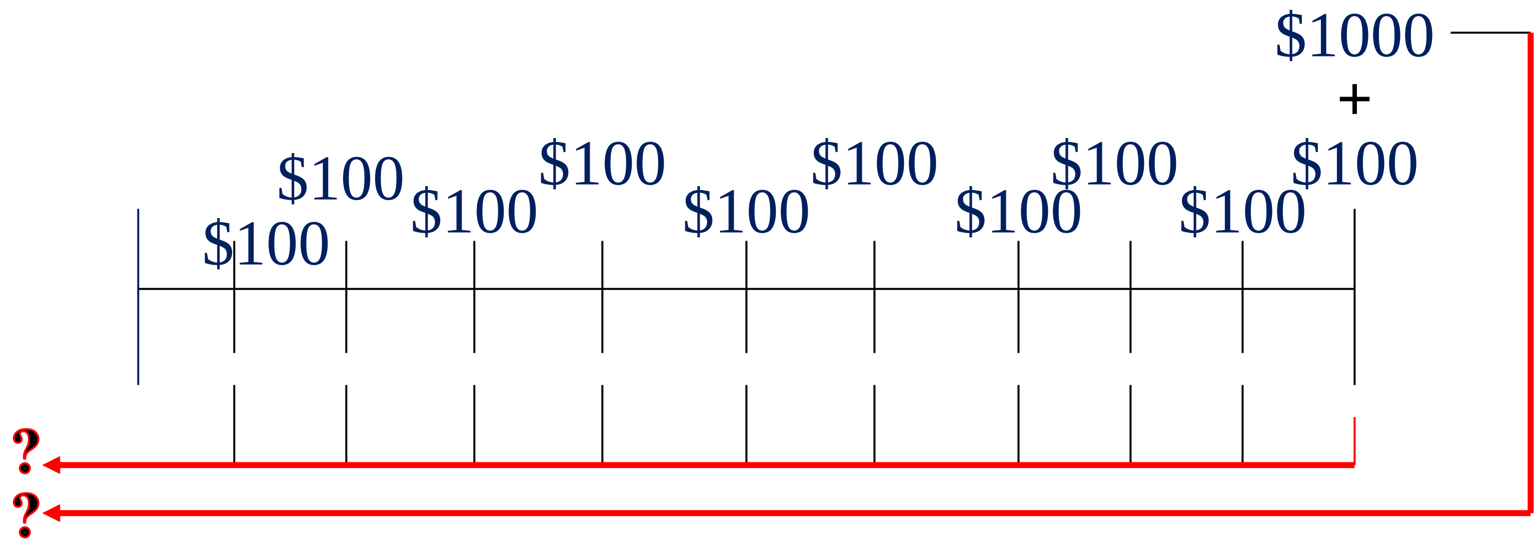
$802.31 + $117.46 = $919.77

Suppose that, instead of an 11% required yield, the required yield is 6.8%.

Price of the bond: $1,347.04

Example

Assume an investor buys a 10-year bond from the KLM corporation on January 1, 2003. The bond has a face value of $1000 and pays an annual 10% coupon. The current market rate of return is 12%. Calculate the price of this bond today.



First, find the value of the coupon stream

Remember to follow the same approach you use in time value of money calculations.

You can find the PV of a cash flow stream

$$PV=\frac{\$100}{(1+0.12)^1}+\frac{\$100}{(1+0.12)^2}+\frac{\$100}{(1+0.12)^3}+\frac{\$100}{(1+0.12)^4}+\frac{\$100}{(1+0.12)^5}+$$

$$\frac{\$100}{(1+0.12)^6}+\frac{\$100}{(1+0.12)^7}+\frac{\$100}{(1+0.12)^8}+\frac{\$100}{(1+0.12)^9}+\frac{\$100}{(1+0.12)^{10}}$$

Or, you can find the PV of an annuity

$$PVA=\$100\times\frac{1-(1+0.12)^{-10}}{0.12}$$

PV = $565.02

Find the PV of the face value

$$PV = \frac{CFt}{(1+r)^t}$$

$$PV = \frac{\$1{,}000}{(1.12)^{10}} = \$321.97$$

Add the two values together to get the total PV

$565.02 + $321.97 = $886.99

Yield to Maturity

Interest rate that equates the present value of cash flows received from a debt instrument with is value today.

Is the yield promised by the bondholder on the assumption that the bond will be held to maturity, that all coupon and principal payments will be made and coupon payments are reinvested at the bond's promised yield at the same rate as invested. It is a measurement of the return of the bond. This technique in theory allows investors to calculate the fair value of different financial instruments. The YTM is almost always given in terms of annual effective rate.

The calculation of YTM is identical to the calculation of internal rate of return.

If a bond's current yield is less than its YTM, then the bond is selling at a discount.
If a bond's current yield is more than its YTM, then the bond is selling at a premium.
If a bond's current yield is equal to its YTM, then the bond is selling at par.

Yields to Maturity on a 10% Coupon rate Bond Maturing in 10 years (Face Value = $ 1,000)

Price of Bond (4)	Yield to Maturity
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

1. When the coupon bond is priced at is face value, the YTM equals the coupon rate
2. The price of a coupon bond and the YTM are negatively related; that is, as the YTM rises, the price of the bond falls. If the YTM falls, the price of the bond rises
3. The YTM is greater than the coupon rate when the bond price is below its face value

Variants of Yield to Maturity

Given that many bonds have different characteristics, there are some variants of YTM:

• Yield to Call: when a bond is callable (can be repurchased by the issuer before the maturity), the market looks also to the Yield to Call, which is the same calculation of the YTM, but assumes that the bond will be called, so the cash flow is shortened.

• Yield to Put: same as Yield to Call, but when the bond holder has the option to sell the bond back to the issuer at a fixed price on specified date.

• Yield to Worst: when a bond is callable, "*puttable*" or has other features, the yield to worst is the lowest yield of Yield to Maturity, Yield to Call, Yield to Put, and others.

Pricing Coupon Paying Securities

Case 1: YTM Known, Price Required

Class Exercise 1: Consider a bond which pays an annual coupon rate of 10% with 5 years to maturity and a face value of $1,000

a) If the bond has a yield to maturity of 8% what price should it be selling for today?

b) What will its price be when there is one year left to maturity and interest rates do not change during this time?

c) What will the bond's price be immediately before maturity?

d) What is the bond's price if this were a zero coupon bond?

Answer to Class Exercise 1

Case 1: YTM Known, Price Required

Given: Coupon rate = 10%, n = 5 years, P_n = $1,000, k_d = 8%

a) P_0 = 100[(1 - 1.08-5)/0.08] + 1000/1.085
P_0 = 399.27 + 680.58 = $1079.85
The bond is selling at a premium of $79.85 (= 1079.85 - 1000) above face value because YTM < Coupon rate promised

b) P_4 = (100 + 1000)/1.08 = $1018.52
Note: Price does not include the coupon paid in that year

c) Price immediately before maturity?

d) Price of a zero coupon bond, P_0 = 1000/1.085 = $680.58

Pricing Coupon Paying Securities

Case 2: Price Known, YTM Required

Class Exercise 2: Consider the same bond in Class Exercise 1 which pays an annual coupon rate of 10% with 5 years to maturity and a face value of $1,000

a) If it is selling for $985.00 today what is the implied YTM on the bond?

b) Redo part (a) assuming that the coupons are paid semiannually

Answer to Class Exercise 2

a) Information given:

Coupon rate = 10%, n = 5 years, P_n = \$1,000, and P_0 = 985.00

Need to solve for kd in the following equation

$$P_0 = 985 = 100[(1 - (1 + k_d)^{-5})/k_d] + 1000/(1 + k_d)^5$$

At r = 10%:	PV(RHS) = \$1,000	(Known)
At r = k_d:	PV(RHS) = Price = \$985	(Given)
At r = 11%:	PV(RHS) = \$963.04	(Computed)

So, 10% < k_d < 11% (Actual YTM = 10.4%)

YTM can be computed using

Trial and error

Interpolation

Financial calculator

Answer to Class Exercise 2

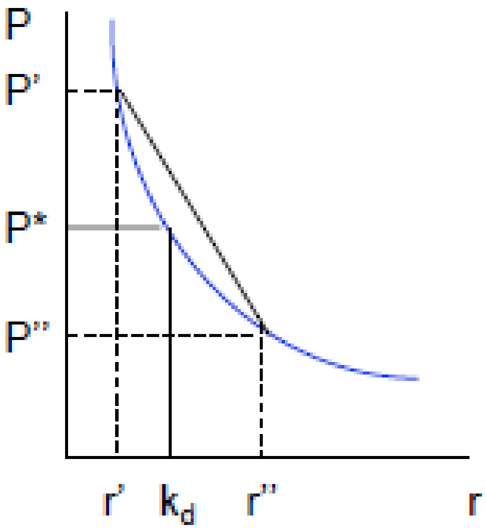
YTM can be computed using the interpolation method

r' = 10%, r'' = 11% and P' = 1000, P'' = 963.04, P* = 985

$(k_d - r'')/(P^* - P'') = (r' - r'')/(P' - P'')$

- $k_d = r'' + (P^* - P'')[(r' - r'')/(P' - P'')]$
- $k_d = 11 + \frac{(985 - 963.04)(10 - 11)}{(1000 - 963.04)}$

$= 10.4\%$



Answer to Class Exercise 2

YTM can also be computed using an approximation

The approximate YTM equals (a) the sum of coupon yield and (b) capital gain per period, both taken as a proportion of the average price over the remaining life of the bond

YTM ≈ Coupon yield + Capital gain per period

$kd \approx I/[(P_0 + P_n)/2] + [(P_n – P_0)/n]/[(P_0 + P_n)/2]$

In Class Exercise 3

$k_d \approx 100/[(985+1000)/2] + [(1000 - 985)/5]/[(985+1000)/2]$

$k_d \approx 0.1038$ or 10.38%

Answer to Class Exercise 2

b) Semi annual coupons implies:
Coupon rate = 10%/2 = 5%
Coupon payment = 100/2 = $50 every six months
$n = 5 \times 2 = 10$ periods
Need to solve for k*

Need in the following equation
$P_0 = 985 = 50[(1 - (1 + k^*_d)\text{-}10)/k^*_d] + 1000/(1 + k^*_d)^{10}$

$k^*_d = 5.196\%$
YTM, $k_d = 5.196 \times 2 = 10.39\% \approx 10.4\%$

Relating Coupon Rates to the YTM

Summary of the relationship between coupon rates and yields to maturity

1. When YTM = coupon rate; Price = Face Value

   Bond is selling at *par*

2. When YTM < coupon rate; Price > Face Value

   Bond is selling at a *premium*

3. When YTM > coupon rate; Price < Face Value

   Bond is selling at a *discount*

Holding Period Yield

The holding period yield (HPY or rh) of a bond is the return realized during the time period the bond is owned

$P_0 = I/(1 + r_h) + I/(1 + r_h)^2 + ... + (Pm + I)/(1 + r_h)^m$
$P_0 = (I/r_h)[1 - (1 + r_h)^{-m}] + Pm/(1 + r_h)^m$

Note: The holding period, $m < n$ (the bond's total life)

Example: Consider a bond paying annual coupons at a rate of 10% with 5 years to maturity and a face value of $1,000 selling for $985 today. If the bond is sold for $995 at the end of the year what is the holding period yield over this period?

$985 = (100 + 995)/(1 + r_h)$
$r_h = (100 + 995)/985 - 1 = 11.17\%$

Risks Associated with Investing in Bonds

Interest-rate risk or market risk

As interest rates rise, the price of a bond fall (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price)

Reinvestment Income or Reinvestment Risk

Calculation of the yield assumes that the CF received are invested

Interest rate at which interim CF can be reinvested will fall.

Greater for longer holding periods, as well as for bonds with large, early CF, such as high coupon bonds

Call Risk (callable bond)

Issuer can retire or “call” all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate)

Investor perspective: i) the CF pattern is not known with certainty, ii) exposed to reinvestment risk (issuers will call the bonds when interests have dropped) and iii) capital appreciation of a bond will be reduced

Risks Associated with Investing in Bonds (cont.)

Credit Risk:
Risk that the issuer of a bond will fail to satisfy the terms of the obligation (coupons and repayment of the amount borrowed)

Inflation Risk: arises because of the variation in the value of cash flows from a security due to inflation.

Exchange-rate Risk: A non-dollar-denominated bond has unknown US dollar cash flows.

Liquidity Risk: Size and spread btw the bid and ask price. The wider the dealer spread, the more the liquidity risk.

Volatility Risk: Value of an option rises when expected interest-rate volatility increases. In the case of a bond that is callable, the price of security falls, because the investor has given away a more valuable option.

Price Sensitivity to Interest Rate Changes

All else being the same, prices of longer maturity bonds are more sensitive to changes in interest rates than are prices of shorter maturity bonds

Investors bear higher interest rate risk and reinvestment risk with longer maturity bonds

Class Exercise 3: Consider two bonds each with an annual coupon rate of 10% and face value of $1,000. Bond A matures in 2 years and Bond B matures in 20 years. Assume both bonds have a yield to maturity of 10% p.a. What happens to the price of these bonds when market interest rates change unexpectedly to 6%, 8%, 12% and 14%? What is the general relationship between prices and yields to maturity?

Answer to Class Exercise 3

Both bonds are currently selling at par (why?)

Price of bonds A and B

$P_0^A = 100[(1 - (1 + k_d)^{-2})/k_d] + 1000/(1 + k_d)^2$

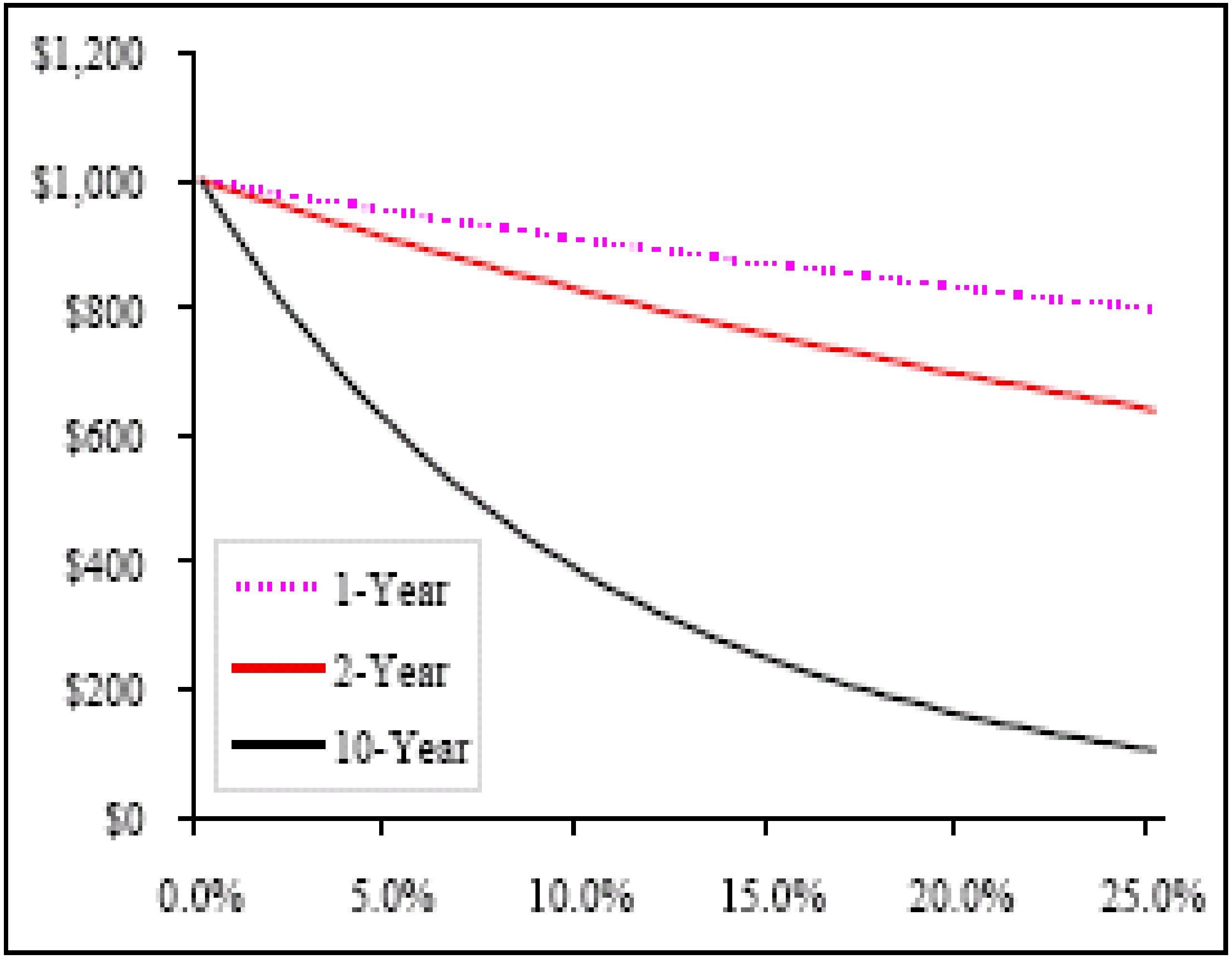
$P_0^B = 100[(1 - (1 + k_d)^{-20})/k_d] + 1000/(1 + k_d)^{20}$

Effect of market interest rate changes on prices

6%:	P_0^A = \$1073.34 (+7.33%)	P_0^B =\$1458.80 (+45.88%)
8%:	P_0^A = \$1035.67 (+3.57%)	P_0^B = \$1196.36 (+19.63%)
12%:	P_0^A = \$966.20 (-3.38%)	P_0^B = \$850.61 (-14.94%)
14%:	P_0^A = \$934.13 (-6.59%)	P_0^B = \$735.07 (-26.49%)

Percentage changes in prices compared to the *original* price of \$1,000 for each bond

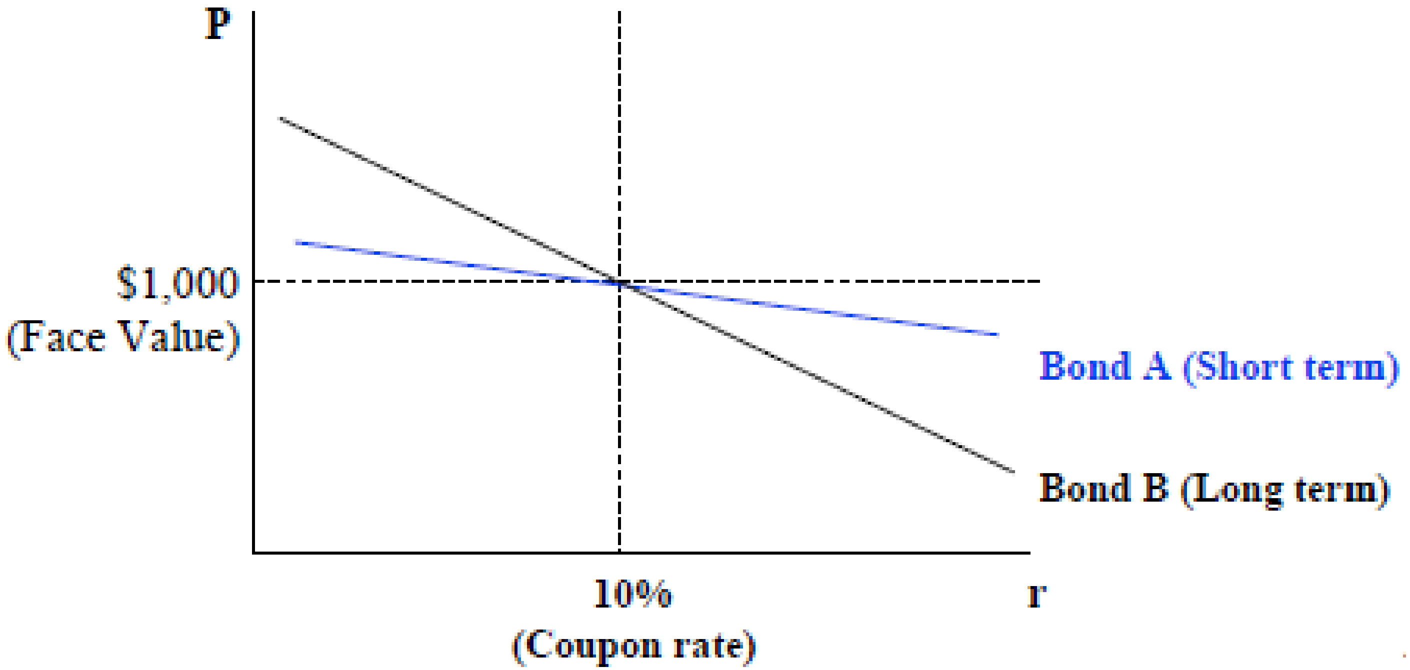
Bond Prices and Interest Rates



- Bond prices are inversely related to Interest Rate
- Longer term bonds are more sensitive to Interest Rate changes than short term bonds
- The lower the Interest Rate, the more sensitive the price

Price Sensitivity to Interest Rate Changes

Shorter maturity bond less sensitive to interest rate changes than similar longer maturity bond



Measuring Interest Rate Sensitivity

We would like to measure the interest rate sensitivity of a bond or a portfolio of bonds.

How much do bond prices change if interest rates change by a small amount? Why is it important?

Yield	10% 5 Years PV	10% 20 years PV
9.96%	100.1546	100.3441
9.97%	100.1159	100.2579
9.98%	100.0773	100.1718
9.99%	100.0386	100.0859
10.00%	100.00	100.00
10.01%	99.9614	99.9143
10.02%	99.9228	99.8286
10.03%	99.8843	99.7431
10.04%	99.8457	99.6578

Example

Consider a 30-year zero coupon bond with a face value of \$100. If the bond is priced at a yield-to-maturity of 10%, it will cost \$5.73 today (the present value of this cash flow). Over the coming 30 years, the price will advance to \$100, and the annualized return will be 10%.

Suppose that over the first 10 years of the holding period, interest rates decline, and the yield-to-maturity on the bond falls to 7%.

With 20 years remaining to maturity, the price of the bond will be \$25.84.

Even though the yield-to-maturity for the remaining life of the bond is just 7%, and the yield-to-maturity bargained for when the bond was purchased was only 10%, the return earned over the first 10 years is 16.26%. This can be found by evaluating:

$$(1+i) = \left(25.84/5.73\right)^{0.1} = 1.1626$$

Over the remaining 20 years of the bond, the annual rate earned is not 16.26%, but 7%

This can be found by evaluating:

$$25.84 = 100/(1+i)^{20}$$

$$(1+i) = \left(100/25.84\right)^{0.05} = 1.07$$

Over the entire 30 year holding period, the original $5.73 invested matured to $100, so 10% annually was made, irrespective of interest rate changes in between

Real and Nominal Interest rates

Inflation - Rate at which prices as a whole are increasing.

Nominal Interest Rate - Rate at which money invested grows.

Real Interest Rate - Rate at which the purchasing power of an investmen increases

$$1+\text{real interest rate}=\frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

Approximation formula

$$\text{Real int. rate}\approx\text{nominal int. rate}-\text{inflation rate}$$

The Fisher Effect And Expected Inflation

The relationship between nominal and real (inflation-adjusted) interest rates and expected inflation called the Fisher Effect (or Fisher Equation).

Nominal rate (*r*) is approximately equal to real rate of interest (*a*) plus a premium for expected inflation (*i*).

If real rate equals 3% (a = 0.03) and expected inflation equals 2% (i = 0.02):

$$r \cong a + i \cong 0.03 + 0.02 \cong 0.05 \cong 5\%$$

True Fisher Effect multiplicative, rather than additive:

$$(1+r) = (1+a)(1+i) = (1.03)(1.02) = 1.0506; \quad \text{so } r = 5.06\%$$

Interest Rates and Returns

Rate of return: payments to the owner plus the change in its value, expressed as a fraction of its purchase price

What would the rate of return be on a bond bought for $1,000 and sold one year later for $800. The bond has a face value of $1,000 and a coupon rate of 8%

$$R=\frac{C+P_{t+1}-P_t}{P_t}$$

Where: R is the rate of return, C the coupon payment, P_{t+1} price of the bond one year later and P_t price of the bond today

$$R=\frac{\$80+(\$800-\$1{,}000)}{\$1{,}000} \Leftrightarrow R=-12\%$$

One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%

(1)	(2)	(3)	(4)	(5)	(6)
Years to Maturity when Bond is Purchased	Initial Current Yield (%)	Initial Price ($)	Price Next Year ($)	Rate of Capital Gain (%)	Rate of Return (2+5)
30	10	1000	503	-49.7	-39.7
20	10	1000	516	-48.4	-38.4
10	10	1000	597	-40.3	-30.3
5	10	1000	741	-25.9	-15.9
2	10	1000	917	-8.3	+1.7
1	10	1000	1000	0.0	+10.0

Rate of Capital Gain

Calculate the rate of capital gain or loss on a ten-year zero coupon bond for which the interest rate has increased from 10% to 20%. The bond has a face value of $1,000

$$g=\frac{P_{t+1}-P_t}{P_t}$$

P_{t+1}= price of the bond one year from now $=\frac{\$1,000}{(1+0.20)^9}=\$193.81$

P= price of the bond today $=\frac{\$1,000}{(1+0.10)^{10}}=\$385.54$

Thus, $g=\frac{\$193.81-\$385.54}{\$385.54}=-0.497=-49.7\%$

But as we already calculated (slide 28), the capital gain on the 10% ten-year coupon bond is -40.3%. The interest rate risk for the ten-year coupon bond is *less* than for the ten-year zero coupon bond. The effective maturity on the coupon bond (which measures interest rate risk) is, as expected, *shorter* than the effective maturity on the zero-coupon bond.

Key Concepts

- Short term debt instruments mature in less than a year and only pay the promised face value at maturity

- Bonds are long term debt instruments issued by firms or the Government

- The price of a security today equals the present value of all future expected cash flows discounted at the "appropriate" required rate of return (or discount rate)

- A bond's yield to maturity is the interest rate an investor would earn if the bond was held until it matured (became due)

- Prices and yields to maturity are inversely related

- Prices of longer maturity bonds are more sensitive to changes in interest rates than are prices of shorter maturity bonds

For small changes in yield (such as one basis point) the percentage change in a bond price is roughly the same whether the yield is increased or decreased.

- For a larger change in yield the percentage rise in the bond price if the yield is decreased is greater than the percentage fall in the bond price if the yield is increased.

Maturity and Price Risk

- Zero coupon bonds have well-defined relationship between maturity and interest rate sensitivity.
- Coupon Bonds can have different sensitivities for the same maturity
- Need concept of “average maturity” of coupon bonds
    - DURATION

Key Relationships

- Price of discount security
  - $P_0 = S_n/[1 + (n/365) \times r]$
- Annualized cost of a BAB
  - $\left(\dfrac{Face\ Value}{P_0 - Costs} - 1\right)\dfrac{365}{n}$
- Price of coupon paying bond
  - $P_0 = [I/k_d][1 - (1 + k_d)^{-n}] + P_n/(1 + k_d)^n$
- Price of a zero coupon bond
  - $P_0 = P_n/(1 + k_d)^n$
- Price of a perpetual (non-maturing) bond
  - $P_0 = I/k_d$