FINA 1082 Financial Management

Dr Cesario MATEUS
Senior Lecturer in Finance and Banking
Room QA257 – Department of Accounting and Finance

c.mateus@greenwich.ac.uk
www.cesariomateus.com

Lecture 3

Valuation of Debt Securities II

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The arbitrage-free Approach to Bond Valuation

The traditional valuation approach is deficient because it uses a single discount rate (the appropriate YTM) to find the present value of the future cash flows with no regard given to the timing of those cash flows.

Cash flows received in year 1 on a 20 year bond are discounted at the same rate as the cash flows received in 20 years!

Arbitrage Free Valuation Model

$$P = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_3)^2} + \frac{CF_3}{(1+r_2)^3} + \ldots + \frac{CF_n}{(1+r_n)^n}$$

This model treats each separate cash flow paid by a fixed-income security as if it were a stand-alone zero-coupon bond. These discount rates are called spot rates.

Example

Give the following Treasury spot rates, calculate the arbitrage-free value of a 5% coupon, 2 year treasury note.

Maturity	Spot Rate
0.5 years	4.0%
1.0	4.4%
1.5	5.0%
2.0	5.2%

The arbitrage-free price of the note is:

$$P = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_3)^2} + \frac{CF_3}{(1+r_2)^3} + \ldots + \frac{CF_n}{(1+r_n)^n}$$

$$P = \frac{\$2.50}{(1.020)} + \frac{\$2.5}{(1.022)^2} + \frac{\$2.5}{(1.025)^3} + \frac{\$102.5}{(1.026)^4} = \$99.66 \text{ per } \$100 \text{ of par value}$$

Profiting from Arbitrage Opportunities: Stripping and Reconstituting Bonds

Stripping

Suppose the same 2-year, 5% coupon Treasury note is priced at $95.00, which is below its arbitrage-free value of $99.66. What action should arbitrageurs take and what will be the affect of their actions?

Because the note is priced below its arbitrage-free value, its zero-coupon cash flow “pieces ”are worth more than the note it self. Therefore, an arbitrage profit could be earned by:

- Buying the undervalued note at $95.00
- Stripping the note of its individual cash flows
- Selling the individual cash flows “pieces” as zero-coupon bonds for $99.66 and earning arbitrage profit of $4.66 per $100 of investment.

As this arbitrage is performed:

• The increased demand for the notes will cause their prices to increase and their yields maturity to fall

• The increased supply of zero-coupon bond "pieces" will cause the prices of zero-coupon bonds to fall and their yields (spot rates) to rise.

• These forces will quickly eliminate the arbitrage opportunity

Reconstituting

The 2-year, 5% coupon Treasury note is priced at $100. Its arbitrage-free value is $99.66. What action should arbitrageurs take and what will be the effect of their actions?

Because the note is priced above its arbitrage-free value, it is overpriced relative to the value of its zero-coupon cash flow "pieces". Therefore, an arbitrage profit can be earned by:

- Buying the zero coupon "pieces" in the zero-coupon treasury market for $99.66
- Reconstituting the note from these zero-coupon Treasuries.
- Selling the reconstituted note for $100 to earn an arbitrage profit of $0.34 for every $99.66 of original investment.

As dealers perform this arbitrage:

• The increased demand for Treasury zero-coupon bonds will drive their prices up and their yields (spot rates) down.

• The increased supply of reconstituted 2-year, 55 coupon treasuries will drive their prices down and their yields-to-maturity up

• These forces will quickly eliminate the arbitrage opportunity

Arbitrage Example

We observe two types of bonds: T-bills and coupon bonds. A one-year T-bills pays 1000 in one year, a two year T-bill pay 1000 in two years and a three year T-bill pays 1000 in three years.

There are no coupon interests on T-bills. The coupon bond is a 5% three-year bond with a face value of 1000. Thus the cash flow from the coupon bond are: In the first year, you receive 50, in the second year 50, and in the last year 1050. We observe the following prices:

Type of Bond	Price	Yield
One year T-bill	943.4	$\frac{100}{1+r}=943{,}4 \Leftrightarrow r=6\%$
Two year T-bill	873.44	$\frac{100}{(1+r)^2}=873{,}44 \Leftrightarrow r=7\%$
Three year T-bill	793.83	$\frac{100}{(1+r)^3}=793{,}83 \Leftrightarrow r=8\%$
Coupon bond	1.000	

Assumption: We can borrow funds at the above rates and short sell the securities without any costs

- No arbitrage profit:
  - Using no wealth
  - No risk
  - Positive return
- The first condition requires a long and short position
- To satisfy the second condition (no risk) we need to match the cash flows from the long and short positions

- Short sell (borrow) 20 coupon bonds and buy 1 one year T-bill, 1 two year T-bill and 21 three year T-bills $\rightarrow$ we do not use any of our own wealth.

Pricing by arbitrage

Cash flows from bond transactions						
			Cash flows at time:			
	Number of Bonds	Price	0	1	2	3
Coupon Bonds	20	1,000	20,000	-1,000	-1,000	-21,000
Short position - Loan						**Long Position**
One year T-bill	1	943,4	-943,4	1,000	0	0
Two year T-bill	1	873,44	-873,44	0	1,000	0
Three year T-bill	21	793,83	-16,670.43	0	0	21,000
TOTAL	3		1,512.73	0	0	0

We have an arbitrage profit of 1,512.73, with no risk and using none of our own funds

What will happen:

- Investors will sell the coupon bonds → prices start to drop
- Investors will buy T-bills → price start to increase
- The price of the coupon bond that is consistent with the no *arbitrage condition* is

924.3635

Pricing using discounted cash flow

$$P = \frac{50}{1+0.06} + \frac{50}{(1+0.07)^2} + \frac{1050}{(1+0.08)^3} = 47.17 + 43.67 + 833.52$$

924.37

• Each period has different discount rates since the opportunity costs for each cash flow (coupon payment) is different, i.e. different T-bills

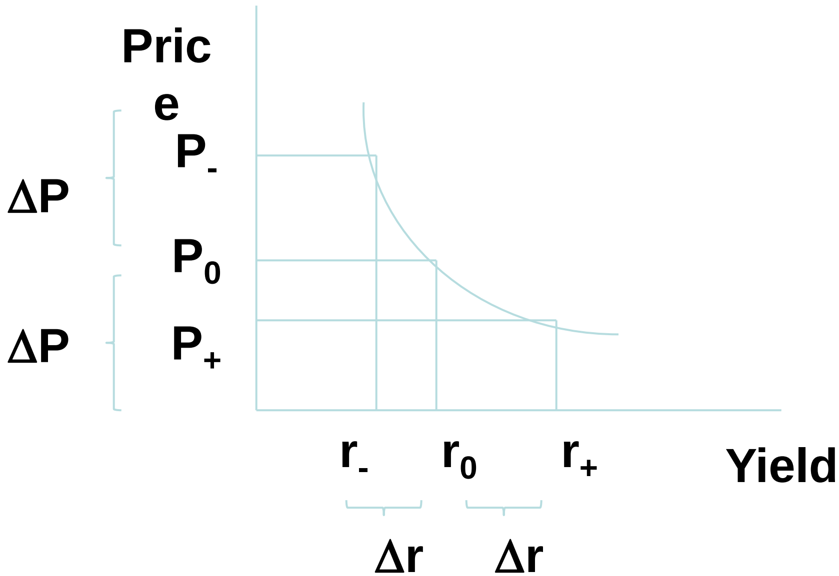
• Do not look for arbitrage profits in markets where is simple to construct arbitrage portfolios

- Bond market
- Option market
- Forward markets for exchange rates

Measuring Interest Rate Risk Using Duration

Easiest way to determine the sensitivity of a bond's price to changes in interest rates is to calculate how much the price of a bond would change if its YTM were to increase and decrease by a small amount, Δr, above an below its current level.

The concept is illustrated below, that depicts the price-yield relationship for a conventional bond whose current price and yield are P_0 and r_0, respectively.



Effective Duration

If the yield increased by a small amount Δr, from r_0 to r_+, the price of the bond will decrease from P_0 to P_-.

A bond's effective duration measures how sensitive the return on the bond (measured as the percentage change in its price) will be to the change in interest rates.

$$DE = \frac{\text{Percentage Change in Price of Bond}}{\text{Change in Interest Rates}}$$

$$= \frac{\Delta P/P}{\Delta \text{ rates}} = \frac{(P_- - P_+)/P_0}{2\Delta r}$$

$$DE = \frac{P_- - P_+}{2P_0 \Delta r}$$

Example:

A 7% coupon, 5-year bond, yielding 6% is priced at 104.265. If its yield declines by 25 basis points to 5.75%, the bond's price will increase to 105.366. On the other hand, if its yield increases by 25 basis points to 6.25%, the price of the bond will decline to 103.179. Compute the effective duration of the bond under these conditions.

$$DE = \frac{P_- - P_+}{2P_0 \Delta r} = \frac{105.366 - 103.179}{2(104.265)(.0025)} = 4.2$$

If this bond's yield increases by 1% (ostensibly because interest rates have increased by 100 basis points), the price of the bond will fall by approximately 4.2%.

Dollar Duration

Measures the dollar market value change resulting from a 100 basis point change in yield:

$$\text{Dollar duration} = -D_E \times (\$\text{ Market Value}) \times \Delta r$$

Example:

A manager has a holding of XYZ bond with a current market value of $25 million and a duration of 5.4. If the bond's yield dropped by 100 basis points, what would be the change in the market value.

$$\text{Dollar duration} = -D_E \times (\$\ \text{Market Value}) \times \Delta r$$
$$= -5.4 \times (\$25{,}000{,}000) \times -0.0100$$
$$= +\$1{,}350{,}000$$

Application of Effective Duration

From the basic formula: percentage change in the price of bond will approximately equal its effective duration times any change that occurs in its yield, but in opposite direction

$$\%\ \text{Change in Price of Bond} = \frac{\Delta P_B}{P_B} = -D_E \times \Delta r$$

Example:

A 7% coupon, 5-year bond, priced at 104.265 with duration of 4.2 has a YTM of 6%. Estimate the percentage change in the price and the new price of the bond if its YTM declines by 50 basis points from 6% to 5.5%.

$$\% \text{ Change in Price of Bond} = \frac{\Delta P_B}{P_B} = -D_E \times \Delta r = -4.2(-.005) = 2.1\%$$

$$P_{NEW} = P_0\left(1 + \frac{\Delta P_B}{P_B}\right) = 104.265 \times (1.021) = 106.455$$

Variations of Duration

Macaulay Duration

Macaulay Duration is the weighted average time-to-maturity of the cash flows of a bond. In the Macaulay, and all other duration measures, the weighting of the cash flows is based on their discounted present value, rather then their nominal value.

$$\text{Macaulay Duration} = \frac{\sum (\text{n})(\text{PV of Cash Flows})}{\text{k} \times \text{Bond Price}}$$

Where:
n is the number of periods until each cash flow is paid
K is the number of time coupon interest is paid per year

Macaulay's Duration

Measure the effective maturity of a coupon bond

Realize that a coupon bond is equivalent to a set of *zero-coupon discount bonds*

A ten-year 10% coupon bond is equivalent to a set of zero-coupon discount bonds

A ten-year 10% coupon bond with $1,000 face value has cash payments identical to the following set of a zero-coupon bonds:

- a $100 one year zero-coupon bond (which pays the equivalent of the $100 coupon payment made by the $1,000 ten-year 10% coupon bond at the end of one year;
- a $100 two-year zero-coupon bond (which pays the equivalent of the $100 coupon payment at the end of two years)
- ………………..
- and a $1,000 ten-year zero-coupon bond (which pays back the equivalent of the coupon bond's $1,000 face value)

Calculating Duration on a $1,000 ten-year 10% Coupon Bond when its Interest Rate is 10%

(1)	(2)	(3)	(4)	(5)
Year	Cash Payments (Zero-Coupon Bonds)	Present Value of Cash Payments	Weights (% of total)	Weighted Maturity (1×4)/100 years
1	**100**	**90.91**	**9.091**	**0.09091**
2	**100**	**82.64**	**8.264**	**0.16528**
3	**100**	**75.13**	**7.513**	**0.22539**
4	**100**	**68.30**	**6.830**	**0.27320**
5	**100**	**62.09**	**6.209**	**0.31045**
6	**100**	**56.44**	**5.644**	**0.33864**
7	**100**	**51.32**	**5.132**	**0.35924**
8	**100**	**46.65**	**4.665**	**0.37320**
9	**100**	**42.41**	**4.241**	**0.38169**
10	**100**	**38.55**	**3.855**	**0.38550**
10	**1000**	**385.54**	**38.554**	**3.85500**
Total		1,000.00	100.00	**6.75850**

Duration is a weighted average of the maturities of the cash payments

Can be written as follows:

$$DUR = \sum_{t=1}^{n} t \frac{CP_t}{(1+i)^t} \Bigg/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

Where:

DUR = Duration

t = years until cash payment is made

CP_t = Cash payment (interest plus principal) at time t

i = interest rate

n = years to maturity of the security

All else being equal, the *longer the term to maturity* of a bond, the *longer its duration*.

All else being equal, when *interest rates rise*, the *duration of a coupon bond falls*.

All else being equal, the *higher the coupon rate* on the bond, the *shorter the bond's duration*.

The duration of a portfolio of securities is the *weighted average of the durations* of the individual securities, with the weights reflecting the proportion of the portfolio invested in each.

The *greater the duration* of a security, the *greater the percentage change* in the market value of the security for a *given change in interest rates*. Therefore, the *greater the duration* of a security, the *greater its interest-rate risk*

Modified Duration

Modified Duration is an adjusted measure of the Macaulay duration that produces a more accurate estimate of how much the percentage change in the price of a bond will be per 100 basis points change in the interest rate.

$$\text{Modified Duration } (D^*) = \frac{\text{Macaulay Duration}}{(1 + \text{yield} / k)}$$

Where
Yield is the yield-to-maturity of the bond
K is the number of periodic payment (compounding) periods per year

Interpretations of Duration

Three interpretations:

1. Effective duration is the first derivative of the price-yield relationship of a security.

$$\mathrm{DE} = -\frac{dP/dr}{\mathrm{P}}$$

While correct, this interpretation only has meaning to the mathematically inclined.

2. Unadjusted duration is a weighted average of time. Indeed Macaulay's (unadjusted) duration is measured in units of time (years) .Thus a Macaulay's duration may be said to be 8 years. This may be meaningful to an investment professional who understands that a bond with a duration of 8 years is more volatile than a bond with a duration of 3 years.

3. Effective duration is a measure of how sensitive the return on a bond is to small changes in interest rates

$$D_E = - \frac{\Delta P/P}{\Delta r}$$

Indicates that if a bond has a duration of 3, its price will rise or fall 3% every time interest rates fall or rise by 100 basis points. A bond of duration of 6, has therefore twice as much interest risk as the first. However, that effective duration only measures interest rate risk – it does not measure any other type of risk (credit risk, currency risk, liquidity risk, and so forth).

Calculating Bond Portfolio Duration

$$D_{Portfolio} = W_1 D_1 + W_2 D_2 + ... + W_n D_n$$

Example:

A barbell portfolio is constructed with 60% of its value invested in a 4-year bond with an effective duration of 3.0 and 40% of its value invested in a 15-year bond with an effective duration of 10.0. What is the effective duration of the portfolio?

$$D_E = W_1 D_{E1} + W_2 D_{E2} = .6(3) + .4(0) = 5.8$$

Duration and Yield Volatility Determine Risk

For a given basis point change in interest rates, the price volatility (sensitivity to interest rate changes) of a standard bond is a function of 3 factors:

1. Bonds with long maturities are more volatile than bonds with short maturities
2. Low-coupon bonds are more volatile than high-coupon bonds
3. The higher interest rates are, the lower the volatility of bonds.

Example

Compare a 10% coupon, B-rated, 10 year corporate bond yielding 11% to a 6% coupon, 10-year Treasury bond, yielding 4%. Which bond will ne more volatile?

•Their maturities are the same, suggesting equal volatility

•The corporate has a higher coupon, suggesting lower volatility

•The corporate trades at a highest interest rate, suggesting lower volatility

Bond Convexity

Measure of the sensitivity of the duration of a bond to changes in interest rates.

There is an inverse relationship between convexity and sensitivity:

- Higher the convexity, the less sensitive the bond price is to interest rate shifts
- The lower the convexity, the more sensitive it is.

Duration: linear measure of 1st derivative of how the price of a bond changes in response to interest rate changes.

- As interest rates change, the price is not likely to change linearly, but instead it would change over some curved function of interest rates.
- The more curved the price function of the bond is, the more inaccurate duration is as a measure of the interest rate sensitivity.

Convexity

Measure of the curvature or 2nd derivative of how the price of a bond varies with interest rate, i.e. *how the duration of a bond changes as the interest rate changes*.

One assumes that the interest rate is constant across the life of the bond and that changes in interest rates occur evenly

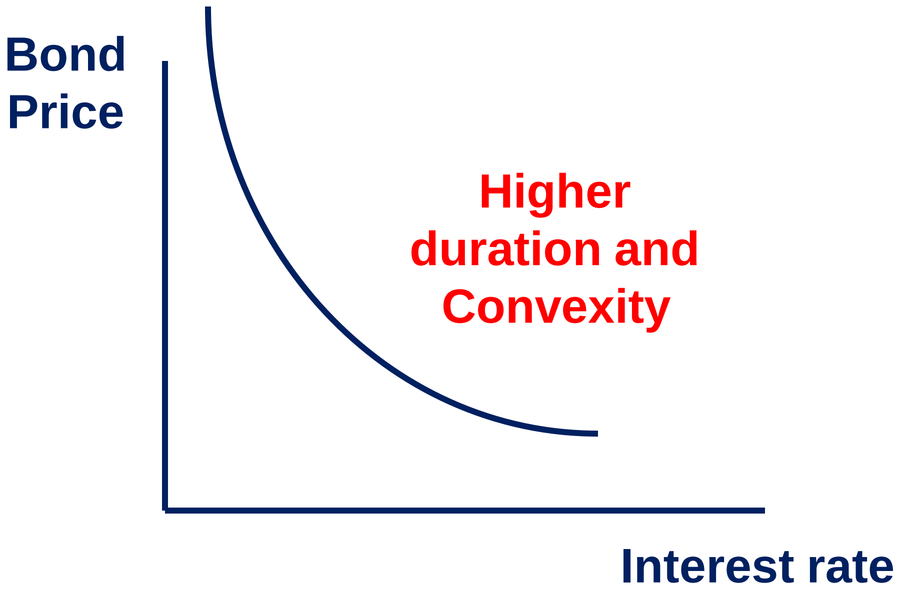
Using these assumptions, duration can be formulated as the first derivative of the price function of the bond with respect to the interest rate in question.

Convexity would be the second derivative of the price function with respect to the interest rate.

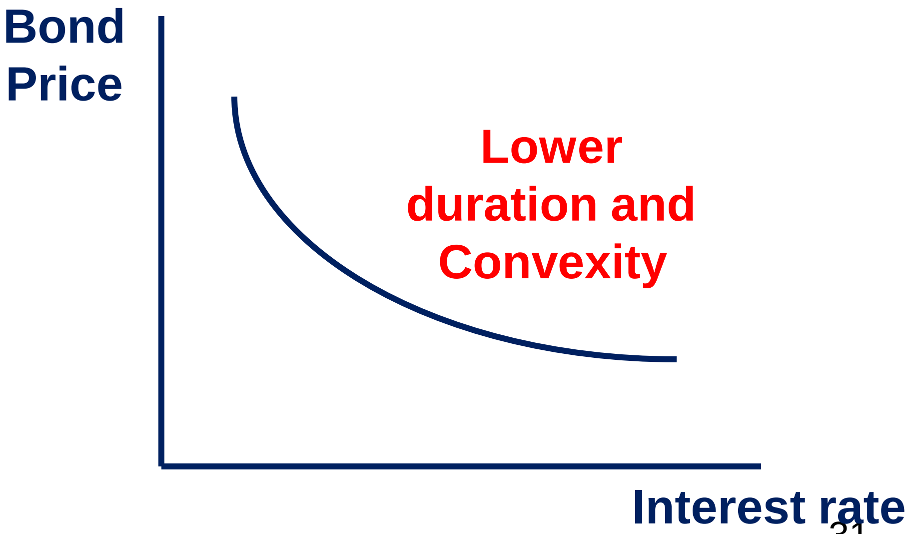
Why bond convexities differ?

The price sensitivity to parallel changes in the term structure of interest rates is highest with a zero-coupon bond and lowest with an amortizing bond.

30-year maturity,
8% coupon
Straight bond



5-year maturity,
8% coupon
Straight bond



The divergence from the straight line is called convexity

As interest rates decline duration projects the bond's price to rise, but the actual increase in price is greater than duration projects.

If interest rates rise, the duration line projects the bond's price will fall, but the actual decline in price is less than duration projects.

Positive convexity: a larger increase in price than decrease in price, for the same change in rates. The upside is greater than downside.

All bonds that are free of embedded options will have positive convexity, but the actual degree of duration and convexity of bonds will vary (see previous graphs)

Observations about Straight Bonds

Generalizations about option-free bonds:

1) The price-yield relationship is negative, meaning that the price and yield move in opposite directions.

2) For small changes in interest rates, the upside and downside price moves are approximately equal in size.

3) For large changes in interest rates, the upside and downside will diverge from the duration life.

4) For option-free bonds, the upside will be greater than the downside price move for the same change in rates.

5) The actual level of duration and convexity will vary with the terms of the bond.

6) As interest rates change from high to low, the duration of the bond will increase (and so will be convexity).

Amortizing and the zero-coupon bond's

Although the amortizing bond's and the zero-coupon bond's have different sensitivities at the same maturity, if their final maturities differ so that they have identical bond durations they will have identical sensitivities.

That is, their prices will be affected equally by small, first-order, (and parallel) yield curve shifts. They will, however, start to change by different amounts with each further incremental parallel rate shift due to their differing payment dates and amounts.

Two bonds with same par value, same coupon and same maturity, convexity may differ depending on at what point on the price yield curve they are located.

Example

Suppose two bonds with at the present the same price yield combination.

Take into consideration the profile, rating, etc of the issue and that they are issue by different entities.

Bond A may be located on a more elastic segment of the price-yield curve compared to bond B.

Means that:

- If yield increases further, price of bond A may fall drastically while price of bond B won't change
    - Bond B holder are expecting a price rise any moment and so reluctant to sell it off
    - Bond A holders are expecting further price-fall and ready to dispose it.

This means:

Bond B has better rating than bond A!

Higher the rating or credibility of the issuer the less the convexity and the less the gain from risk-return game or strategies; after all less convexity means less price-volatility or risk, less risk means less return.

Calculating Convexity

$$C_E = \frac{P_- + P_+ - 2P_0}{2P_0 \Delta r^2}$$

Where:

P_0 is the initial price of the bond

P_- is the price if the yields decline by Δr

P_+ is the price if the yields increases by Δr

Δr is the change in the bond's yield, which can also be viewed as being a change in interest rates if the bond's yield spread is assumed to be constant

Bond Sensitivity to Interest Rate Changes

Duration and convexity can be used to project how the price of a bond, or the value of a portfolio of bonds, will change for a given change in interest rates

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2$$

Where:

P_0 is the initial price of the bond

ΔP is the change in the price of the bind associated with Δr

D_E is the effective duration of the bond

C_E is the effective convexity of the bond

Δr is the change in the bond's yield, which can also be viewed as being a change in interest rates if the bond's yield spread is assumed to be constant

Note: duration and convexity are simply variations on the first and second derivative of the price function of the bond.

Example:

A 10-year, 8% coupon bind is selling at 93.5000, with a yield-to-maturity of 9.0%. A bond valuation model indicates that if the yield on the bond is increased by 50 basis points to 9.5%, its price will fall to 90.4520, and if the bond's yield is shocked downward by 50 basis points to 8.5%, the bond's value will increase to 96.6764.

What is the effective duration and effective convexity of this bond at the current price of 93.50000? If interest rates decreased by 100 basis points, what is the estimated percentage change in decrease by 100 basis points, what is the estimated percentage change in the price of the bond?

$$D_E = \frac{P_- - P_+}{2P_0 \Delta r^2} = \frac{96.6764 - 90.4520}{2(93.5000)(0.0050)} = 6.657$$

$$C_E = \frac{P_- + P_+ - 2P_0}{2P_0 \Delta r^2} = \frac{96.6764 + 90.4520 - 2(93.5000)}{2(93.5000)(0.0050)^2} = 27.465$$

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2 = -6.657(-0.01) + 27.465(0.01)^2 = 6.93\%$$

Magnitude of changes in interest rates

If Δr is small, Δr^2 will approach zero.

The convexity adjustment $C_E \Delta r^2$ will be insignificant

Convexity is not important in estimating the price change of a bond for small changes in yields.

Example

Bond with and effective duration of 3.94 and a convexity of 9.685

For a large change in rates convexity matters:

If interest rates rise by 1% (100 basis points)

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2 = -3.940(0.010) + 9.685(0.010)^2 = -3.84\%$$

If interest rates decrease by 1% (100 basis points)

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2 = -3.940(0.010) + 9.685(-0.010)^2 = 4.04\%$$

For a small change in rates convexity does not matter:

If interest rates increase by 1 basis point (0.0001)

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2 = -3.940(0.0001) + 9.685(0.0001)^2 = -0.04\%$$

If interest rates decrease by 1 basis point (0.0001)

$$\%\Delta P = \frac{\Delta P}{P} = -D_E \Delta r + C_E \Delta r^2 = -3.940(-0.0001) + 9.685(-0.0001)^2 = +0.04\%$$

$C_E \Delta r^2$ is virtually zero (at four decimal places), so the upside and downside percentage price movements are virtually equal.

Duration and Yield Volatility Determine Risk

For a given basis point change in interest rates, the price volatility (sensitivity to interest rate changes) of a standard bond is a function of three factors:

1. Bonds with long maturities are more volatile than bonds with short maturities.
2. Low-coupon bonds are more volatile than high-coupon bonds.
3. The higher interest rates are, the lower the volatility of bonds.

Example

Compare a 10% coupon, B-rated, 10-year corporate bond yielding 11% to a 6% coupon, 10-year Treasury bond, yielding 4%. Which bond will be more volatile?

- Their maturities are the same, suggesting equal volatility.
- The corporate has a higher coupon, suggesting lower volatility.
- The corporate trades at a higher interest rate, suggesting lower volatility.

- Considering 3 factors, corporate bond will be less volatile if interest rates move by the same amount for both bonds.
- Corporate bond has the lower duration.
- However, if yields in the corporate bond market moves by more basis points than Treasury yields, then the corporate could turn out to be more (or less) volatile than the Treasury, even though the corporate bond has a lower duration.

Duration, tells what happens when interest rates move by the same amount for both securities.

If they do not, you must project the percentage price change using both the duration and the interest rate change.

$$\%\Delta P = -D_E \Delta r$$

Term structure of Interest Rates

- Effect of economic factors
  - real growth rate
  - tightness or ease of capital market
  - expected inflation
  - or supply and demand of loanable funds

- Impact of bond characteristics
  - credit quality
  - term to maturity
  - foreign bond risk including exchange rate risk and country risk

The Term Structure of Interest Rates

Term structure

- bonds with the same characteristics, but different maturities
- focus on Treasury yields
  - same default risk, tax treatment
  - similar liquidity
  - many choices of maturity

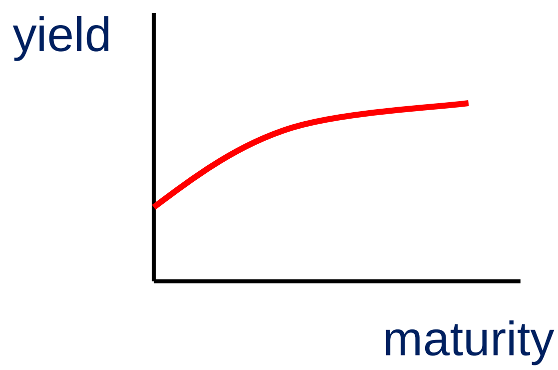
Relationship between yield & maturity is NOT constant

- sometimes short-term yields are highest,
- most of the time long-term yields are highest

The Yield Curve

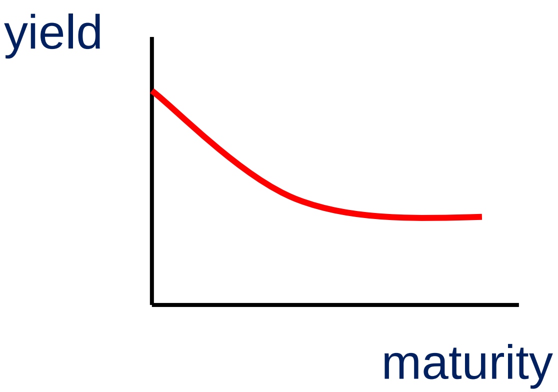
- plot of maturity vs. yield
- slope of curve indicates relationship between maturity and yield

upward sloping



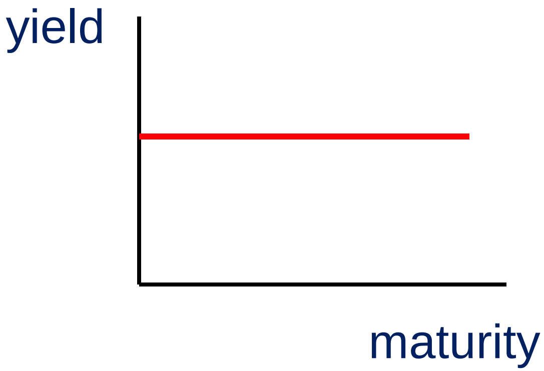
yields rise w/ maturity (common)
July 1992, currently

downward sloping (inverted)



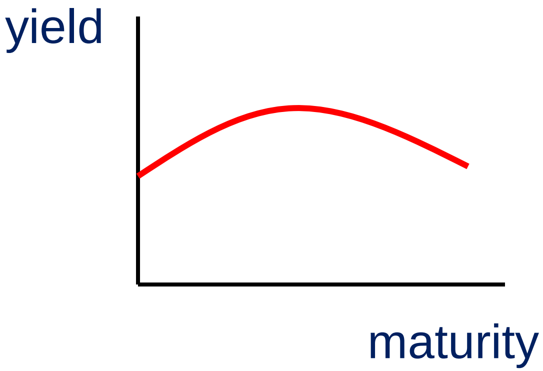
yield falls w/ maturity (rare)
April 1980

Flat



yields similar for all maturities
June 2000

humped



intermediate yields are highest
May 2000