



333-201 Business Finance

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Lecture 5

Risk and Return

Objectives

- Measure returns earned on securities
- Examine realized returns and their variability over time
- Describe the probability distribution approach
- Explain the concept of standard deviation as a measure of risk
- Interpret risk and return measures
- Illustrate how investor risk preferences can be represented

Returns on Financial Assets

In any period, the observed (or realized) returns of an asset over that holding period are measured as the change in cash flows divided by the initial investment

Realized returns can be measured discretely or continuously

Discrete returns

$$R_t = (P_t + D_t - P_{t-1})/P_{t-1} = (P_t + D_t)/P_{t-1} - 1$$

Continuously compounded returns

$$r_t = \ln [(P_t + D_t)/P_{t-1}]$$

$\ln[\cdot]$ is the natural logarithm

Discrete returns used more commonly rather than continuous returns



Returns on Financial Assets

Example: You purchased the shares of Cozco Ltd for \$5.00 and received a dividend of \$0.50. Compute the discrete and continuous returns earned if you sold the shares for (a) \$5.50 and (b) \$4.30

Discrete returns

$$R_t = (5.50 + 0.50)/5.00 - 1 = 20.0\%$$

$$R_t = (4.30 + 0.50)/5.00 - 1 = -4.0\%$$

Continuous returns

$$r_t = \ln [(5.50 + 0.50)/5.00] = 18.2\%$$

$$r_t = \ln [(4.30 + 0.50)/5.00] = -4.1\%$$

Discrete returns typically **exceed** continuous returns

Discrete returns tend to be **more volatile** (variable) over time than continuous returns

Geometric vs Arithmetic Average Returns

The **arithmetic** average (r_a) measures returns earned from a **single, one period** investment over that time horizon

$$r_a = (R_1 + R_2 + \dots + R_n)/n$$

The **geometric** average (r_g) measure returns earned **per period** from an investment over its **entire** time horizon

$$r_g = [(1+R_1)(1+R_2) \dots (1+R_n)]^{1/n} - 1$$

Only the geometric average can be used to link the starting and ending values of an investment

$$\begin{aligned} X_n &= X_0 (1 + r_g)^n \\ X_n &\neq X_0 (1 + r_a)^n \end{aligned}$$

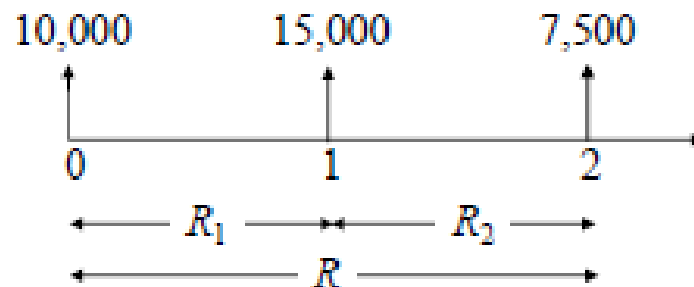
where X_0 and X_n are the cash flows at time 0 and n , respectively.

Arithmetic averages can give **incorrect** or nonsensical results

Geometric vs Arithmetic Average Returns

Example: An investment's value changed from \$10,000 to \$15,000 from year 0 to the end of year 1 and then to \$7,500 in year 2.

- a) Compute the investment's arithmetic and geometric average returns.
- b) What average **annual return** has the investor earned over the entire two year period? Relate this return to the investment's value



- a) Annual returns over year 1, $R_1 = (15000 - 10000)/10000 = +50\%$
Annual returns over year 2, $R_2 = (7500 - 15000)/15000 = -50\%$
The total return over years 0 to 2, $R = (7500 - 10000)/10000 = -25\%$

Geometric vs Arithmetic Average Returns

Arithmetic average return = $(+0.5 - 0.5)/2 = 0\%$

Note that $X_n \neq X_0(1 + r_a)^n$

So, $7500 \neq 10000(1 + 0.0)^2$

Geometric average = $[(1 + 0.5)(1 - 0.5)]^{1/2} - 1 = -13.4\%$

Note that $X_n = X_0(1 + r_g)^n$

So, $7500 = 10000(1 - 0.134)^2$

Also, the total return, $R = (1 - 0.134)^2 - 1 = -25\%$

Geometric vs Arithmetic Average Returns

Example: For the following investments (a) compare the arithmetic and geometric average rates of returns, and (b) outline the general relationship between these averages.

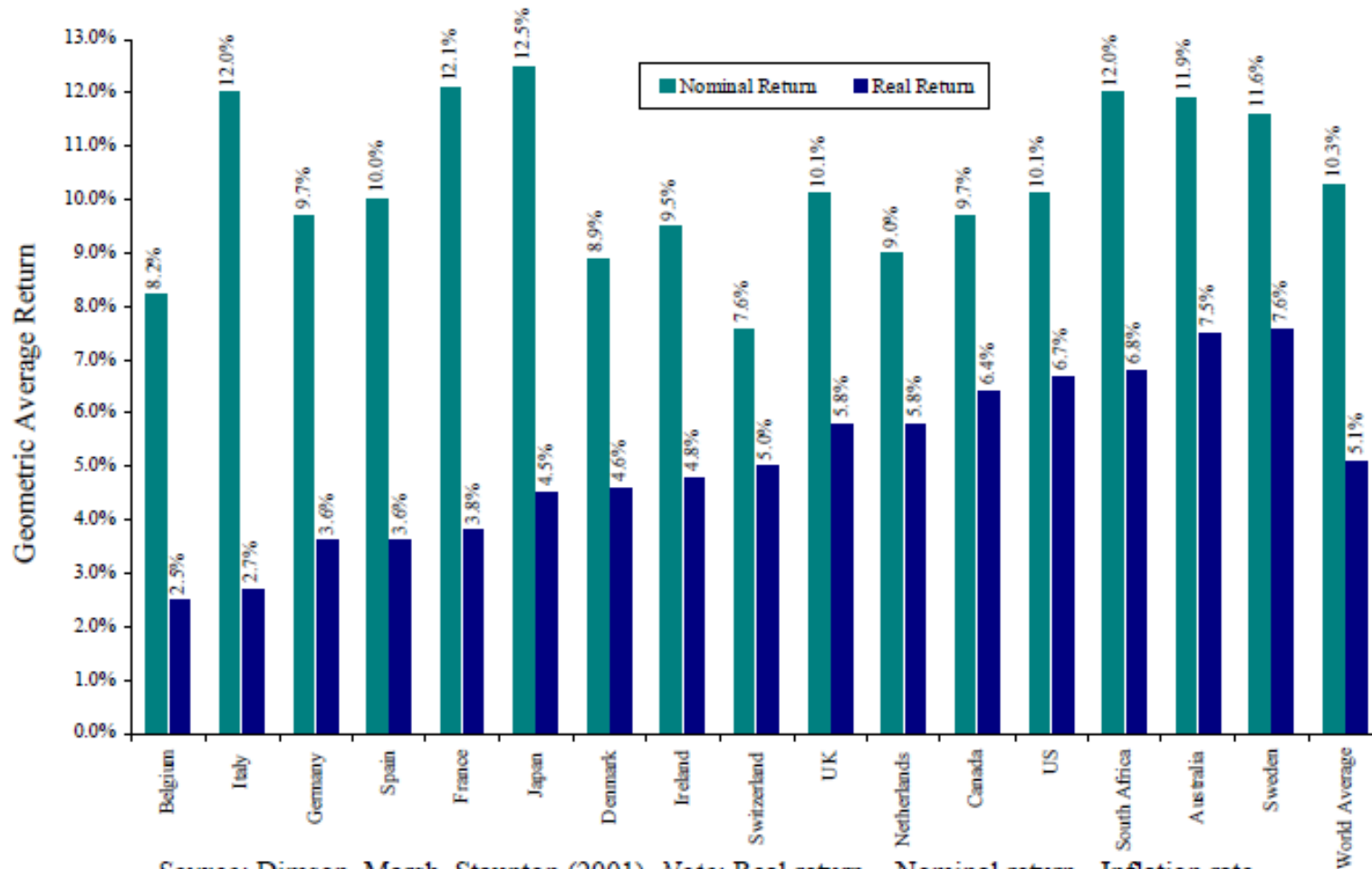
Year	Investment A		Investment B	
	Value	Return	Value	Return
0	100.0	-	100.0	-
1	120.0	20.0%	110.0	10.0%
2	90.0	-25.0%	121.0	10.0%
3	135.0	50.0%	140.0	15.7%
4	148.0	9.6%	150.0	7.1%

$$R_t = (X_t - X_{t-1}) / X_{t-1}$$

<u>Average Return</u>	<u>Investment A</u>	<u>Investment B</u>
<i>Arithmetic</i>	$(0.20 - 0.25 + 0.50 + 0.096) / 4 = 13.66\%$	10.71%
<i>Geometric</i>	$[(1.2)(0.75)(1.5)(1.1)]^{1/4} - 1 = 10.30\%$	10.67

Arithmetic averages **exceed** geometric averages but these differences **decline** as the **volatility of returns decline**.

Nominal and Real Equity Returns: 1900 - 2000



Source: Dimson, Marsh, Staunton (2001). Note: Real return \approx Nominal return - Inflation rate

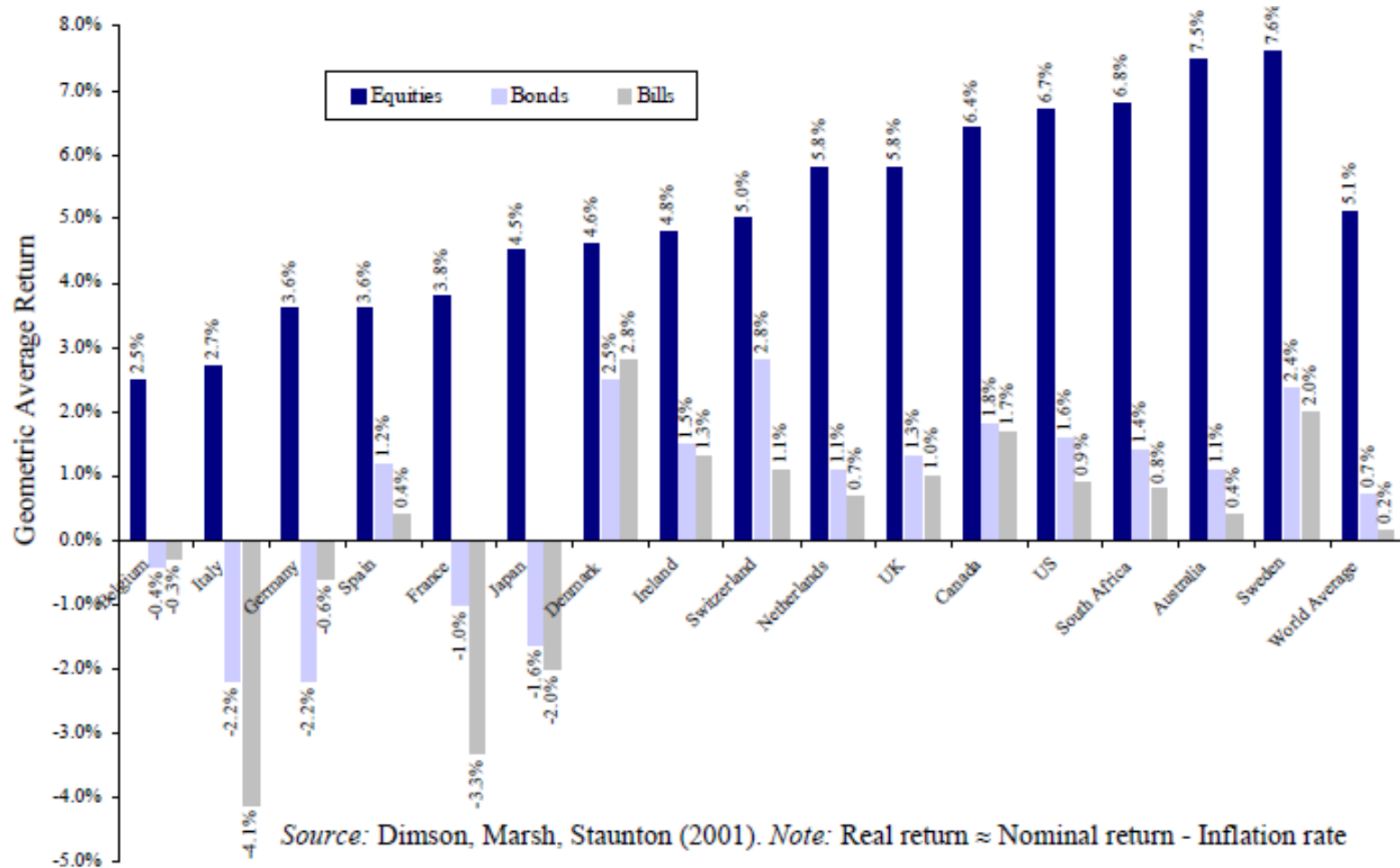
Real Equity Returns Around the World: 1900 - 2000

Country	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)	Minimum Return (%)	Minimum Year	Maximum Return (%)	Maximum Year
Australia	7.5	9.0	17.7	-34.2	1974	53.5	1983
Belgium	2.5	4.8	22.8	-40.9	1947	100.5	1940
Canada	6.4	7.7	16.8	-32.0	1974	55.2	1933
Denmark	4.6	6.2	20.1	-28.4	1974	106.1	1983
France	3.8	6.3	23.1	-37.5	1947	66.1	1954
Germany	3.6	8.8	32.3	-89.6	1948	155.9	1949
Ireland	4.8	7.0	22.2	-54.3	1974	69.9	1977
Italy	2.7	6.8	29.4	-72.9	1945	120.7	1946
Japan	4.5	9.3	30.3	-84.0	1946	119.6	1952
Netherlands	5.8	7.7	21.0	-34.9	1941	101.6	1940
South Africa	6.8	9.1	22.8	-52.2	1920	102.9	1933
Spain	3.6	5.8	22.0	-43.3	1977	98.9	1986
Sweden	7.6	9.9	22.8	-43.0	1918	89.5	1905
Switzerland*	5.0	6.9	20.4	-37.8	1974	56.2	1985
United Kingdom	5.8	7.6	20.0	-57.1	1974	96.7	1975
United States	6.7	8.7	20.2	-38.0	1931	56.8	1933

* Swiss equities are from 1911.

Source: Dimson, Marsh, Staunton (2001). Note: Real return \approx Nominal return - Inflation rate

Real Returns on Major Asset Classes: 1900 - 2000



Real Returns on Major Asset Classes: 1900 - 2000

Country	Equities (%)		Bonds (%)		Bills (%)	
	Arithmetic Mean	Standard Deviation	Arithmetic Mean	Standard Deviation	Arithmetic Mean	Standard Deviation
Australia	9.0	17.7	1.9	13.0	0.6	5.6
Belgium	4.8	22.8	0.3	12.1	0.0	8.2
Canada	7.7	16.8	2.4	10.6	1.8	5.1
Denmark	6.2	20.1	3.3	12.5	3.0	6.4
France	6.3	23.1	0.1	14.4	-2.6	11.4
Germany*	8.8	32.3	0.3	15.9	0.1	10.6
Ireland	7.0	22.2	2.4	13.3	1.4	6.0
Italy	6.8	29.4	-0.8	14.4	-2.9	12.0
Japan	9.3	30.3	1.3	20.9	-0.3	14.5
The Netherlands	7.7	21.0	1.5	9.4	0.8	5.2
South Africa	9.1	22.8	1.9	10.6	1.0	6.4
Spain	5.8	22.0	1.9	12.0	0.6	6.1
Sweden	9.9	22.8	3.1	12.7	2.2	6.8
Switzerland**	6.9	20.4	3.1	8.0	1.2	6.2
United Kingdom	7.6	20.0	2.3	14.5	1.2	6.6
United States	8.7	20.2	2.1	10.0	1.0	4.7

*Bond and bill statistics for Germany exclude the years 1922-23.

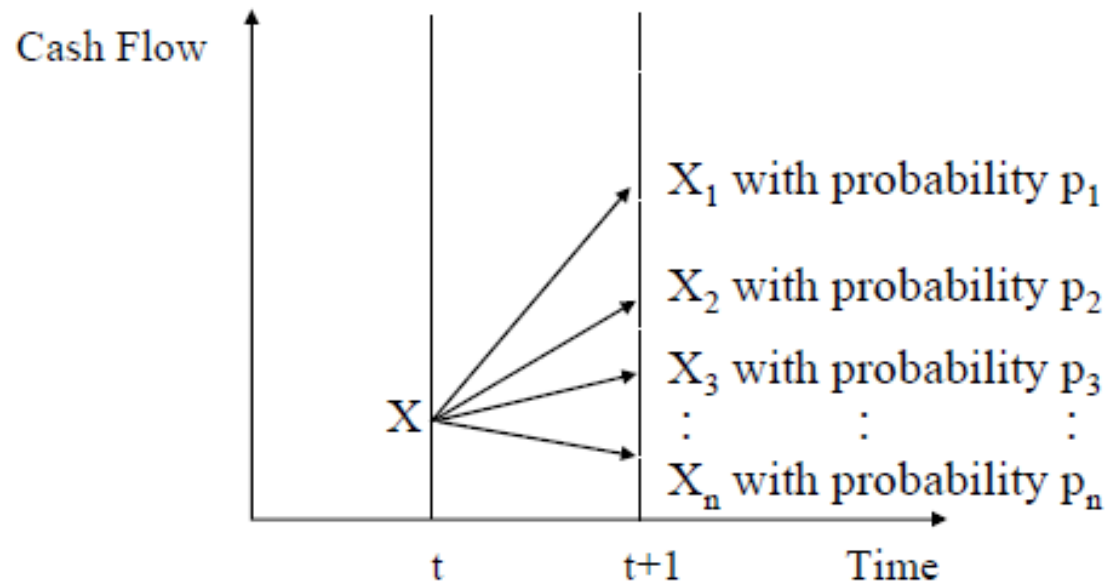
** Swiss equities are from 1911.

Source: Dimson, Marsh, Staunton (2001). Note: Real return \approx Nominal return - Inflation rate

The Probability Distribution Approach

We assume that investors can specify the possible outcomes (X_1 through X_n) and associate probabilities or likelihoods (p_1 through p_n) with these outcomes

Note: X_1 through X_n are not known with certainty



The Probability Distribution Approach

We assume that investors can specify the possible outcomes (X_1 through X_n) and associate probabilities or likelihoods (p_1 through p_n) with these outcomes. For each state, convert the cash flows into rates of returns

State	Probability	Cash Flows	Rate of Return
1	p_1	X_1	$R_1 = (X_1 - X)/X$
2	p_2	X_2	$R_2 = (X_2 - X)/X$
3	p_3	X_3	$R_3 = (X_3 - X)/X$
:	:	:	:
n	p_n	X_n	$R_n = (X_n - X)/X$

Note: $p_1 + p_2 + \dots + p_n = 1$, because one of these states of the world will be realized

Measures of Risk and Expected Return

The **expected return** is the expected outcome measured as the weighted average of the individual outcomes

$$E(r) = R = \mu = p_1R_1 + p_2R_2 + \dots + p_nR_n$$

The **variance or standard deviation of returns** is the measure of dispersion around the expected return. Greater the dispersion, higher the risk and uncertainty

$$\text{Var}(r) = \sigma^2 = p_1(R_1 - \mu)^2 + p_2(R_2 - \mu)^2 + \dots + p_n(R_n - \mu)^2$$
$$\text{SD}(r) = \sigma$$

Note: Variance and standard deviation take into account returns above and below the expected return. Investor concern is typically with returns below the expected return (**downside risk**)



Measures of Risk and Expected Return

Class Exercise 1: The current price of Stock X is \$10. The stock does not pay a regular dividend and market analysts expect the following three states to occur one year from now

State	Probability	Price
Bad	0.3	\$9.00
Good	0.4	\$11.00
Awesome	0.3	\$12.00

Compute the expected return and standard deviation of return for the above investment.

Answers to Class Exercise 1

- Convert the cash flows into rates of return for each state
- The return distribution for Stock X is

State	Probability	Rate of Return
Bad	0.3	$(9 - 10)/10 = -10\%$
Good	0.4	$(11 - 10)/10 = 10\%$
Awesome	0.3	$(12 - 10)/10 = 20\%$

$$E(r) \text{ or } \mu = 0.3(-0.1) + 0.4(0.1) + 0.3(0.2) = 7.0\%$$

$$\text{Var}(r) = 0.3(-0.1-0.07)^2 + 0.4(0.1-0.07)^2 + 0.3(0.2-0.07)^2 = 0.0141$$

$$\text{SD}(r) \text{ or } \sigma = (0.0141)^{1/2} = 0.1187 \text{ or } 11.9\%$$

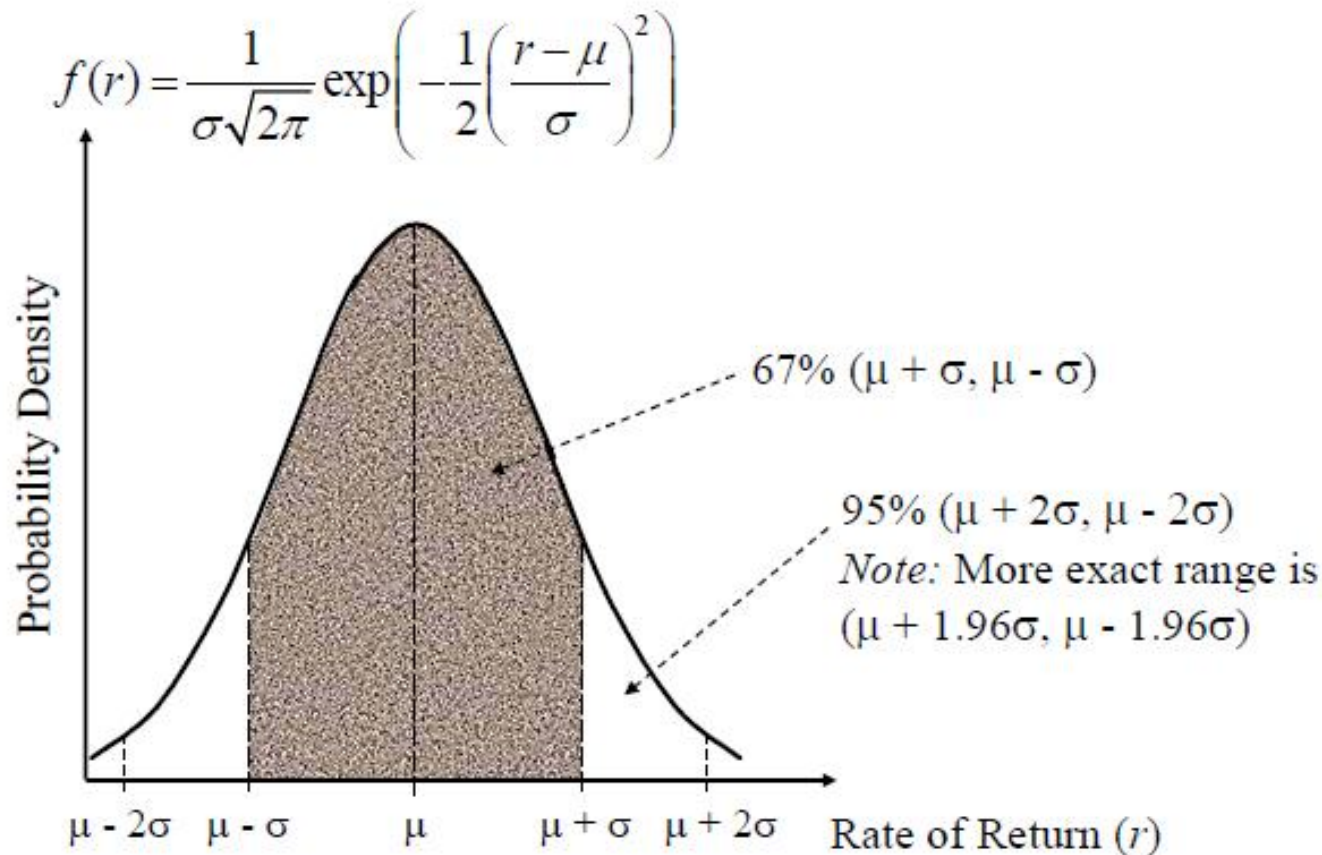
Interpreting Return and Risk Measures

- A general interpretation of the expected return and standard deviation of return requires assuming returns are **continuously** and **normally** distributed.
 - Only the expected (or mean) return and standard deviation of return are needed to fully describe the distribution of returns

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{r - \mu}{\sigma}\right)^2\right)$$

Note: A normal distribution implies an unlimited downside loss potential - unrealistic assumption because the most you can lose on a security purchase is 100%!

Interpreting Return and Risk Measures



Interpreting Return and Risk Measures

- In the previous example, $\mu = 7\%$ and $\sigma = 11.9\%$
- Assume you purchased 100 shares of Stock X, investing \$1,000
 - The expected value of this security next period is
$$1000(1 + \mu) = 1000(1.07) = \$1,070$$
 - There is a 67% probability that the **realized** return will lie in the range $(\mu - \sigma, \mu + \sigma)$ or $(-4.9\%, 18.9\%)$
 - There is a 95% probability that the **realized** return will lie in the range $(\mu - 2\sigma, \mu + 2\sigma)$ or $(-16.8\%, 30.8\%)$
- There is a 95% probability that a \$1000 investment in this security will be worth between $1000(1 - 0.168) = \$832$ and $1000(1 + 0.308) = \$1,308$ next period
 - The **range of final wealth** is \$832 - \$1,308 with a 95% likelihood of the **actual/realized wealth** lying in this range



Specifying Investor Preferences

Class Exercise 2: You're given the following information on four risky securities. Which securities would a risk averse investor prefer?

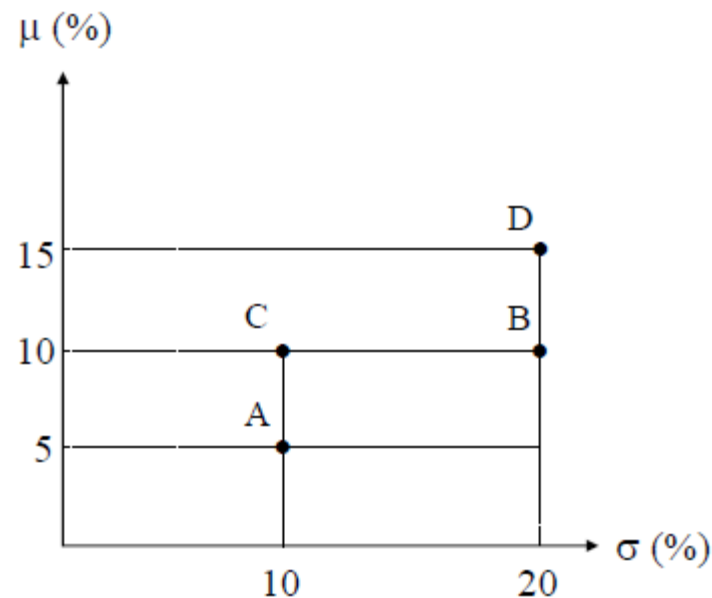
Security	Expected Return	Standard Deviation of Returns
A	5%	10%
B	10%	20%
C	10%	10%
D	15%	20%

Answers to Class Exercise 2

- Risk averse investors would prefer the **highest expected returns** and the **lowest risk levels**

- C is preferred to A
- C is preferred to B
- D is preferred to B

- What about A and B, C and D, A and D?



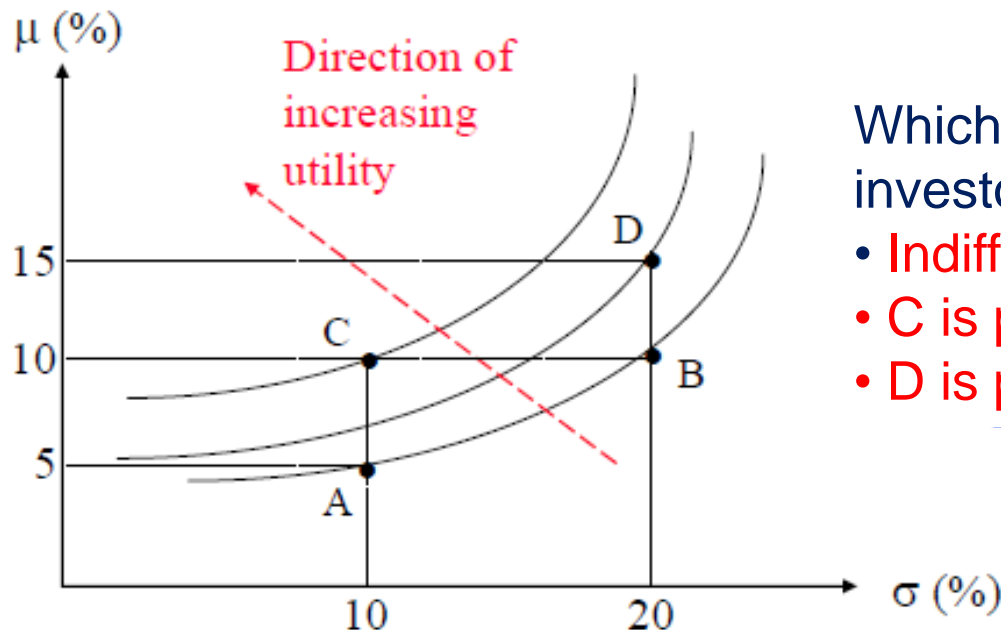
Specifying Investor Preferences

Assuming asset returns are normally distributed, investor utility can be written as a function of μ and σ^2 : $U = f(\mu, \sigma^2)$

Example: $U = \mu - 0.3\sigma^2$ (this is just an example of a utility function)

$\Delta U / \Delta \mu > 0$ (Higher expected return preferred to lower expected return)

$\Delta U / \Delta \sigma^2 < 0$ (Lower risk preferred to higher risk)



Which securities would a risk averse investor **now** prefer?

- Indifferent between A and B
- C is preferred to A, B and D
- D is preferred to A and B

Specifying Investor Preferences

- Example: Evaluate securities A, B and C based on their coefficients of variation

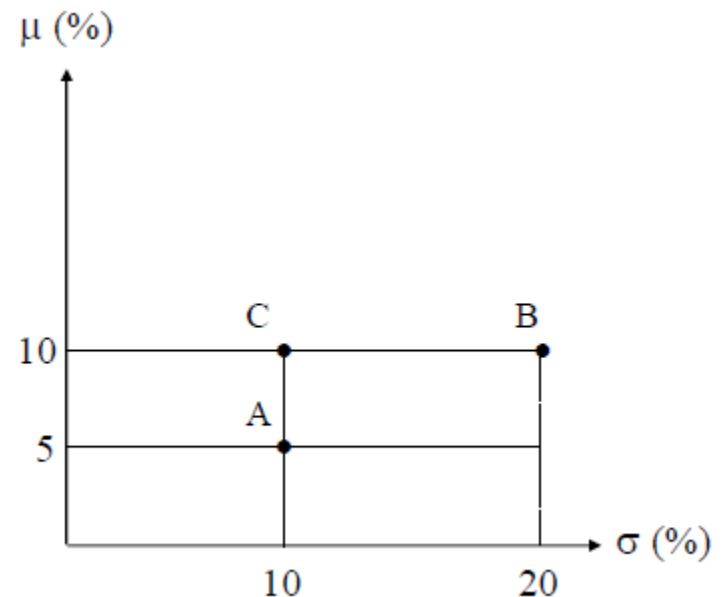
- $CV = \sigma / \mu$

- $CV_A = 0.10/0.05 = 2.0$

- $CV_B = 0.20/0.10 = 2.0$

- $CV_C = 0.10/0.10 = 1.0$

- C is preferred to A and B (as before)
- Investor is **indifferent** between A and B - same risk per unit expected return



Key Concepts

- Arithmetic average return indicates returns from a single, one period investment over that period.
- Geometric average return indicates returns per period from an investment over the whole period
- Probability distributions represent specifications of possible outcomes in the future associated with probabilities of these outcomes occurring
- Expected return is the expected return outcome measured as the weighted average of the individual outcomes
- Standard deviation of returns is the measure of dispersion around the expected return
- Risk averse investors prefer securities with the highest expected returns and lowest risk levels



Key Relationships

- Discrete returns: $R_t = (P_t + D_t - P_{t-1})/P_{t-1}$
- Continuously compounded returns: $r_t = \ln [(P_t + D_t)/P_{t-1}]$
- Arithmetic average return: $r_a = (R_1 + R_2 + \dots + R_n)/n$
- Geometric average return: $r_g = [(1+R_1)(1+R_2) \dots (1+R_n)]^{1/n} - 1$
- Expected return on a security: $E(r) = p_1R_1 + p_2R_2 + \dots + p_nR_n$
- Return variance: $\sigma^2 = p_1(R_1 - \mu)^2 + p_2(R_2 - \mu)^2 + \dots + p_n(R_n - \mu)^2$
- Return standard deviation: $SD(r)$ or σ