
FINA 1082 Financial Management

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Lecture 6

Risk and Return

Modern Portfolio Theory I

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Objectives

- Describe the probability distribution approach
- Explain the concept of standard deviation as a measure of risk
- Interpret risk and return measures
- Illustrate how investor risk preferences can be represented

Measures of Risk and Expected Return

The **expected return** is the expected outcome measured as the weighted average of the individual outcomes

$$E(r) = R = \mu = p_1R_1 + p_2R_2 + \dots + p_nR_n$$

The **variance or standard deviation of returns** is the measure of dispersion around the expected return. Greater the dispersion, higher the risk and uncertainty.

$$\begin{aligned} \text{Var}(r) = \sigma^2 &= p_1(R_1 - \mu)^2 + p_2(R_2 - \mu)^2 + \dots + p_n(R_n - \mu)^2 \\ \text{SD}(r) &= \sigma \end{aligned}$$

Note: Variance and standard deviation take into account returns above and below the expected return. Investor concern is typically with returns below the expected return (**downside risk**)

Measures of Risk and Expected Return

Class Exercise 1: The current price of Stock X is \$10. The stock does not pay a regular dividend and market analysts expect the following three states to occur one year from now

State	Probability	Price
Bad	0.3	\$9.00
Good	0.4	\$11.00
Awesome	0.3	\$12.00

Compute the expected return and standard deviation of return for the above investment.

Answers to Class Exercise 1

- Convert the cash flows into rates of return for each state
- The return distribution for Stock X is

State	Probability	Rate of Return
Bad	0.3	$(9 - 10)/10 = -10\%$
Good	0.4	$(11 - 10)/10 = 10\%$
Awesome	0.3	$(12 - 10)/10 = 20\%$

$$E(r) \text{ or } \mu = 0.3(-0.1) + 0.4(0.1) + 0.3(0.2) = 7.0\%$$

$$\text{Var}(r) = 0.3(-0.1-0.07)^2 + 0.4(0.1-0.07)^2 + 0.3(0.2-0.07)^2 = 0.0141$$

$$\text{SD}(r) \text{ or } \sigma = (0.0141)^{1/2} = 0.1187 \text{ or } 11.9\%$$

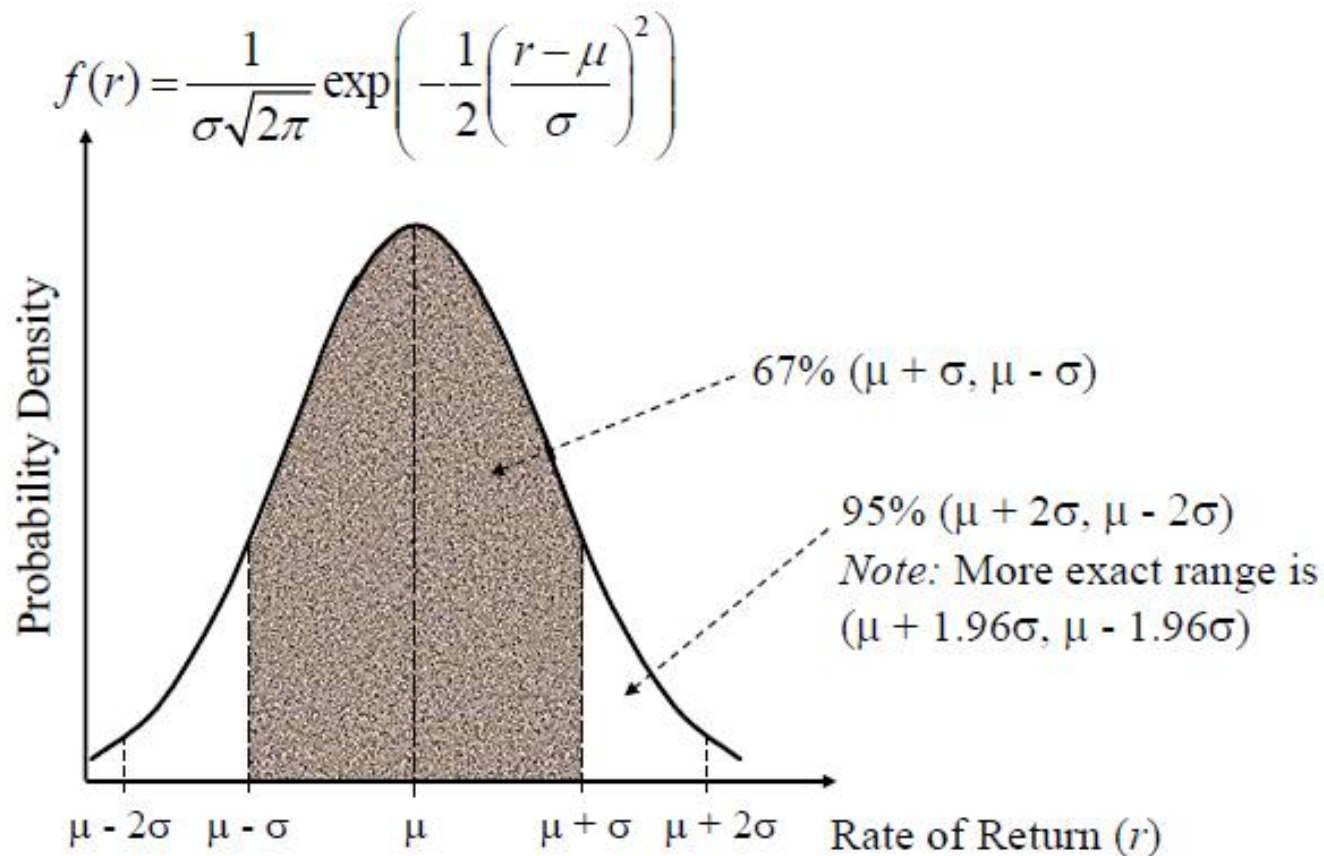
Interpreting Return and Risk Measures

- A general interpretation of the expected return and standard deviation of return requires assuming returns are **continuously** and **normally** distributed.
 - Only the expected (or mean) return and standard deviation of return are needed to fully describe the distribution of returns

$$f(r) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{r - \mu}{\sigma}\right)^2\right)$$

Note: A normal distribution implies an unlimited downside loss potential - unrealistic assumption because the most you can lose on a security purchase is 100%!

Interpreting Return and Risk Measures



Interpreting Return and Risk Measures

- In the previous example, $\mu = 7\%$ and $\sigma = 11.9\%$
- Assume you purchased 100 shares of Stock X, investing \$1,000
 - The expected value of this security next period is
$$1000(1 + \mu) = 1000(1.07) = \$1,070$$
 - There is a 67% probability that the **realized** return will lie in the range $(\mu - \sigma, \mu + \sigma)$ or $(-4.9\%, 18.9\%)$
 - There is a 95% probability that the **realized** return will lie in the range $(\mu - 2\sigma, \mu + 2\sigma)$ or $(-16.8\%, 30.8\%)$
- There is a 95% probability that a \$1000 investment in this security will be worth between $1000(1 - 0.168) = \$832$ and $1000(1 + 0.308) = \$1,308$ next period
 - The **range of final wealth** is \$832 - \$1,308 with a 95% likelihood of the **actual/realized wealth** lying in this range

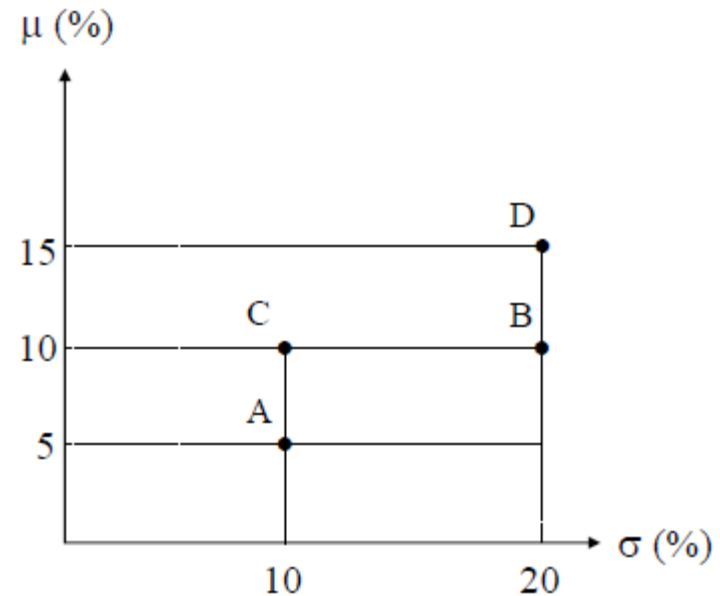
Specifying Investor Preferences

Class Exercise 2: You're given the following information on four risky securities. Which securities would a risk averse investor prefer?

Security	Expected Return	Standard Deviation of Returns
A	5%	10%
B	10%	20%
C	10%	10%
D	15%	20%

Answers to Class Exercise 2

- Risk averse investors would prefer the **highest expected returns** and the **lowest risk levels**
- C is preferred to A
- C is preferred to B
- D is preferred to B
- What about A and B, C and D, A and D?



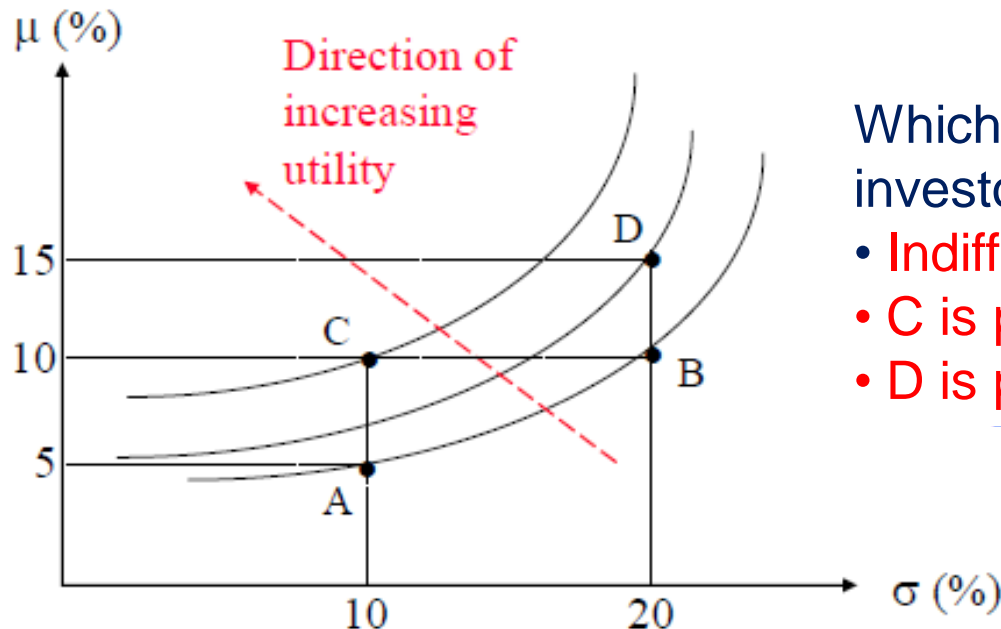
Specifying Investor Preferences

Assuming asset returns are normally distributed, investor utility can be written as a function of μ and σ^2 : $U = f(\mu, \sigma^2)$

Example: $U = \mu - 0.3\sigma^2$ (this is just an example of a utility function)

$\Delta U / \Delta \mu > 0$ (Higher expected return preferred to lower expected return)

$\Delta U / \Delta \sigma^2 < 0$ (Lower risk preferred to higher risk)



Which securities would a risk averse investor **now** prefer?

- Indifferent between A and B
- C is preferred to A, B and D
- D is preferred to A and B

Specifying Investor Preferences

- Example: Evaluate securities A, B and C based on their **coefficients of variation**

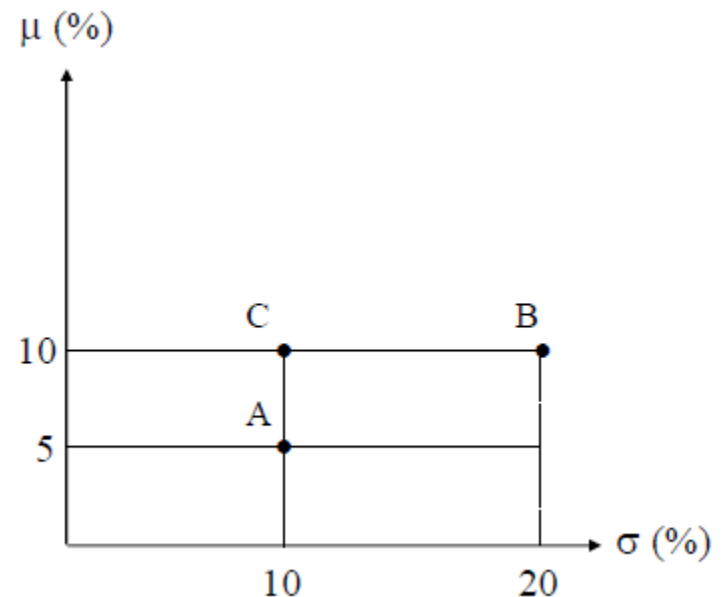
- $CV = \sigma / \mu$

- $CV_A = 0.10/0.05 = 2.0$

- $CV_B = 0.20/0.10 = 2.0$

- $CV_C = 0.10/0.10 = 1.0$

- C is preferred to A and B (as before)
- Investor is **indifferent** between A and B - same risk per unit expected return



Key Concepts

- Probability distributions represent specifications of possible outcomes in the future associated with probabilities of these outcomes occurring
- Expected return is the expected return outcome measured as the weighted average of the individual outcomes
- Standard deviation of returns is the measure of dispersion around the expected return
- Risk averse investors prefer securities with the highest expected returns and lowest risk levels

Key Relationships

- Expected return on a security: $E(r) = p_1R_1 + p_2R_2 + \dots + p_nR_n$
- Return variance: $\sigma^2 = p_1(R_1 - \mu)^2 + p_2(R_2 - \mu)^2 + \dots + p_n(R_n - \mu)^2$
- Return standard deviation: $SD(r)$ or σ

Objectives

- Explain the concept of risk diversification
- Explain the concepts of covariance and correlation of returns
- Compute and interpret the expected return and standard deviation of a two asset portfolio
- Examine the effects on risk and expected return of short selling and portfolio leveraging

Portfolios and Risk Diversification

- A risk averse investor's objective is to
 - Minimize the risk of portfolio of investments, given a desired level of expected return, or
 - Maximize the expected return of portfolio of investments, given a desired level of risk
- The easiest way to minimize risk is to diversify across different assets by forming a portfolio (collection) of assets
 - Portfolio risk falls as the number of assets in the portfolio increases
 - Portfolio risk cannot be entirely eliminated using this method
 - The risk not eliminated is called **systematic risk**
- More on this in Lecture 7

Portfolio Risk and Return: Two Assets

Portfolio's expected return is the weighted average of the expected returns of its component assets - weights are the percentage of investor's original wealth invested in each asset

$$E(r_p) = \mu_p = w_1 E(r_1) + w_2 E(r_2)$$

w_j = Amount invested in asset j / Total invested in all assets

Note: $w_1 + w_2 = 1$ and $w_1 = 1 - w_2$ (or $w_2 = 1 - w_1$)

Portfolio's variance is the weighted average of the variance of its component assets and the **covariance** between the assets' returns

$$\text{Var}(r_p) = \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\sigma_{12}$$

$\sigma_{12} = \text{Cov}(r_1, r_2) = \text{Covariance between assets 1 and 2}$

Standard deviation of portfolio, $\text{SD}(r_p) = \sigma_p$

Covariance Between Asset Returns

Covariance measures the level of **comovement** between assets

- $\sigma_{12} = p_1(r_{11} - \mu_1)(r_{21} - \mu_2) + \dots + p_n(r_{1n} - \mu_1)(r_{2n} - \mu_2)$
 - r_{jk} = Return on asset $j = 1, 2$ in state $k = 1, 2, \dots, n$
 - $\sigma_{12} > 0$: Above (below) average returns on asset 1 tend to occur with above (below) average returns on asset 2
 - $\sigma_{12} < 0$: Above (below) average returns on asset 1 tend to occur with below (above) average returns on asset 2
- The magnitude of covariance may change depending on how returns are measured
 - Dollars versus cents
 - Percentages versus decimals

Correlation Between Asset Returns

Correlation coefficient is a “standardized” measure of comovement between two assets

$$\rho_{12} = \sigma_{12} / \sigma_1 \sigma_2$$
$$-1 \leq \rho_{12} \leq +1$$

Note: The sign of the correlation coefficient is the same as the sign of the return covariance

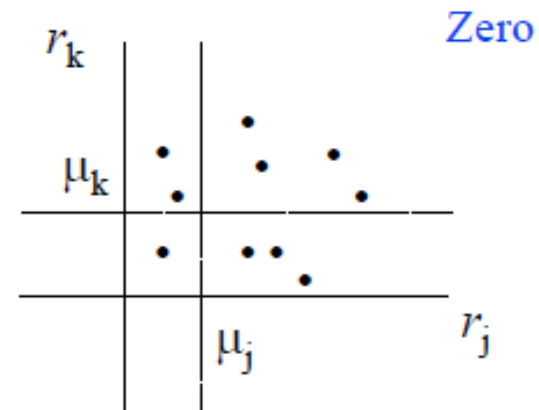
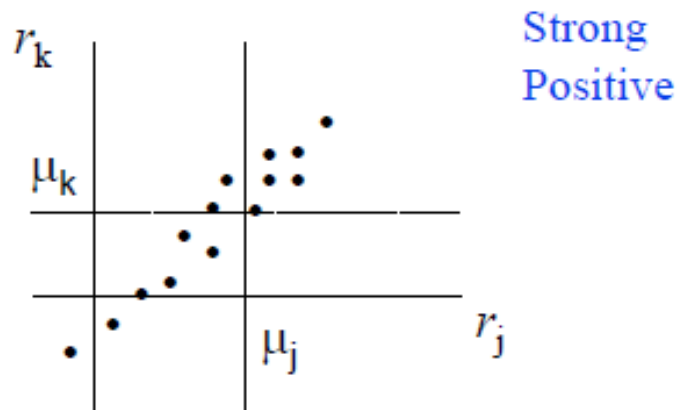
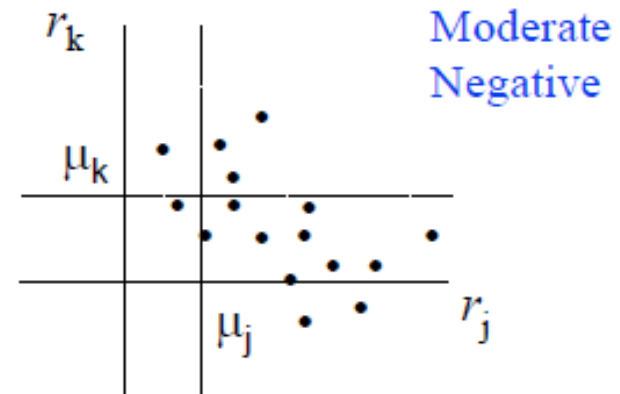
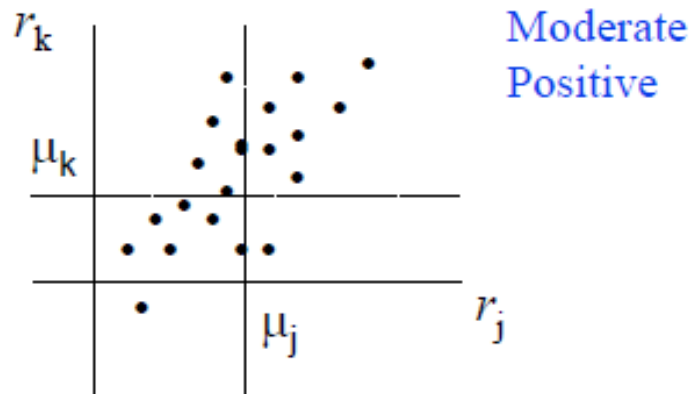
The covariance of returns can be rewritten as

$$\sigma_{12} = \sigma_1 \sigma_2 \rho_{12}$$

A portfolio's variance can be rewritten as the weighted average of the variance of its component assets and the correlation between the assets' returns

$$\text{Var}(r_p) = \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

Examples of Asset Correlations



Return Correlations of Selected Stocks: 1993-2003

	AOI	ANZ	BHP	CBA	CML	FGL	NAB	SRP
AOI	1.00	0.62	0.61	0.59	0.33	0.41	0.62	0.39
ANZ		1.00	0.29	0.68	0.22	0.27	0.61	0.38
BHP			1.00	0.22	0.12	0.16	0.34	0.28
CBA				1.00	0.28	0.25	0.69	0.28
CML					1.00	0.11	0.22	0.30
FGL						1.00	0.26	0.25
NAB							1.00	0.30
SRP								1.00
Mean	6.5%	17.6%	4.8%	15.3%	4.3%	5.9%	14.4%	-0.9%
Median	9.5%	16.5%	0.0%	14.0%	-1.7%	0.0%	22.0%	-8.5%
Std Dev	12.9%	21.3%	22.8%	19.8%	20.4%	17.3%	21.2%	28.7%

Note: The sample period is Jan 1993 - Jun 2003. AOI refers to the All Ordinaries Index which is a proxy for the “market” portfolio. Stocks tend to be positively correlated with each other and with the market portfolio. The mean, median and standard deviation of returns are based on continuously compounded monthly returns which have been annualized

Portfolios and Risk Diversification

Key implications

- As long as individual securities are **not perfectly positively correlated**, diversification benefits will always exist in the form of reduction of portfolio risk
- Diversification benefits can be **maximized** if securities are negatively correlated with each other
- As the number of securities in a portfolio increases, the **covariance** of returns between different securities determines the portfolio's **total risk**
- **Importance of portfolio diversification in practice**
 - See next two slides

The Importance of Portfolio Diversification

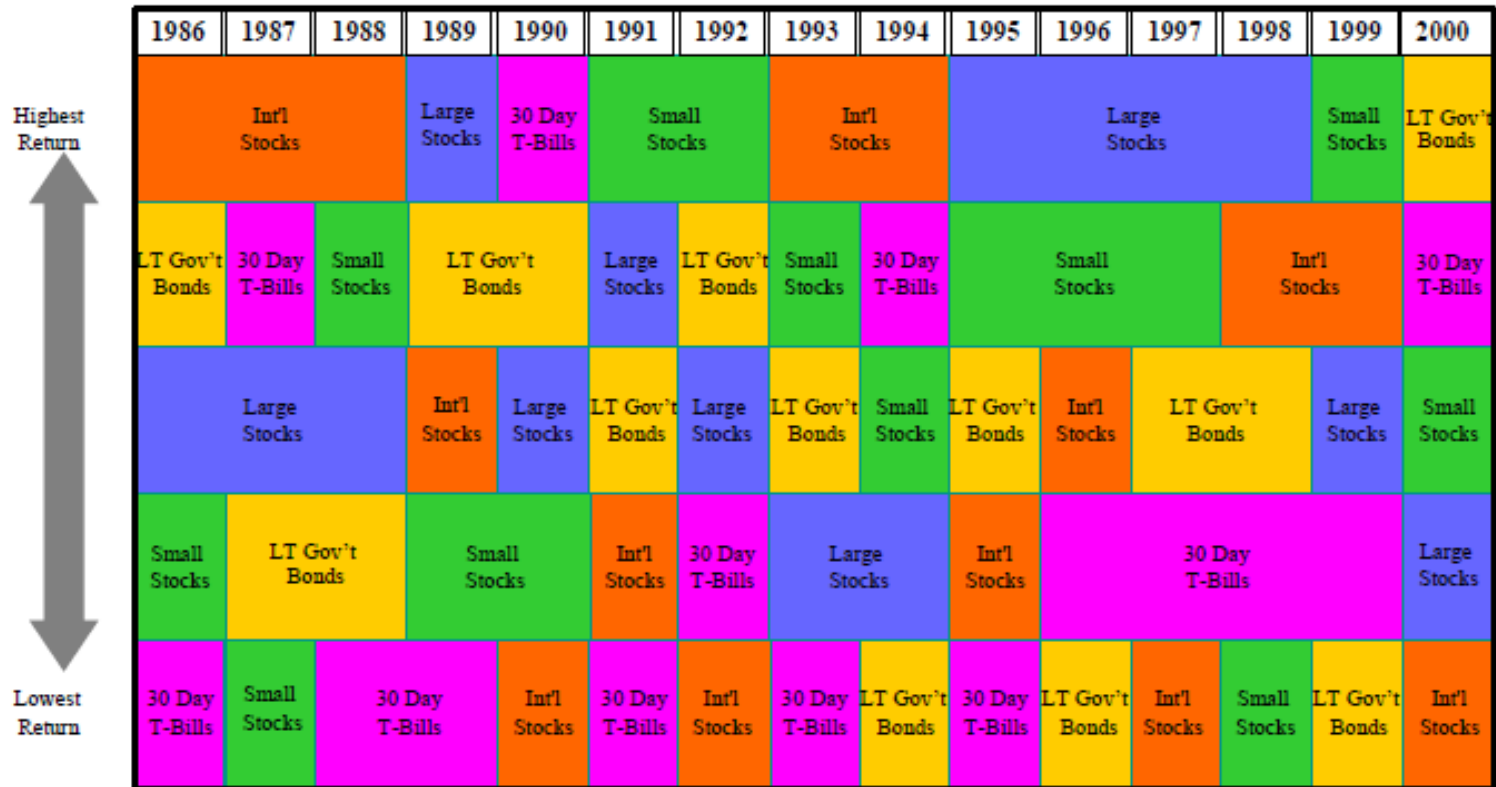
Downturn	US Market Downturns and Recovery: 1926-2001		Recovery
34 months -83.4%	Sept. 1929 - June 1932	July 1932 - Jan. 1945	151 months
6 months -21.8%	June 1946 - Nov. 1946	Dec. 1946 - Oct. 1949	35 months
7 months -10.2%	Aug. 1956 - Feb. 1957	March 1957 - July 1957	5 months
5 months -15.0%	Aug. 1957 - Dec. 1957	Jan. 1958 - July 1958	7 months
6 months -22.3%	Jan. 1962 - June 1962	July 1962 - April 1963	10 months
8 months -15.6%	Feb. 1966 - Sept. 1966	Oct. 1966 - March 1967	6 months
19 months -29.3%	Dec. 1968 - June 1970	July 1970 - March 1971	9 months
21 months -42.6%	Jan. 1973 - Sept. 1974	Oct. 1974 - June 1976	21 months
14 months -14.1%	Jan. 1977 - Feb. 1978	March 1978 - July 1978	5 months
20 months -16.9%	Dec. 1980 - July 1982	Aug. 1982 - Oct. 1982	3 months
3 months -29.5%	Sept. 1987 - Nov. 1987	Dec. 1987 - May 1989	18 months
5 months -14.7%	June 1990 - Oct. 1990	Nov. 1990 - Feb. 1991	4 months
2 months -15.4%	July 1998 - Aug. 1998	Sept. 1998 - Nov. 1998	3 months
13 months -30.5%	Sept. 2000 - Sep. 2001		

Source: Ibbotson and Associates, Inc. - www.ibbotson.com

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The Importance of Portfolio Diversification

US Asset Class Winners and Losers: 1986-2000



Source: Ibbotson and Associates, Inc. - www.ibbotson.com

The Importance of Portfolio Diversification

Australian Asset Class Winners and Losers: 1986-2000



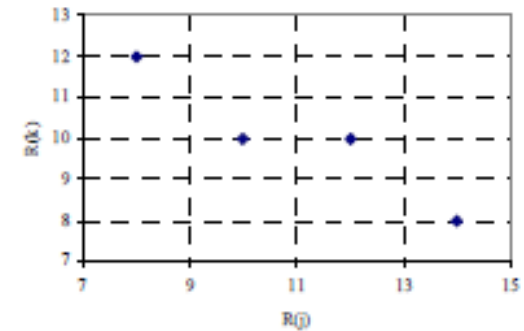
	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Highest Return	Domestic Stocks	Fixed Interest	Domestic Stocks	Int'l Stocks	Fixed Interest	Domestic Stocks	Fixed Interest	Domestic Stocks	Cash	Int'l Stocks	Domestic Stocks	Int'l Stocks	Int'l Stocks	Int'l Stocks	Listed Property
	Int'l Stocks	Cash	Listed Property	Cash	Cash	Diversified	Listed Property	Diversified	Fixed Interest	Diversified	Listed Property	Listed Property	Listed Property	Domestic Stocks	Fixed Interest
	Diversified	Int'l Stocks	Diversified	Diversified	Listed Property	Fixed Interest	Cash	Listed Property	Listed Property	Domestic Stocks	Fixed Interest	Diversified	Diversified	Diversified	Diversified
	Listed Property	Listed Property	Cash	Domestic Stocks	Diversified	Int'l Stocks	Int'l Stocks	Int'l Stocks	Diversified	Fixed Interest	Diversified	Domestic Stocks	Domestic Stocks	Cash	Cash
	Fixed Interest	Diversified	Fixed Interest	Fixed Interest	Int'l Stocks	Listed Property	Diversified	Fixed Interest	Int'l Stocks	Listed Property	Cash	Fixed Interest	Fixed Interest	Listed Property	Domestic Stocks
Lowest Return	Cash	Domestic Stocks	Int'l Stocks	Listed Property	Domestic Stocks	Cash	Domestic Stocks	Cash	Domestic Stocks	Cash	Int'l Stocks	Cash	Cash	Fixed Interest	Int'l Stocks

Source: Adapted from Macquarie Bank

Portfolio Risk and Return: Two Assets

Example: Securities X and Y have the following distributions

State	Probability	Security X	Security Y
Poor	0.25	8%	12%
Fair	0.25	10%	10%
Good	0.25	12%	10%
Groovy	0.25	14%	8%



$E(r_X) = 11\%$ and $E(r_Y) = 10\%$, $\sigma_X = 2.24\%$ and $\sigma_Y = 1.41\%$

$$\sigma_{XY} = 0.25(0.08-0.11)(0.12-0.10) + 0.25(0.10-0.11)(0.10-0.10) + 0.25(0.12-0.11)(0.10-0.10) + 0.25(0.14-0.11)(0.08-0.10) = -0.0003$$

$$\rho_{XY} = -0.0003 / (0.0224)(0.0141) = -0.95$$

Portfolio Risk and Return: Two Assets

The expected return of a portfolio with \$5,000 in asset X and \$20,000 in asset Y is

- ♦ $w_X = 5000/(5000 + 20000) = 0.20$
 $w_Y = 20000/25000 = 1 - w_X = 0.80$
- ♦ $E(r_p) = 0.2(0.11) + 0.8(0.10) = 0.102$ or 10.2%

❖ The standard deviation of the portfolio is

- ♦ $\sigma_p = [0.2^2(0.0224)^2 + 0.8^2(0.0141)^2 + 2(0.2)(0.8)(-0.0003)]^{1/2}$
 $\sigma_p = 0.00716$ or 0.72%
or

- ♦ $\sigma_p = [0.2^2(0.0224)^2 + 0.8^2(0.0141)^2 + 2(0.2)(0.8)(0.0224)(0.0141)(-0.95)]^{1/2} = 0.72\%$

$$\underbrace{\sigma_X \times \sigma_Y \times \rho_{XY}}$$

Equals σ_{XY}

Portfolio Risk and Return: Short Selling

- It is possible to increase a portfolio's expected return above the expected return of its component assets, but at a cost
- **Strategy:** Sell short the security with a lower expected return and invest the proceeds in the security with a higher expected Return
- **Short selling** refers to borrowing (via a broker) shares, selling them now with a promise to buy them back later at an expected lower price
 - **Short selling** is like **risky** borrowing to **leverage** a portfolio
 - **This leveraging increases portfolio risk!**
 - Short selling security A and investing proceeds in security B implies $w_A < 0\%$ and $w_B > 100\%$ such that $w_A + w_B = 1$

Portfolio Risk and Return: Short Selling

Example: Assume shares of ABX Ltd are selling for \$2.00 and you expect the price to fall to \$1.70 in the near future. Assume you can sell this security short, reinvest all the proceeds from the sale, and transactions costs are zero

Borrow and sell ABX today	+\$2.00
Repurchase ABX when price falls	-\$1.70
Dollar return per share	$-\$1.70 + \$2.00 = \$0.30$
Realized rate of return	$0.30/2.00 = 15\%$

Where's the risk in this strategy? Assume the price rises to \$2.50

Borrow and sell ABX today	+\$2.00
Repurchase ABX at \$2.50	-\$2.50
Dollar return per share	$-\$2.50 + \$2.00 = -\$0.50$
Realized rate of return	$-0.50/2.00 = -25\%$

Portfolio Risk and Return: Short Selling

Example: You are given the following information on two stocks which have a correlation of +0.5

Stock	Expected Return	Standard Deviation of Returns
A	10%	15%
B	20%	20%

Base case: An investor with \$1,000 invests \$500 each in stocks A and B implying that $w_A = w_B = 0.5$

$$E(rp) = 0.5(0.10) + 0.5(0.20) = 0.15 \text{ or } 15\%$$

$$\sigma_p^2 = 0.5^2(0.15)^2 + 0.5^2(0.2)^2 + 2(0.5)(0.5)(0.15)(0.2)(0.5)$$

$$\sigma_p = (0.0231)^{1/2} = 0.152 \text{ or } 15.2\%$$

Note: Even though stocks A and B are positively correlated there is still reduction in risk

Portfolio Risk and Return: Short Selling

Short selling case 1: An investor with \$1,000 borrows stock A and sells short \$500 worth of A, investing \$1,500 in stock B

$$w_A = -500/1000 = -0.5 \text{ and } w_B = (500 + 1000)/1000 = 1.5$$

$$E(r_p) = -0.5(0.10) + 1.5(0.20) = 0.25 \text{ or } 25\%$$

$$\sigma_p^2 = (-0.5)^2(0.15)^2 + 1.5^2(0.2)^2 - 2(-0.5)(1.5)(0.15)(0.2)(0.5)$$

$$\sigma_p = (0.0731)^{1/2} = 0.27 \text{ or } 27.0\%$$

Short selling case 2: What happens to the portfolio's risk and expected return when the investor with \$1,000 sells short \$1,000 worth of stock A and invests \$2,000 in stock B?

$$w_A = -1000/1000 = -1.0 \text{ and } w_B = (1000 + 1000)/1000 = 2.0$$

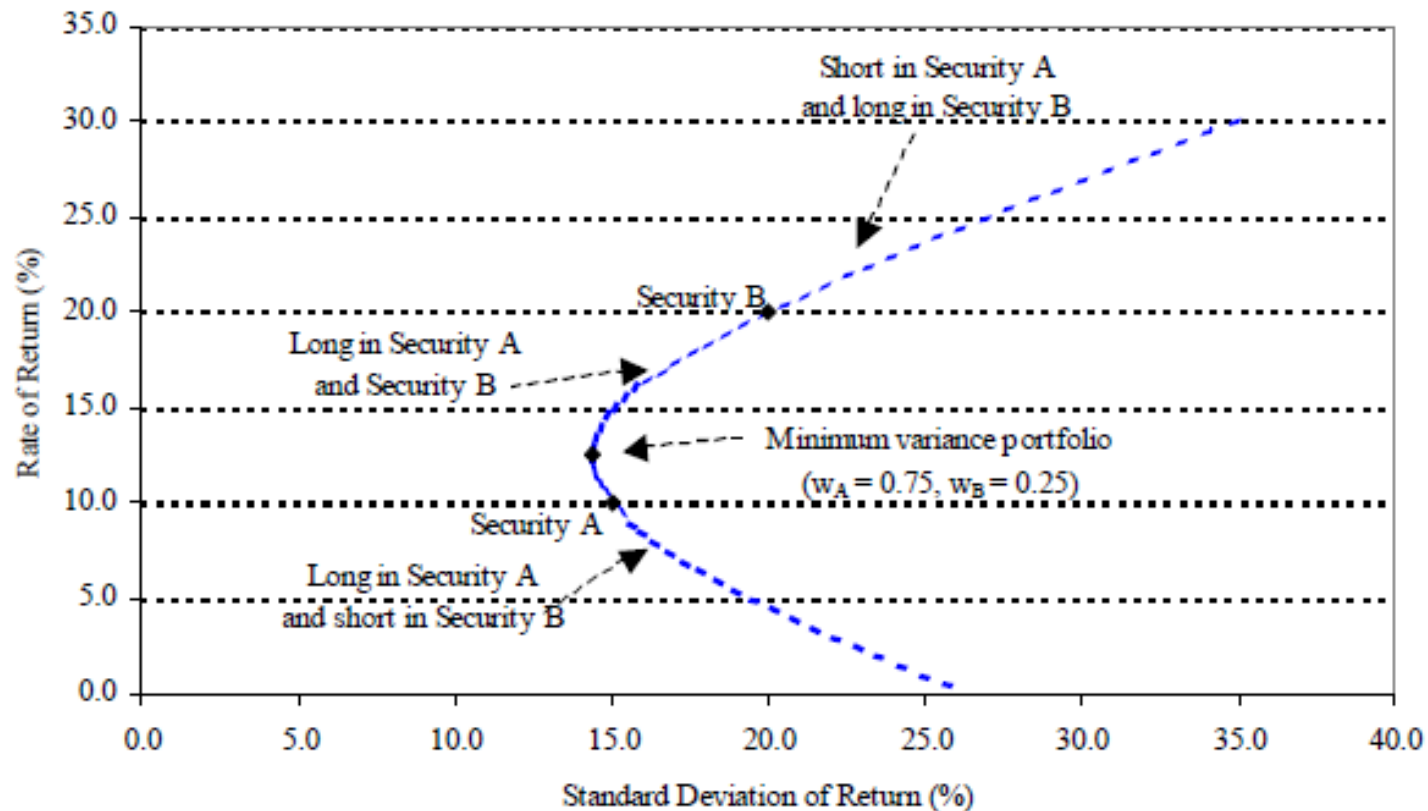
$$E(r_p) = -1.0(0.10) + 2.0(0.20) = 0.30 \text{ or } 30\%$$

$$\sigma_p^2 = (-1.0)^2(0.15)^2 + 2.0^2(0.2)^2 - 2(-1.0)(2.0)(0.15)(0.2)(0.5)$$

$$\sigma_p = (0.1225)^{1/2} = 0.35 \text{ or } 35.0\%$$

Both return and risk are higher in Case 2!

Portfolio Risk and Return: Short Selling



Portfolio Risk and Return: Short Selling

The portfolio's expected return and risk increase as more of stock A is borrowed (as a percentage of initial wealth) and sold short to invest in stock B

Base case: $E(r_p) = 15\%$ and $\sigma_p = 15.2\%$

There is a 95% probability that the realized return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or $(-15.4\%, +45.4\%)$

Short selling case 1: $E(r_p) = 25\%$ and $\sigma_p = 27\%$

There is a 95% probability that the realized return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or $(-29\%, +79\%)$

Short selling case 2: $E(r_p) = 30\%$ and $\sigma_p = 35\%$

There is a 95% probability that the realized return will lie in the range $(E(r_p) - 2\sigma_p, E(r_p) + 2\sigma_p)$ or $(-40\%, +100\%)$

Key Concepts

- Risk minimization can be achieved by diversifying across different securities or assets
- A portfolio's expected return is the weighted average of the returns of its component securities
- A portfolio's variance is the weighted average of the variances of its component securities and the covariance between pairs of securities
- Short selling refers to borrowing shares, selling them at the current market price with a promise to repurchase them later at an expected lower price
- The overall effect of short selling is to increase the expected return of a portfolio as well as the risk associated with the portfolio

Key Relationships

- Expected portfolio return: $E(r_p) = w_1 E(r_1) + w_2 E(r_2)$
- Portfolio return variance
 - $\sigma_p^2 = (w_1\sigma_1)^2 + (w_2\sigma_2)^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$
 - $\sigma_p^2 = (w_1\sigma_1)^2 + (w_2\sigma_2)^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$
- Covariance of returns
 - $\sigma_{12} = p_1(r_{11} - \mu_1)(r_{21} - \mu_2) + \dots + p_n(r_{1n} - \mu_1)(r_{2n} - \mu_2)$
- Correlation coefficient: $\rho_{12} = \sigma_{12} / \sigma_1\sigma_2$
- Covariance of returns: $\sigma_{12} = \sigma_1\sigma_2\rho_{12}$