FINA 1082 Financial Management

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Lecture 9

Asset Pricing Models

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Objectives

- Explain the relationship between systematic risk and expected return using the capital asset pricing model

- Use the security market line relationship to value securities

- Estimating beta

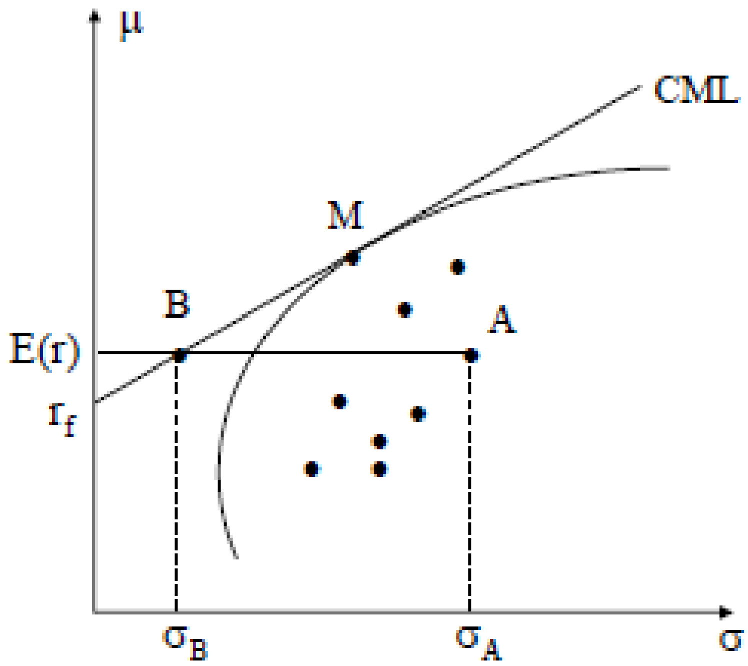
- Summarize the uses of beta

The Capital Asset Pricing Model

- CAPM is used to "price" individual securities given the expected return on the market portfolio and riskfree rate

- CAPM relates a security's expected return to its nondiversifiable or systematic risk and market premium for bearing that risk

- CAPM explains why asset A has the same expected return but a higher (total) risk than (the efficient) portfolio B on the capital market line



The CAPM's Main Assumptions

- Investors are risk averse individuals who maximize the expected utility of their end-of-period wealth
    - Investors make portfolio decisions on the basis of mean (expected return) and variance

- Investors have identical expectations about asset returns

- The returns on these assets follow a normal distribution

- Capital markets are perfect

- Borrowing and lending at the riskfree rate is possible

- The set of risky assets is fixed and all assets are traded

The CAPM's Intuition

- All investors hold efficient portfolios comprising M and r_f
  - Investors who don't diversify gain no increase in expected return for bearing this additional (diversifiable) risk

- Any asset A is held in an efficient portfolio as part of M, the market portfolio

- The excess return (above the riskfree rate) expected on A reflects A's contribution to the non-diversifiable risk of M

- The contribution of A to the non-diversifiable risk of the market portfolio depends on the covariance between A and M, not on A's own risk (standard deviation)
  - Note: σ_A does have a (negligible) effect on σ_M since asset A is part of M

The CAPM's Intuition

- The CAPM can be written as:
  - Expected return on asset A = Riskfree return + Risk premium

- Risk premium = Amount of risk × Market price of risk
  - The amount of risk is measured by the covariance of the asset with the market portfolio (the beta of the asset, βj)
  - The market price of risk is the return above the riskfree rate that investors earn for holding the (risky) market portfolio
  - The risk premium can be thought of as a "price" times "quantity" relationship
    - Higher the market price of risk and/or higher the amount of risk, greater the risk premium

$$E(r_j) = r_f + [E(r_m) - r_f\,]\beta_j$$

CAPM and the Security Market Line

- $E(r_j) = r_f + [E(r_m) - r_f\,]\beta_j$
  - This is the security market line (SML) relationship

- The market "price" of risk is measured as $E(r_m) – r_f$

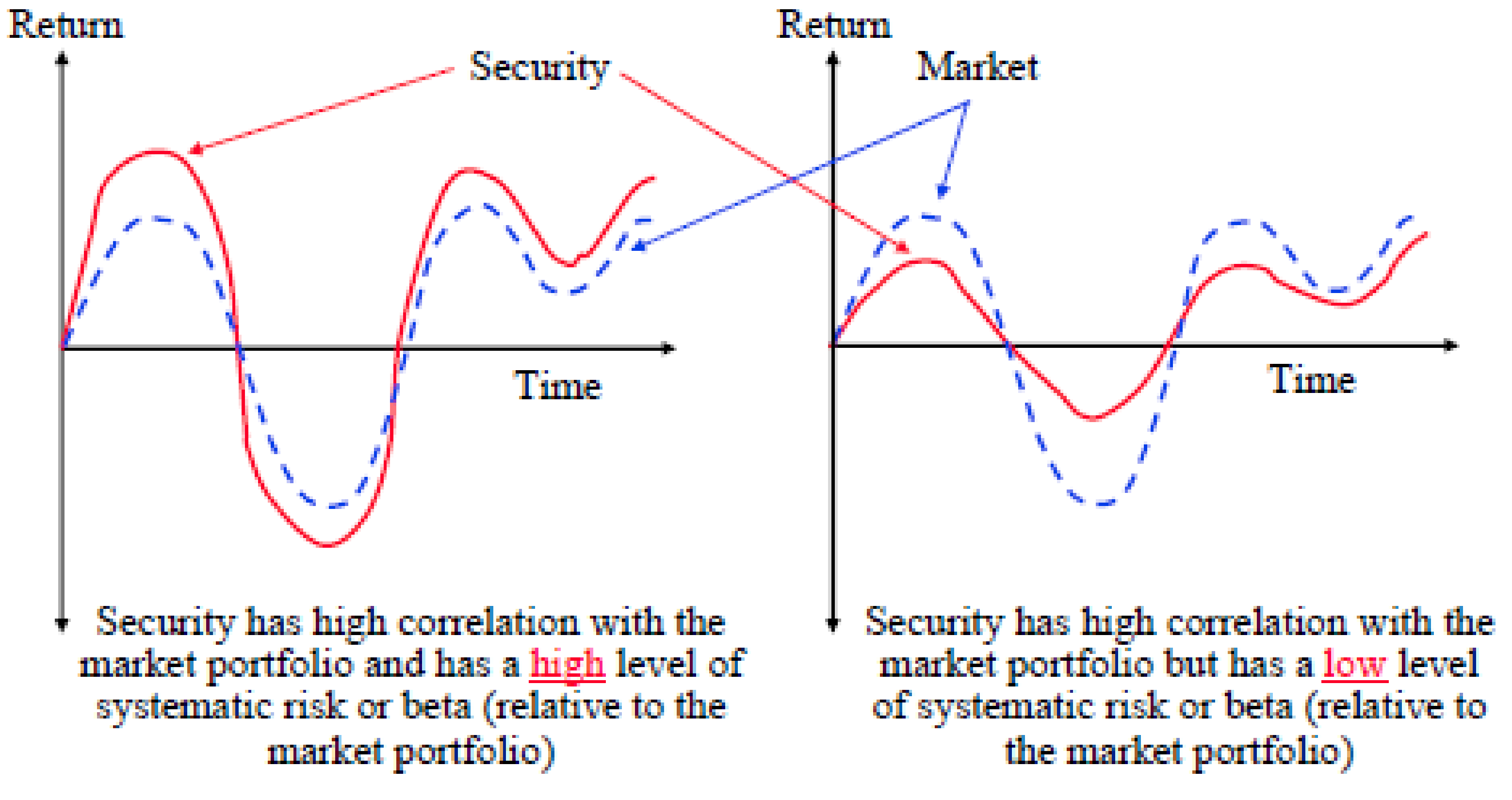
- The systematic risk is measured by the beta, β_j
  - $\beta j = Cov(r_j, r_m)/\sigma_m^2 = \sigma_{jm}/\sigma_m^2$

- Since $\sigma_j^m = \rho_{jm}\sigma_m\sigma_j$ we have
  - $\beta j = \rho_{jm}(\sigma_j /\sigma_m)$
  - $\beta_m = 1.0$ and $\beta_f = 0.0$ (by definition)

- $\beta_j = 1$: Security (portfolio) is as risky as the market portfolio
- $\beta_j = 0$: Security (portfolio) is riskfree
- $\beta_j < 1$: Security (portfolio) is less risky than the market portfolio
- $\beta_j > 1$: Security (portfolio) is more risky than the market portfolio

Betas and Correlations

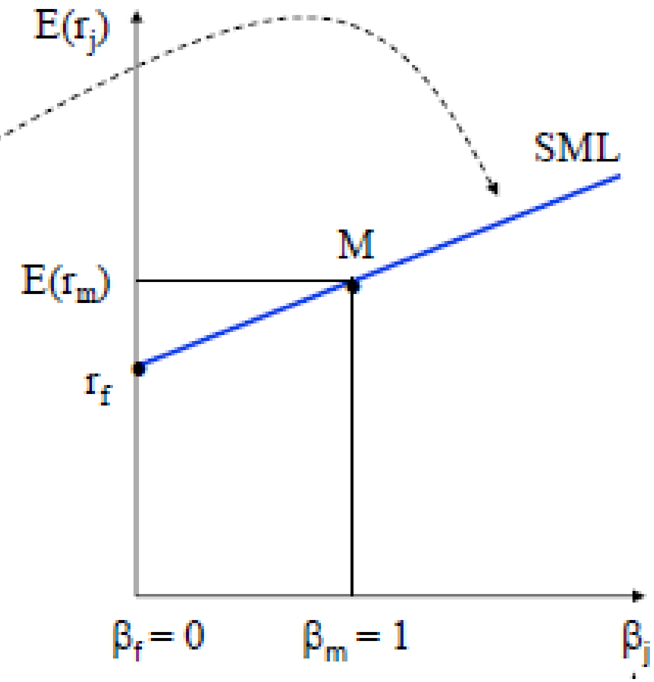
Beta is not the same as the correlation between a security (portfolio) and the market portfolio



The Security Market Line

In equilibrium, all risky securities are priced so that their expected returns plot on the SML

$$E(r_j) = r_f + \beta_j \, [E(r_m) - r_f \,]$$



Assets with β_j less (more) than 1 earn an expected return lower (higher) than the market Portfolio

Note: The x-axis of the CML (used to "price" efficient portfolios) differs from the x-axis of the SML (used to "price" individual assets)

The CML Versus the SML

- The CML's equation is: $E(r_p) = r_f + [E(r_m) - r_f][\sigma_p / \sigma_m]$
- The SML's equation is: $E(r_p) = r_f + [E(r_m) - r_f]\beta_p$
  - $\beta_p = \sigma_{pm}/\sigma_m^2 = \rho_{pm}(\sigma_p / \sigma_m)$
- Rewrite the SML as: $E(r_p) = r_f + [E(r_m) - r_f][\sigma_p / \sigma_m]\rho_{pm}$
- Using the CML we can only price efficient portfolios
  - Portfolios which have a perfect positive correlation with the market portfolio, i.e., $\rho_{pm} = 1.0$
- Using the SML we can price any portfolio or security, regardless of its correlation with the market portfolio and its level of unsystematic risk

Using the Security Market Line

Class Exercise 1: Assume that the riskfree rate is 7% and the expected market return is 12%

a) Locate the expected returns for securities with the following betas on the SML

- $\beta_A = 1.5$
- $\beta_B = 0.5$
- $\beta_C = -0.5$

b) Will an investor ever invest in a security like security C?
Why or why not?

Answer to Class Exercise 1

a) *Given:* $r_f = 7\%$, $E(r_m) = 12\%$

$E(r_m) - r_f = 12 - 7 = 5\%$

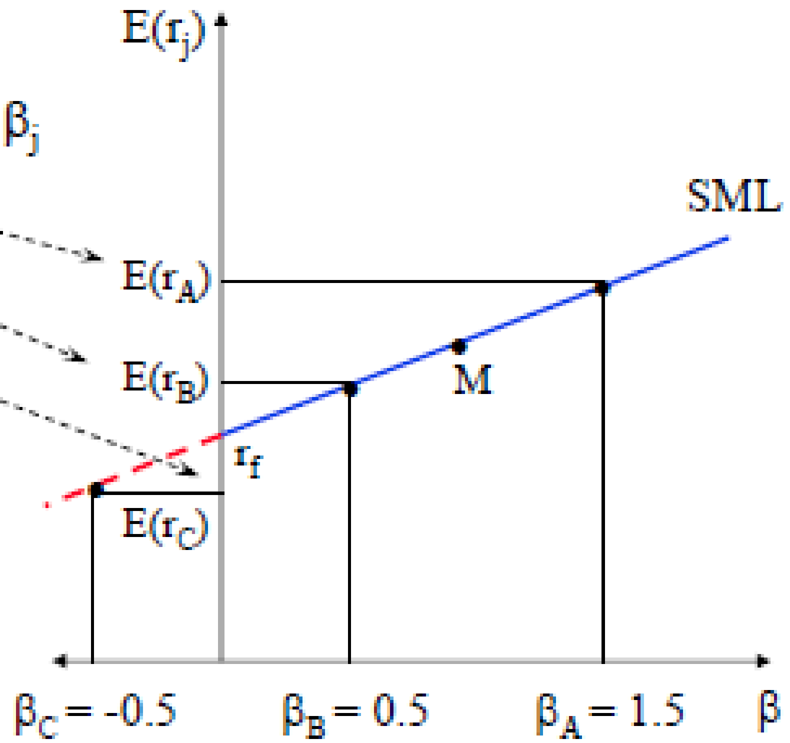
$E(r_j) = r_f + [E(r_m) - r_f]\beta_j = 7 + 5\beta_j$

$E(r_A) = 7+(5)1.5 = 14.5\%$

$E(r_B) = 7+(5)0.5 = 9.5\%$

$E(r_C) = 7-(5)0.5 = 4.5\%!$

b) Why would an investor ever choose security C when its expected return is below the riskfree rate?



Using the Security Market Line

Class Exercise 2: You are given the following incomplete information

Security/ Portfolio	Beta	Expected Return	Standard Deviation
X	?	10%	10%
Y	1.0	12%	20%
Riskfree	?	7%	?
Market	?	?	15%

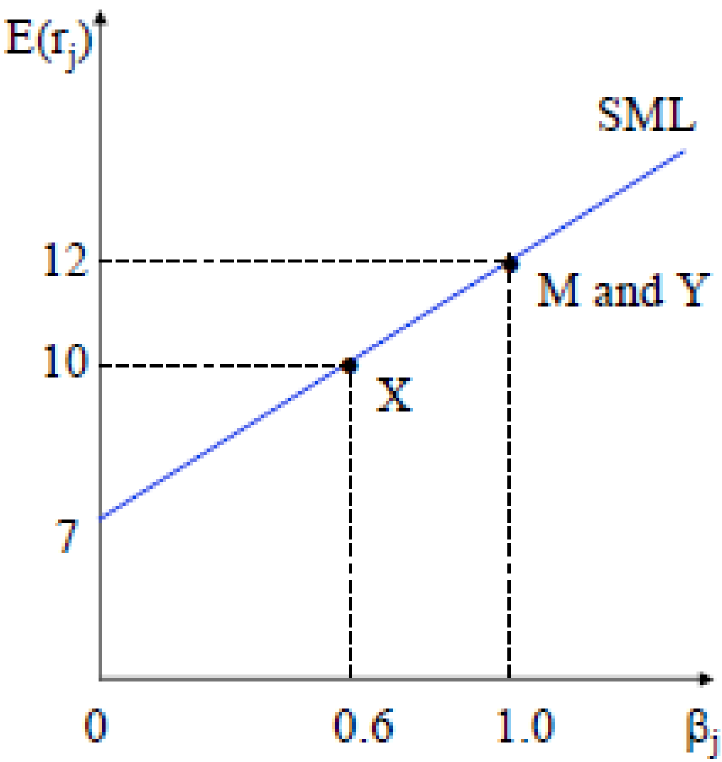
- Complete the above table and show all your calculations
- Draw a graph of the security market line and locate the above securities and portfolio on it
- Compute the beta of a portfolio with $7,500 in X and $2,500 in Y
- Compute the required return on this portfolio
- Evaluate the risk of this portfolio

Answer to Class Exercise 2

Completed table

Security	Beta	E(r)	SD(r)
X	0.6	10%	10%
Y	1.0	12%	20%
Riskfree	0.0	7%	0%
Market	1.0	12%	15%

- $E(r_m) = E(r_y) = 12\%$ ($\beta_y = \beta_m = 1$)
- $E(r_x) = 10 = 7 + (12 - 7)\ \beta_x$
- $\beta_x = (10 - 7)/(12 - 7) = 0.6$
- $\beta_p = w_X \times \beta_X + w_Y \times \beta_Y$
- So, $\beta_p = 0.75(0.6) + 0.25(1.0) = 0.7$
- $E(r_p) = 7 + (12 - 7)0.7 = 10.5\%$
- Risk evaluation of portfolio?



Answer to Class Exercise 2

- Beta of a portfolio with \$7,500 in X and \$2,500 in Y
  - $w_x = 7500/(2500 + 7500) = 0.75$ and $w_y = 1 - w_x = 0.25$
  - $\beta_p = w_x\beta_x + w_y\beta_y = 0.75(0.6) + 0.25(1) = 0.7$
  - Security X dominates portfolio beta

- Required return on this portfolio
  - $E(r_p) = 7 + (12 - 7)\ \beta_p = 7 + 5(0.7) = 10.5\%$

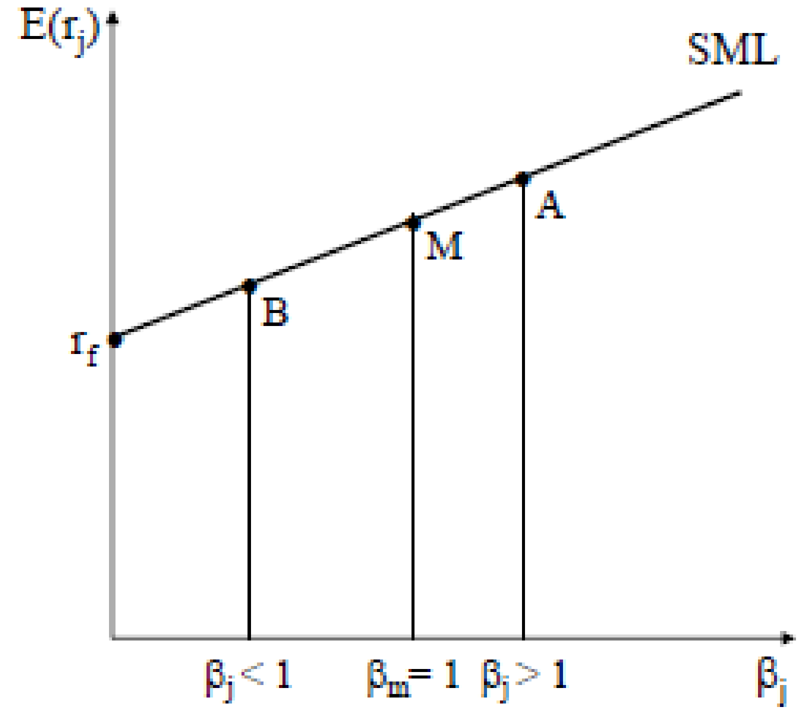
- Risk evaluation of this portfolio
  - Beta less than 1.0 - portfolio is less risky than the market
  - Portfolio's expected return expected to change by 0.7% when the market portfolio's expected return changes by 1%

Movements in the Security Market Line

Application: What happens to the SML in the following cases

a) There is an unexpected increase in the market risk premium

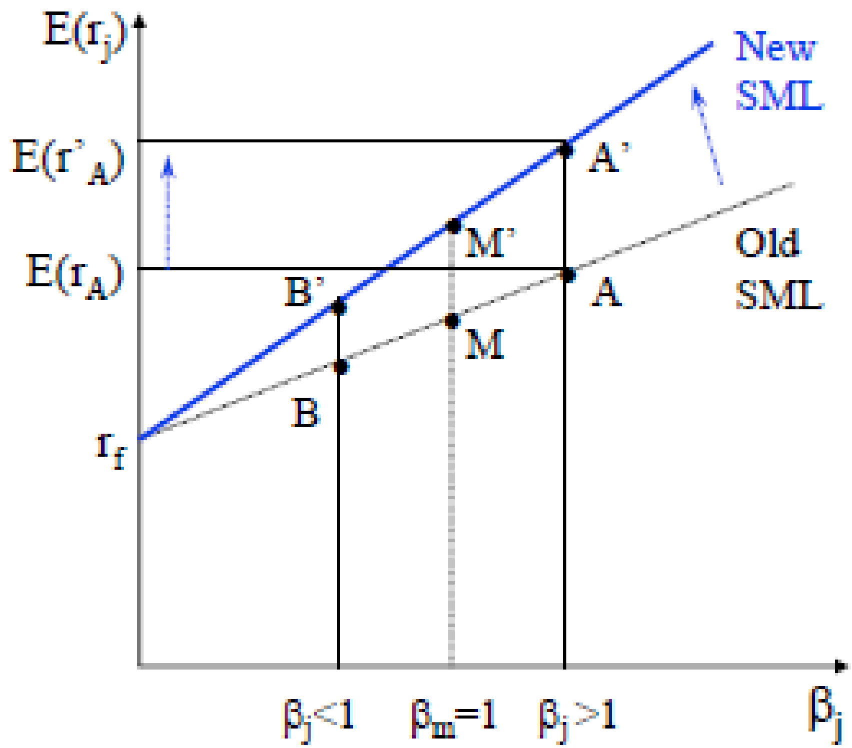
b) There is an unexpected decrease in the riskfree rate



Movements in the Security Market Line

a) An unexpected increase in the market risk premium

- $[E(r_m) - r_f]$ increases
- The SML is steeper (assuming r_f unchanged)
- E(r) of asset A increases so A's price will fall
    - $E(r_m)$ also increases so the market will fall in value

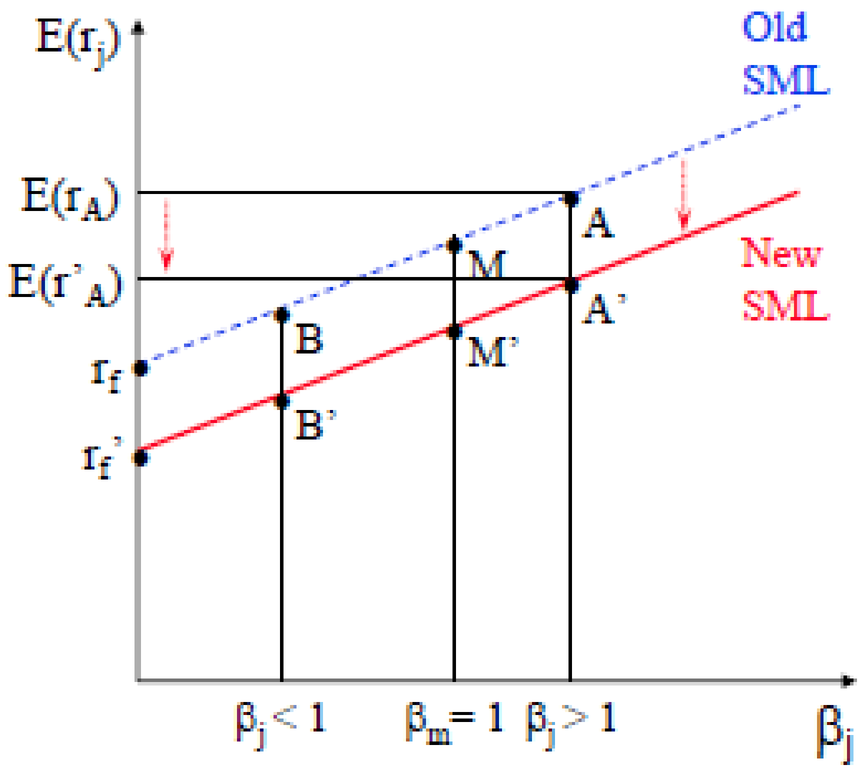


- E(r) of the lower risk asset B will rise less than the E(r) of the higher risk asset A

Movements in the Security Market Line

b) An unexpected decrease in the riskfree rate

- rf decreases - assume no change in the market risk premium, [E(rm) - rf]
- Implies a downward, parallel shift in the SML
- E(r) of asset A decreases so the price of A will rise
- E(rm) also falls so the market will rise in value



- Expected fall in E(rB) = Expected fall in E(rA)

Estimating Betas

- Historical betas can be estimated using the following market model regression
  - $R_{jt} = \alpha_j + \beta_j R_{mt} + e_t \quad t = 1, 2, \ldots T$
  - R_{jt} is the return on asset j at time t and R_{mt} is the return on market at time t
  - α_j is the intercept, β_j is the regression's slope and e_{jt} is the error term

- The market model is obtained by rewriting the CAPM equation

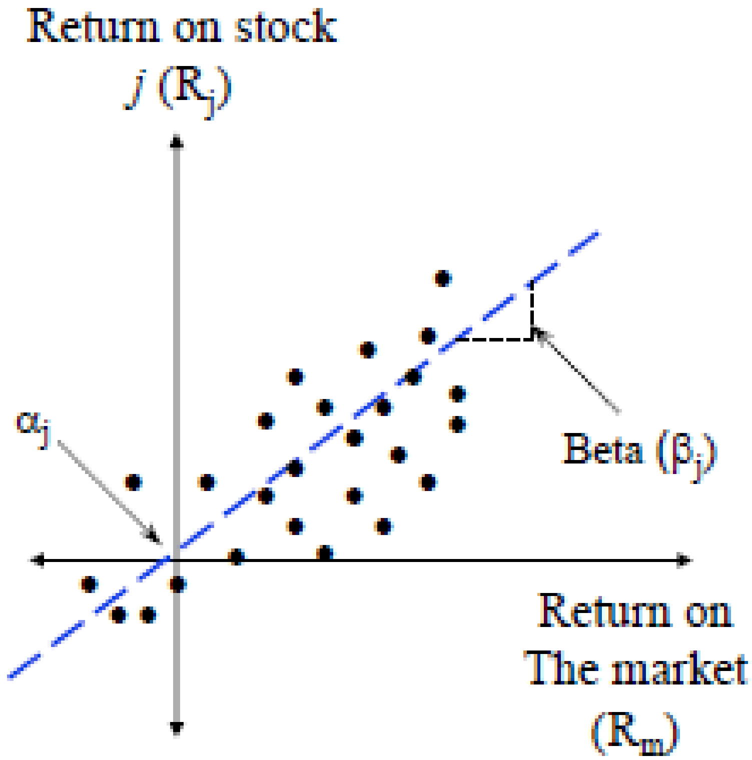
Estimating Betas

- Beta is the slope of the characteristic line - the line of best fit between asset *j* returns and market portfolio returns

- Slope of line = $\beta_j = \sigma_{jm} / \sigma_m^2$

- βj is a measure of asset j's systematic risk

- Intercept αj is expected to be equal to $r_f (1 - \beta_j)$ under the CAPM



Using Betas

- Design portfolios suited to investors' risk preferences
    - Low beta portfolios for less risk tolerant investors
    - High beta portfolios for more risk tolerant investors

- Evaluate portfolio performance
    - On average, high beta portfolios should outperform the market portfolio
    - If market return increases by 10%, a portfolio with beta 2.0 should experience an increase in returns of 20%

- Estimate cost of equity capital - more in later lectures
    - Estimates better reflect market's risk and expected return expectations

Key Concepts

- The beta of an asset reflects its return’s sensitivity to the market portfolio’s return

- In equilibrium, all risky assets will be priced so that their expected returns plot on the security market line

- Beta is the slope of the line of best fit between excess asset j returns and excess market portfolio returns

- Beta estimates are prone to various estimation problems associated with thin (infrequent) trading, sample period used, and reversion to 1.0 over time

Key Relationships

- The SML: $E(r_j) = r_f + [E(r_m) - r_f]\beta_j$
- Security beta: $\beta_j = Cov(r_j, r_m)/Var(r_m) = \sigma_{jm} / \sigma_m^2$
- Security beta: $\beta_j = \rho_{jm}(\sigma_j / \sigma_m)$
- The CML and SML compared
  - CML: $E(r_p) = r_f + [E(r_m) - r_f][\sigma_p / \sigma_m]$
  - SML: $E(r_p) = r_f + [E(r_m) - r_f]\beta_p$
- Beta of a two asset portfolio
  - $\beta_p = w_a \times \beta_a + w_b \times \beta_b$

Objectives

- Examine the relationship between prices and expected returns
- Apply the CAPM to value ordinary shares
- Examine the CAPM's testability

Relationship Between Prices and Returns

Prices and expected returns are inversely related

Using the constant dividend growth model

$P_0 = D_1/(k_e - g)$ or $P_0 = D_1/[E(r) - g]$

If D_1 and g are unchanged then an increase in E(r) implies a decrease in P_0, and vice versa.

• Intuition: If investors require a higher expected return on a security than it currently provides, they will invest in other similar risk financial securities that offer them that higher expected return.

- Investors sell the security they own, invest funds elsewhere
- Selling pressure implies prices will fall; expected returns will rise
- Prices fall until E(r) rises to a point where investors earn the higher expected return

Relationship Between Prices and Returns

Example: Oz Ltd's dividend is expected to be \$1.00 per share next year and remain unchanged in the future (i.e., $g = 0$). The following information is given:

Oz Ltd's beta = 1.2

Riskfree rate, $r_f = 6\%$

Expected market risk premium, $E(r_m) - r_f = 7\%$

a) What price should Oz Ltd be selling for today?

b) What will happen to Oz price if, after a market crash, analysts change their estimate of Oz beta to 1.5 and no other change occurs? Explain

c) What general relationship between prices and returns is being illustrated here?

Relationship Between Prices and Returns

a) Based on the CAPM

$E(r) = 0.06 + 0.07(1.2) = 0.144$ or 14.4%

$P_0 = 1.00/0.144 = \$6.94$

b) Based on the new beta estimate of 1.5, we have

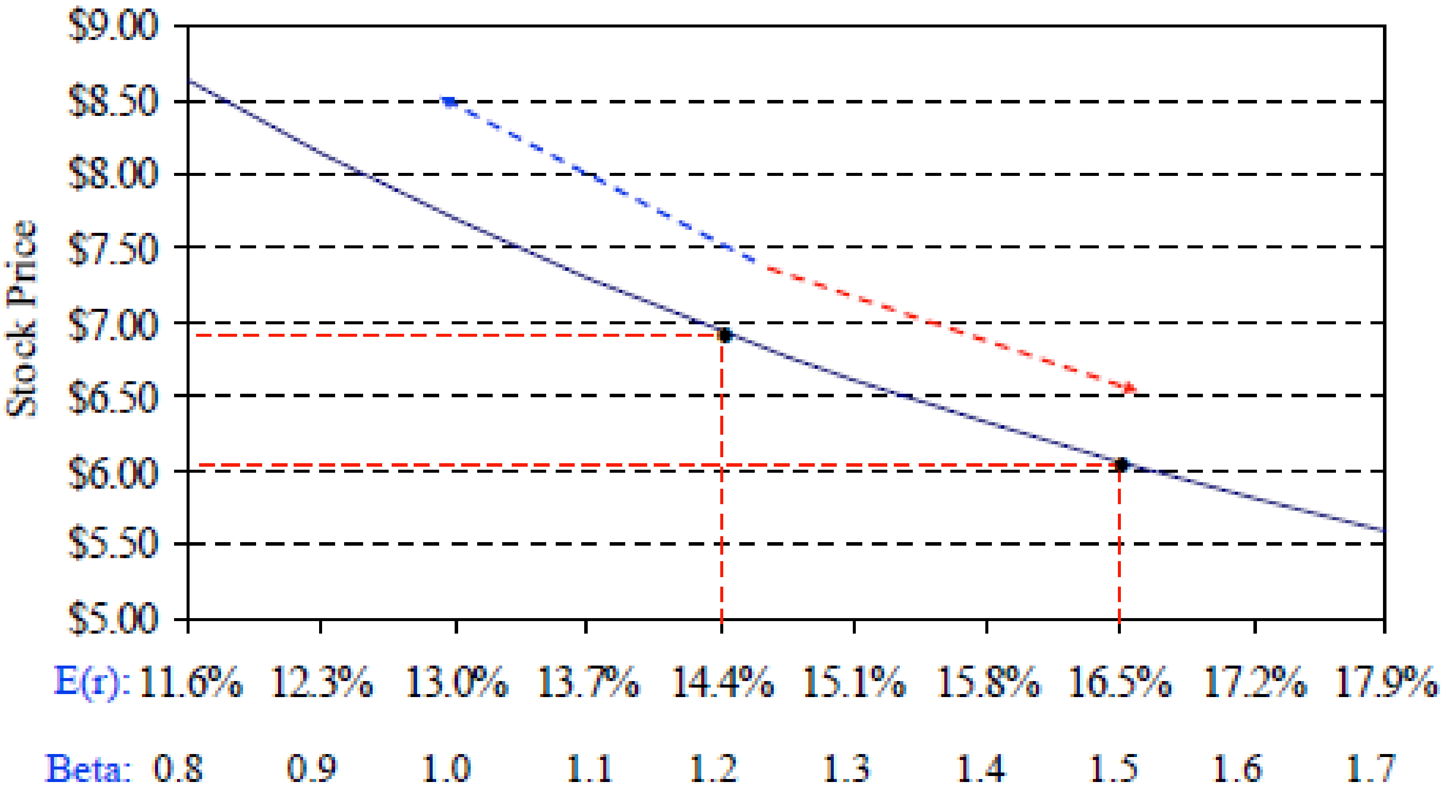
Revised $E(r) = 0.06 + 0.07(1.5) = 0.165$ or 16.5%

E(r) has increased but at $6.94 investors earn only 14.4% Investors will move funds to other similar risk securities which offer a higher expected return of 16.5%

The selling pressure results in a new price

New $P_0 = 1.00/0.165 = \$6.06$

Relationship Between Prices and Returns



Applying the CAPM

- Inputs needed to use the CAPM
  - Beta, β_i
  - Riskfree rate, r_f
  - Market return, E(rm) or market risk premium, $[E(r_m) - r_f]$

- Betas typically estimated using historical return data
  - Adjusted for infrequent (thin) trading, non-stationarity, etc

- Riskfree rate typically estimated as the return on short-term treasury instruments - e.g. the T-Note rate

- Market risk premium typically estimated as a historical (longrun) average
  - Market risk premium tends to be more stable over time

Applying the CAPM

Application 1: An online broker's current estimate of the beta of BHP Billiton (ASX code: BHP) is 1.0. Assume that the riskfree return is 6% and the expected market risk premium is 6%

a) What is BHP's expected return using the CAPM?

b) In mid-February 2004, BHP's price was $11.80 and its expected dividend in 2004 is $0.222. What constant annual growth rate in dividends is implied by this information?

Applying the CAPM

a) CAPM states that: $E(rj) = r_f + \beta j[E(r_m) - r_f]$
Given: rf = 6% and $E(r_m) - r_f = 6\%$ = Market risk premium
$E(r_{BHP}) = 0.06 + (0.06)1.0 = 0.12$ or 12.0%

b) $D_1 = \$0.222$ and $P_0 = \$11.80$
Recall from Lecture 4: $P_0 = D_1 /(ke - g)$

In equilibrium, the expected return [k_e] will be equal to the return required by investors [$E(r_{BHP})$] for bearing the systematic risk inherent in BHP shares

So, $k_e = E(r_{BHP})$
$P0 = 11.80 = 0.222/(0.12 - g)$
Solving for the growth rate in dividends, we get

$g = 0.12 - 0.222/11.80 =$ 10.1%

Applying the CAPM

Application 2: The estimated beta for NAB shares is 0.88 (see Class 9 notes). Assume: r_f = 6% and $[E(r_m) - r_f]$ = 6%

$E(r_{NAB}) = rf + [E(r_m) - r_f]\beta_{NAB} = 6 + (6)0.88 =$ 11.3%

Assume that NAB's current dividend of $1.47 is expected to grow at 8% forever. At the closing price in December 2003 of $31.50 what is the implied expected return?

$P_0 = D_1/(k_e - g)$ where $k_e = E(_r)$ in equilibrium

$E(r_{NAB}) = D_1/P_0 + g = [1.47(1.08)/31.50] + 0.08 =$ 13.0%

Since the current expected return of 13.0% will fall to the equilibrium required return of 11.3%, NAB's price should rise.

NAB is currently underpriced or undervalued

Applying the CAPM

- New $P_0 = D_1/[E(r_{NAB})-g] = 1.47(1.08)/(0.113 - 0.08) =$ \$48.11
  - What should you do now?
- Need to be careful when using this analysis
  - What if g was estimated as 6% instead of 8%?
- Now, $E(r_{NAB}) = D_1/P_0 + g = [1.47(1.06)/31.50] + 0.06 =$ 11.0%

- Expected return of 11.0% should now rise to the equilibrium level of 11.3% - NAB is now overpriced so its price should fall

- New $P_0 = D_1/(E(r_{NAB}) - g) = 1.47(1.06)/(0.113 - 0.06) =$ \$29.40
  - What would you do now?

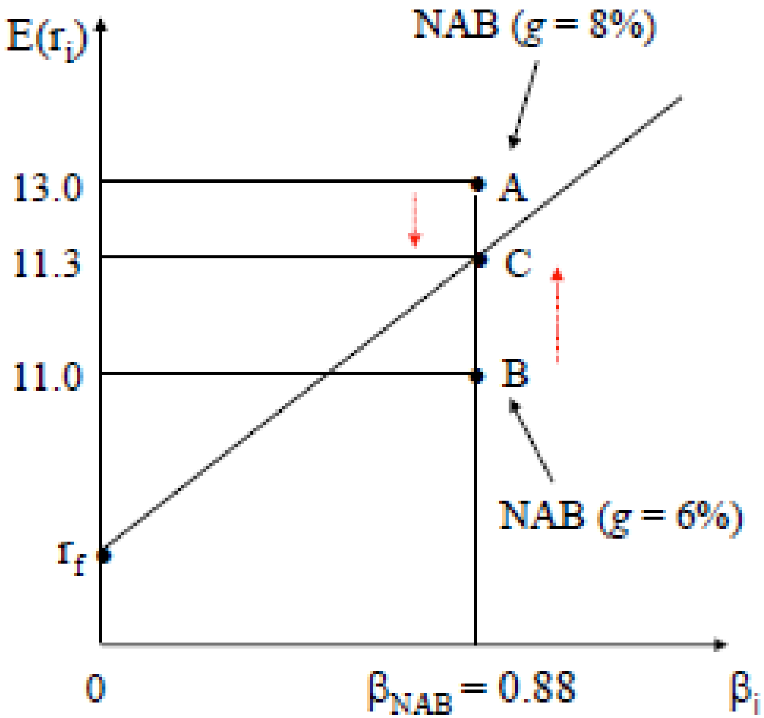
- Better to do a "what if" analysis using different future expected dividends and growth rates to get a range of possible future prices.
  - Compare estimates with other valuation models as well.

Applying the CAPM

- At A, NAB is undervalued (price "too low"; expected return "too high") - NAB's price should rise so its expected return would fall to 11.3%

- At B, NAB is overvalued (price "too high"; expected return "too low") - NAB's price should fall so its expected return would rise to 11.3%

- In equilibrium, NAB must plot on the SML (at C) earning an expected return of 11.3%



Questions about the CAPM

1. Are the CAPM's assumptions realistic?
   - A model's assumptions are simplifications of reality
   - The real concern is how the CAPM performs empirically

2. Can the CAPM be tested?
   - CAPM refers to expected returns - not realized or observed returns
   - Market portfolio comprises all risky assets - is it measurable?
   - β is a measure of systematic risk which is expected to be applicable in the future
     - Does past history give us a good estimate of the future beta?

3. Is the CAPM empirically valid?

Actual Versus Expected Returns

- Illustration: Assume market risk premium, $E(r_m)$- r_f = 5%.

Investors expect the market portfolio to pay a perpetual dividend of $15.
r_f = 10%, $E(r_m)$ = 15%

- The market portfolio's value today = 15/0.15 = $100

- Assume in year 1, r_f rises to 13%, $E(r_m)$ = 18%

- The market portfolio's value in year 1 = $15/0.18 = $83.33
- Actual or observed return on the market portfolio is
  - Observed rm = (Capital gain + Dividend)/Initial value
    = (83.33 - 100 + 15)/100 = -1.67 %

- Plotting observed returns against β gives a negatively sloped SML because rf = 10% and observed rm = -1.67% !

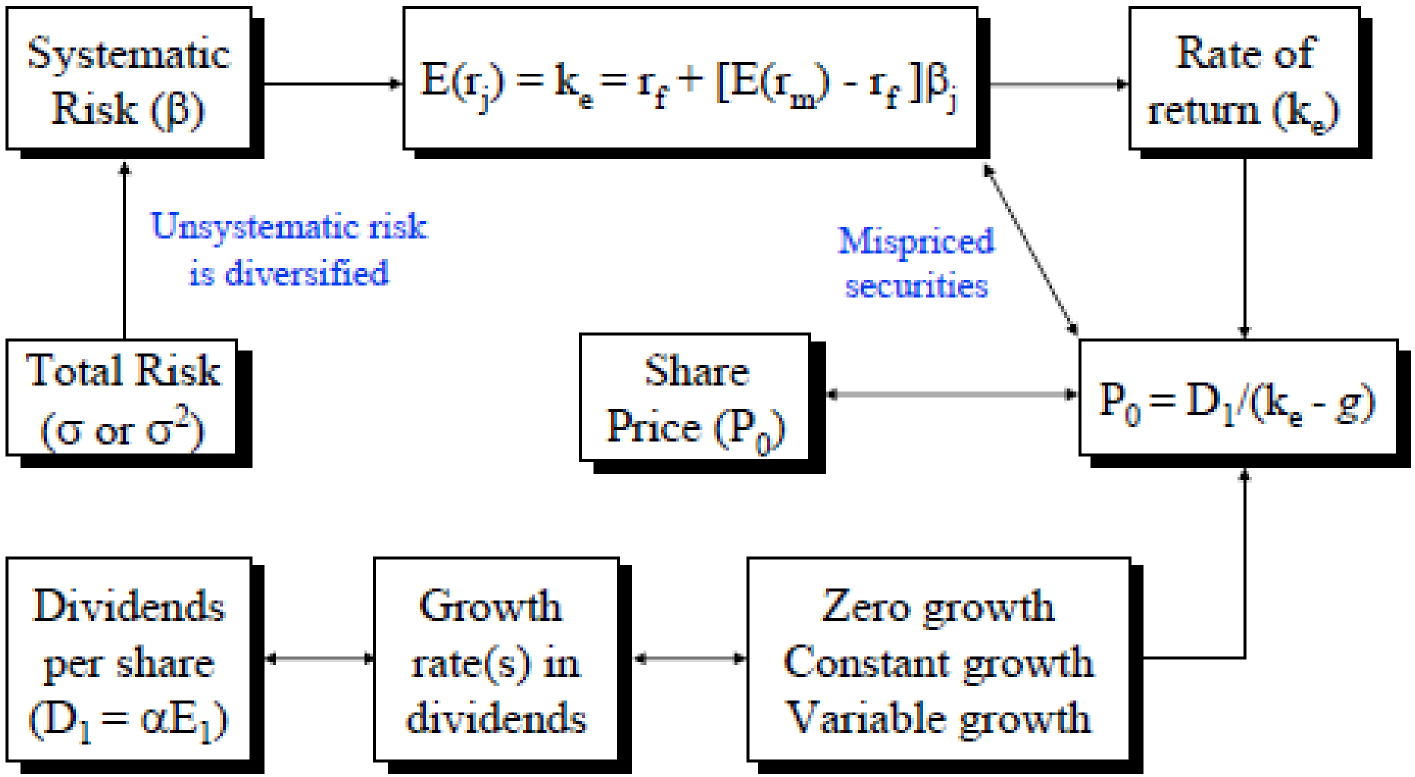
CAPM and Market Anomalies

- The existence of market anomalies is inconsistent with the CAPM

- Some findings across time
  - Returns lower on Mondays than on other days
  - Returns higher in January compared to other months (especially for small firms)
  - Returns higher the day before a holiday
  - Returns higher at the beginning and end of the trading day

- Some findings across securities (holding β constant)
  - Returns higher for firms with “low” price-earnings ratios
  - Returns higher for smaller firms compared to larger firms
  - Returns higher for firms with higher book-to-market value of equity ratios

Putting it all Together



Key Concepts

- Beta can be used in portfolio design and evaluation and in estimation of cost of equity capital

- In equilibrium a security would lie on the SML

- A security lying above the SML is currently underpriced and a security lying below the SML is currently overpriced

- CAPM’s validity has been questioned by empirical evidence
    - In the short run investors may be penalized for taking on more risk via higher beta securities
    - In the long run, investors may not always be rewarded for high systematic risk

Objectives

- Outline the intuition behind multifactor asset pricing models
- Examine the Arbitrage Pricing Theory as an alternative to the CAPM
- Outline the problems with the APT
- Use the APT to price securities and portfolios
- Comparing the APT with the CAPM

Integrative Example:

Integrative Example: Suppose the market portfolio and securities A and B have the probability distributions given in the next slide. You plan to invest $5,000 each in securities A and B to form a portfolio P.

a) Obtain the probability distribution of returns for this portfolio

b) Compute the following:

i) The expected return of securities A and B

ii) The expected return of the market portfolio and portfolio P

iii) The standard deviation of return of securities A and B

iv) The standard deviation of return of the market portfolio and portfolio P

v) The covariance and correlation coefficient between the market portfolio and securities A, B and portfolio P

vi) The beta for securities A, B and portfolio P

vii) The required return on this portfolio. Is this portfolio correctly priced?

Why or why not? Identify the source(s) of mispricing

Integrative Example:

a) Since \$5,000 is invested in securities A and B, we have $w_A = w_B = 0.5$. The portfolio's probability distribution is as follows:

- State 1: 0.5(35) + 0.5(45) = 40%
- State 2: 0.5(25) + 0.5(5) = 15%
- State 3: 0.5(-5) + 0.5(-15) = -10%
- State 4: 0.5(-30) + 0.5(20) = -5%

State	Probability	Market	Security A	Security B	Portfolio	T-Note
1	0.2	40%	35%	45%	40%	5%
2	0.3	15%	25%	5%	15%	5%
3	0.3	0%	-5%	-15%	-10%	5%
4	0.2	-20%	-30%	20%	-5%	5%

Integrative Example

b.i) Expected return of securities A and B

$E(r_a) = 0.2(0.35) + \ldots + 0.2(-0.30) = 0.07$ or 7.0%

$E(r_b) = 0.2(0.45) + \ldots + 0.2(0.20) = 10.0\%$

b.ii) Expected return of the market portfolio and portfolio P

$E(r_p) = 0.2(0.40) + \ldots + 0.2(-0.05) = 8.5\%$

$E(r_m) = 0.2(0.40) + \ldots + 0.2(-0.20) = 8.5\%$

b.iii) Standard deviation of return of securities A and B

$\sigma_a = [0.2(0.35 - 0.07)2 + \ldots + 0.2(-0.30 - 0.07)2]0.5 = 23.9\%$

$\sigma_b = [0.2(0.45 - 0.10)2 + \ldots + 0.2(0.20 - 0.10)2]0.5 = 21.5\%$

b.iv) Standard deviation of return of the market portfolio and portfolio P

$\sigma_p = [0.2(0.40 - 0.085)2 + \ldots + 0.2(-0.05 - 0.085)2]0.5 = 18.7\%$

$\sigma_m = [0.2(0.40 - 0.085)2 + \ldots + 0.2(-0.20 - 0.085)2]0.5 = 19.9\%$

Integrative Example:

b.v) Covariance and correlation coefficient between the market portfolio and securities A, B and portfolio P

σ_{am} = 0.2(0.35-0.07)(0.40-0.085) + ... + 0.2(-0.30 -0.07)(-0.20-0.085) = 0.0453

Similarly, σ_{bm} = 0.0218 and σ_{pm} = 0.0335

$\rho_{am} = \sigma_{am} / (\sigma_a \times \sigma_m) = 0.0453/(0.2390 \times 0.199) = 0.95$

$\rho_{bm} = 0.0218/(0.215 \times 0.199) = 0.51$

$\rho_{pm} = 0.0335/(0.187 \times 0.199) = 0.90$

b.vi) Beta for securities A, B and portfolio P

$\beta_a = \sigma_{am}/\sigma_m^2 = 0.0453 / 0.0395 = 1.15$

$\beta_b = 0.0218/0.0395 = 0.55$

$\beta_p = 0.0335/0.0395 = 0.85$

Note: $\beta_p = w_a \times \beta_a + w_b \times \beta_b = 0.5 \times 1.15 + 0.5 \times 0.55 = 0.85$

Integrative Example:

b.vii) Required return on portfolio P

- $E(r_p) = rf + [E(r_m) - r_f\,]\beta_p = 0.05 + (0.085 - 0.05)(0.85) = 7.975\%$
- Portfolio is mispriced - expected return is 8.5% but the required return is 7.975%
  - Portfolio's expected return would fall to 7.975% - the prices of securities A, B or both would change

- $E(r_a) = r_f + [E(r_m) - r_f\,]\beta_a = 0.05 + (0.085 - 0.05)(1.15) = 9.025\%$

- $E(r_b) = r_f + [E(r_m) - r_f\,]\beta_b = 0.05 + (0.085 - 0.05)(0.55) = 6.925\%$
  - Security A's expected return is 7% which is lower than the required return - A is currently overpriced
  - Security B's expected return is 10% which is higher than the required return - B is currently underpriced

Integrative Example:

State	Probability	Market	Security A	Security B	Portfolio	T-Note
1	0.2	40%	35%	45%	40%	5%
2	0.3	15%	25%	5%	15%	5%
3	0.3	0%	-5%	-15%	-10%	5%
4	0.2	-20%	-30%	20%	-5%	5%

E[r]	8.5%	7.0%	10.0%	8.5%	5.0%
Var[r]	0.0395	0.0571	0.0460	0.0350	0.0000
SD[r]	19.9%	23.9%	21.4%	18.7%	0.0%

Cov[M, A]	0.0453	Corr[M, A]	0.95	Beta[A]	1.15
Cov[M, B]	0.0218	Corr[M, B]	0.51	Beta[B]	0.55
Cov[M, P]	0.0335	Corr[M, P]	0.90	Beta[P]	0.85

CAPM: A Recap

- The total risk of an asset's returns reflects two sources

- Unsystematic (unique or diversifiable) risk is firm-specific and can be avoided by holding a well-diversified portfolio
  - Investors will not be rewarded for bearing unsystematic risk

- Systematic (market or non-diversifiable) risk affects all assets and cannot be avoided by diversification
  - Beta measures how sensitive an asset's return is to changes in the market portfolio's return
  - From the SML: Greater the sensitivity of an asset's returns to this risk (i.e., higher the beta), higher its expected return

- CAPM implies only systematic risk (i.e., beta) is "priced"

Multifactor Asset Pricing Models

- What determines the returns on a firm over a given time period?

- The return can be thought of as consisting of two parts
  1. The normal or expected return component which depends on all the information available on the security
  2. The surprise or unexpected return component which is due to new, unexpected “news” announcements

- r_{jt} = Et-1[r_j] + u_{jt}
  - u_{jt} is the unexpected surprise due to “news” announcements
  - E_{t-1}[•] is the current (time t-1) expected value

Multifactor Asset Pricing Models

- Need to be careful about how much of the new announcement is a surprise
    - Example: Interest rate announcements are often anticipated by the market

- The unexpected component is the true risk to any investment
    - If what you expect occurs then there is no surprise!

- Announcements about interest rate, inflation or GDP have implications for nearly all companies - these represent systematic (or market) risks

- Announcements about the appointment of a new CEO, a new product, etc represent unsystematic (or firm-specific) risks

- The two components to the surprise are the market surprise (m) and firm-specific surprise (ε) - $u_j = m_j + \varepsilon_j$

Multifactor Asset Pricing Models

- Unsystematic risks are unlikely to be related to each other
  - The firm-specific risk of Firm A will be unrelated to the firmspecific risk of Firm B
  - That is, Cov(ε_a, ε_b) = 0

- But their systematic risks are related and influence their returns
  - For example, both firms are most likely influenced by inflation

- The influence of a systematic risk like inflation can be captured by a beta coefficient
  - If risk increases with increased inflation the inflation beta is positive
  - If risk decreases with increased inflation the inflation beta is negative
  - If risk is unchanged with increased inflation the inflation beta is zero

Multifactor Asset Pricing Models

- To summarize - observed returns depend on a number of "factors"
  - r_{jt} = Et-1[r_j] + u_j
  - r_{jt} = Et-1[r_j] + m_j + ε_J
  - $r_{jt} = E_{t-1}[r_j] + \beta_{1j} F_1 + \beta_{2j} F_2 + ... + \beta_{nj} F_n + \varepsilon_j$
  - F_j is j_{th} factor, such as returns on industry indices, economic variables like economic growth, inflation, exchange rates
  - E(r_j) is the part of returns unaffected by these factors and is the expected return on security j
  - ε_j is the firm-specific return

- The CAPM can be thought of as a "one-factor" model in which returns primarily reflect the market index "factor"

Estimating Total Returns

Example: Assume a one-year investment horizon. Also assume that the returns on XYX Ltd are influenced by three factors with the following betas associated with the factors

- Inflation rate $\beta INF = 1.5$
- Interest rates $\beta INT = 1.0$
- GDP growth $\beta GDP = -0.9$

- Interpretation: A 1% surprise increase in GDP would produce a 0.9% decline in the returns on XYX Ltd
- Assume that the initial expectations of these factors are
  - Inflation rate 3%
  - Interest rates 6%
  - GDP growth 4%

Estimating Total Returns

During the year the following occurs

- Inflation rate is 5%
- Interest rates are 7%
- GDP growth is 4%

- Also, XYX Ltd has quickly developed a new business strategy which caused returns to increase by 4%
  - This was an unanticipated development, $\varepsilon_j = 4\%$

- We need to calculate the surprises as follows
  - Surprise = Actual – Expectation
    - Inflation surprise = 5 - 3 = 2%
    - Interest rates surprise = 7 - 6 = 1%
    - GDP growth surprise = 4 - 4 = 0%

Estimating Total Returns

- The total effect of the systematic components is
  - $m_j = \beta_{INF} F_{INF} + \beta_{INT} F_{INT} + \beta_{GDP} F_{GDP}$
  - $m_j = (1.5 \times 2\%) + (1.0 \times 1\%) + (-0.9 \times 0\%) = 4\%$

- Adding the unsystematic component, we get
  - $m_j + \varepsilon_j = 4\% + 4\% = 8\%$

- If the expected component of returns on XYX Ltd was 5%, then the total return will be
  - $r_{XYX,t} = Et\text{-}1[r_j] + m_j + \varepsilon_j = 5\% + 8\% = 13\%$

- The APT uses arbitrage arguments to relate the actual return on any asset j to a number of factors
  - Note: One of these factors may be the market portfolio return, but not necessarily

The APT's Main Assumptions

1. Capital markets are competitive

2. Investors prefer more wealth to less wealth

3. The process generating asset returns can be represented by an N-factor model

- Observed returns (rj) depend on N different factors
- $r_{jt} = Et\text{-}1[r_j] + \beta_{1j} F_1 + \beta_{2j} F_2 + ... + \beta_{nj} F_n + \varepsilon_j$

- APT uses fewer assumptions, and is also more flexible, than the CAPM
  - Returns depend on several factors, not just one factor
  - No special role for the market portfolio
  - No requirement that asset returns should be normally distributed

The Arbitrage Pricing Theory

- The APT uses arbitrage arguments to relate the observed return on any asset j to a number of factors

- Using these arbitrage arguments, the APT implies
  - $E(r_j) = \lambda_0 + \lambda_1 \beta_{1j} + \lambda_2 \beta_{2j} + ... + \lambda_n \beta_{nj}$
  - λ_0 is similar to the riskfree rate of return
  - λ_1 through λ_n are market "prices" of particular types of risk factors
  - β_s are the exposures of asset j to particular types of risk factors

Problems With the APT

The APT does not prespecify

- The number of factors
- Precisely what these factors might be
- Specify the sign or magnitude of the coefficients in the pricing relationship

• Researchers have identified (at least) four risk factors as being “priced” by the market

1. Unanticipated changes in the inflation rate
2. Unanticipated changes in industrial production
3. Unanticipated changes in default risk premium on securities
4. Unanticipated changes in the term structure of interest rates

Using the APT

Application: You own a well-diversified portfolio with a CAPM beta of 1. Assume rf = 8% and E(rm) - rf = 6%.

•The CAPM implies: $E(_{rj}) = 0.08 + (0.06)1.0 = 14.0\%$

• Now assume that a two-factor APT describes asset returns
  • Factor 1: Unexpected changes in industrial production
  • Factor 2: Unexpected changes in the inflation rate
  • How can investors vary the portfolio's sensitivity to the factors?

•The APT relationship: $E(r_j) = \lambda_0 + \lambda_1 \beta_{1j} + \lambda_2 \beta_{2j}$
  • Assume: $E(r_j) = 0.08 + (0.05)\beta_{1j} + (0.11)\beta_{2j}$
  • $\lambda_0 = 0.08$ (riskfree rate), $\lambda_1 = 0.05$, $\lambda_2 = 0.11$
  • In equilibrium, $E(r_j) = 14.0\%$ (why?)

Using the APT

If the portfolio's sensitivity to factor 1 is -0.5 ($\beta p1 = -0.5$), what is its sensitivity to factor 2?

- Using the APT, $E(r_j) = 0.14 = 0.08 + (0.05)(-0.5) + (0.11)\beta_{p2}$
- $\beta_{p2} = (0.14 - 0.08 + 0.025)/0.11 = 0.77$

- If you rebalance the portfolio to keep the same expected return but eliminate the exposure to factor 2 ($\beta_{p2} = 0$), what is β_{p1}?
  - $E(r_j) = 0.14 = 0.08 + (0.05)(\beta_{p1}) + (0.11)(0)$
  - $\beta_{p1} = (0.14 - 0.08)/0.05 = 1.2$

- Interpretation: The portfolio's sensitivity to unexpected changes in inflation (factor 2) has been eliminated but it has now become more sensitive to unexpected changes in industrial production (factor 1)

CAPM and the APT

The CAPM may be viewed as a one-factor APT *under certain conditions*

- $r_j = E(r_j) + \beta_{1j} F_1 + e_j$
- CAPM has one factor which is the unexpected return on the market portfolio, $r_m - E(r_m)$
- The "price" of the market factor risk is the expected excess return on the market portfolio, $\lambda 1 = E(r_m) – r_f$

- $E(r_j) = \lambda_0 + \lambda_1 \beta_{1j} = r_f + [E(r_m) - r_f] \beta_{1j}$
  - This is the SML relationship with β1j measuring the sensitivity of the stock to the "market factor", its beta

Key Concepts

- The (total) risk of an asset's returns reflects two sources
    - Unsystematic or firm-specific risk which can be diversified
    - Systematic or market risk which cannot be diversified

- Beta measures how sensitive an asset's return is to the return on the market portfolio

- CAPM implies only market risk (beta) is priced

- The APT explains observed returns as depending on a number of risk factors

- Some risk factors suggested by evidence include unanticipated changes in inflation, industrial production, the default risk premium, and the term structure of interest rates

- The CAPM can be thought of as a one-factor model in which returns reflect the market index factor

Key Relationships

- Observed return in a multifactor model
  - $r_{jt} = E_{t-1}[r_j\ ] + \beta_{1j} F_1 + \beta_{2j} F_2 + ... + \beta_{nj} F_n + \varepsilon_j$

- The APT model
  - $E(r_j) = \lambda_0 + \lambda_1\ \beta_{1j} + \lambda_2\ \beta_{2j} + ... + \lambda_n\ \beta_{nj}$