Fixed Income

Session 1

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Purpose of the Capital Market

Firms and individuals use the capital markets for long-term investments

Primary reason that individuals and firms choose to borrow long-term is to reduce the risk that interest rates will rise before they pay off their debt**.**

Capital Market Participants

Primary users: federal and local governments and corporations

Largest purchasers: households. Frequently, individuals and households deposit funds in financial institutions that use the funds to purchase capital market instruments such as bonds or stock

Capital Market Trading

Occurs in either primary market or the secondary market

Primary market: where new issues of stocks and bonds are introduced. Investment funds, corporations, and individual investors can all purchase securities offered in the primary market. (*IPO – initial public offering*).

Secondary market: where the sale of previously issued securities takes place, and it is important because most investors plan to sell long-term bonds before they reach maturity.

Types of Bonds

- Bonds are Securities that represent a debt owned by the issuer to the investor.
- Bonds obligate the issuer to pay a specified amount at a given date, generally with periodic payments
- The par, face, or maturity value of the bond is the amount that the issuer must pay at maturity
- The coupon rate is the rate of interest that the issuer must pay

- This rate is usually fixed for the duration of the bond and does not fluctuate with market interest rates,
- If the repayment terms of a bond are not met, the holder of a bond has a claim on assets of the issuer

Par or Face Value - The amount of money that is paid to the bondholders at maturity. For most bonds this amount is $1,000. It also generally represents the amount of money borrowed by the bond issuer.

Coupon Rate -The coupon rate, which is generally fixed, determines the periodic coupon or interest payments. It is expressed as a percentage of the bond's face value. It also represents the interest cost of the bond to the issuer.

Coupon Payments - The coupon payments represent the periodic interest payments from the bond issuer to the bondholder. The annual coupon payment is calculated by multiplying the coupon rate by the bond's face value. Since most bonds pay interest semiannually, generally one half of the annual coupon is paid to the bondholders every six months.

Maturity Date - The maturity date represents the date on which the bond matures, *i.e.,* the date on which the face value is repaid. The last coupon payment is also paid on the maturity date.

Original Maturity -The time from when the bond was issued until its maturity date.

Remaining Maturity -The time currently remaining until the maturity date.

Call Date - For bonds which are callable, *i.e.,* bonds which can be redeemed by the issuer prior to maturity, the call date represents the earliest date at which the bond can be called.

Call Price - The amount of money the issuer has to pay to call a callable bond (there is a premium for calling the bond early). When a bond first becomes callable, *i.e.*, on the call date, the call price is often set to equal the face value plus one year's interest.

Required Return - The rate of return that investors currently require on a bond.

Yield to Maturity - The rate of return that an investor would earn if he bought the bond at its current market price and held it until maturity. Alternatively, it represents the discount rate which equates the discounted value of a bond's future cash flows to its current market price.

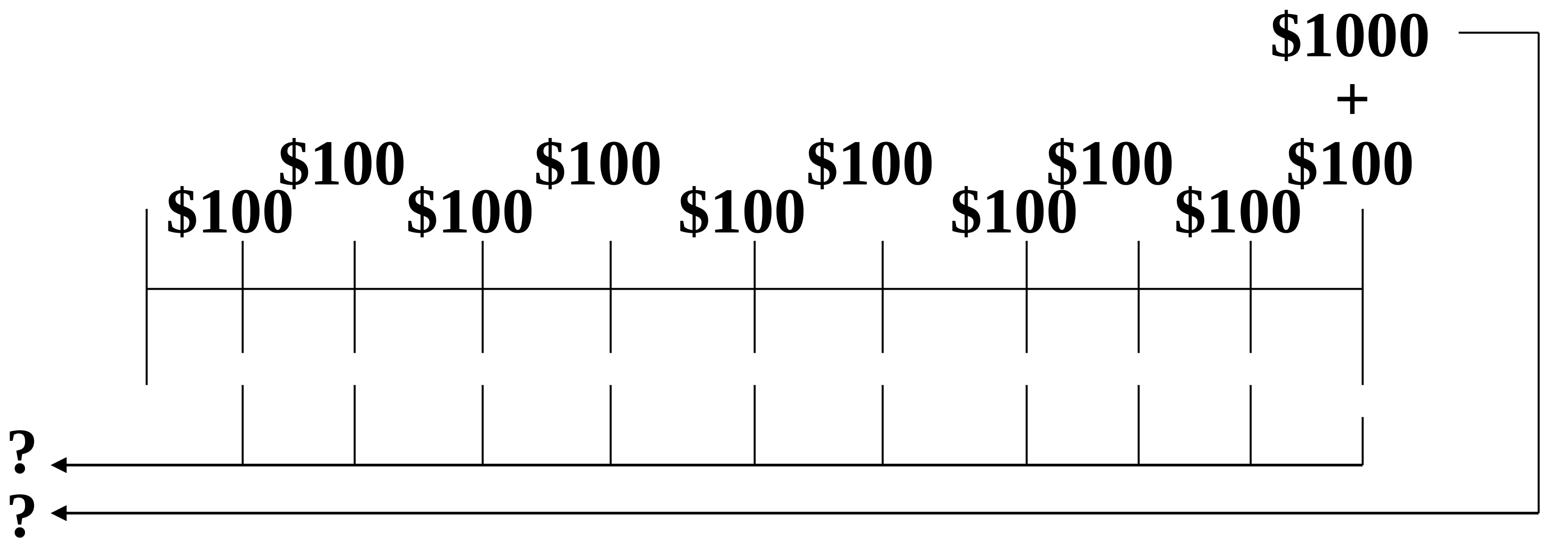
Yield to Call - The rate of return that an investor would earn if he bought a callable bond at its current market price and held it until the call date given that the bond was called on the call date.

Treasury Securities

Type	Maturity
Treasury Bill	Less than 1 year
Treasury Note	1 to 10 years
Treasury Bond	10 to 20 years

Example

Assume an investor buys a 10-year bond from the KLM corporation on January 1, 2003. The bond has a face value of $1000 and pays an annual 10% coupon. The current market rate of return is 12%. Calculate the price of this bond today.



First, find the value of the coupon stream

Remember to follow the same approach you use in time value of money calculations.

You can find the PV of a cash flow stream

$$PV = \frac{\$100}{(1+0.12)^1} + \frac{\$100}{(1+0.12)^2} + \frac{\$100}{(1+0.12)^3} + \frac{\$100}{(1+0.12)^4} + \frac{\$100}{(1+0.12)^5} +$$

$$\frac{\$100}{(1+0.12)^6} + \frac{\$100}{(1+0.12)^7} + \frac{\$100}{(1+0.12)^8} + \frac{\$100}{(1+0.12)^9} + \frac{\$100}{(1+0.12)^{10}}$$

Or, you can find the PV of an annuity

$$PVA = \$100 \times \frac{1 - (1+0.12)^{-10}}{0.12}$$

PV = $565.02

Find the PV of the face value

$$PV = \frac{CFt}{(1+r)^t}$$

$$PV = \frac{\$1{,}000}{(1.12)^{10}} = \$321.97$$

Add the two values together to get the total PV

$565.02 + $321.97 = $886.99

Risks Associated with Investing in Bonds

Interest-rate risk or market risk

As interest rates rise, the price of a bond fall (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price)

Reinvestment Income or Reinvestment Risk

Calculation of the yield assumes that the CF received are invested

Interest rate at which interim CF can be reinvested will fall.

Greater for longer holding periods, as well as for bonds with large, early CF, such as high coupon bonds

Call Risk (callable bond)

Issuer can retire or "call" all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate)

Investor perspective: i) the CF pattern is not known with certainty, ii) exposed to reinvestment risk (issuers will call the bonds when interests have dropped) and iii) capital appreciation of a bond will be reduce

Risks Associated with Investing in Bonds (cont.)

Credit Risk:
Risk that the issuer of a bond will fail to satisfy the terms of the obligation (coupons and repayment of the amount borrowed)

Inflation Risk: arises because of the variation in the value of cash flows from a security due to inflation.

Exchange-rate Risk: A non-dollar-denominated bond has unknown US dollar cash flows.

Liquidity Risk: Size and spread btw the bid and ask price. The wider the dealer spread, the more the liquidity risk.

Volatility Risk: Value of an option rises when expected interest-rate volatility increases. In the case of a bond that is callable, the price of security falls, because the investor has given away a more valuable option.

Interest Rate Risk

Riskiness of an asset's return that results from interest-rate changes

Changes in interest rates make investments in long-term bonds more risky.

Prices and returns for long-term bonds are more volatile than those for shorter-term bonds

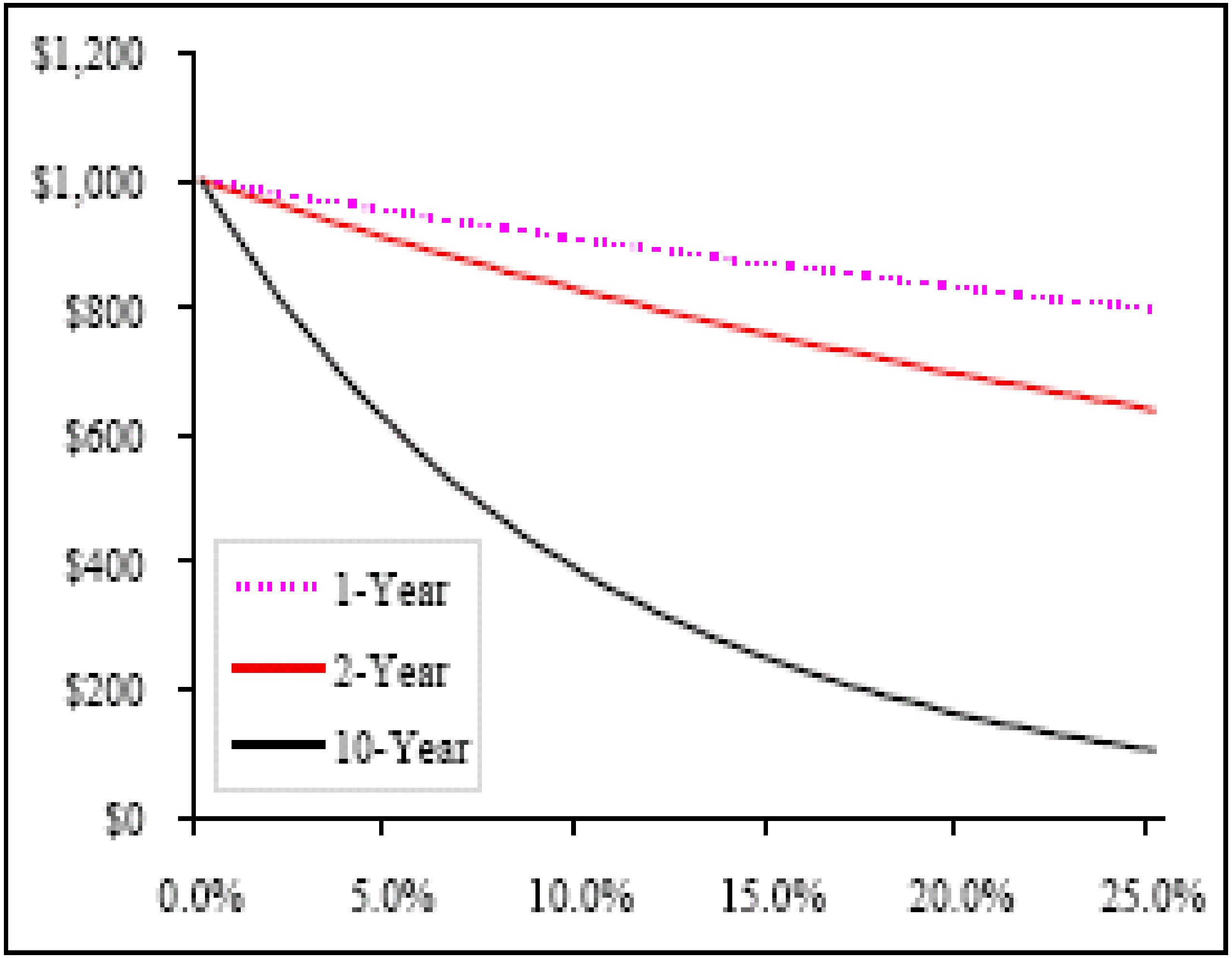
What happens to bond values if required return is not equal to the coupon rate?

The bond's value will differ from its par value

R > Coupon Interest Rate → P_0 < par value = DISCOUNT

R < Coupon Interest Rate → P_0 > par value = PREMIUM

Bond Prices and Interest Rates



- Bond prices are inversely related to Interest Rate
- Longer term bonds are more sensitive to Interest Rate changes than short term bonds
- The lower the Interest Rate, the more sensitive the price

Measuring Interest Rate Sensitivity

We would like to measure the interest rate sensitivity of a bond or a portfolio of bonds.

How much do bond prices change if interest rates change by a small amount? Why is it important?

Yield	10% 5 Years PV	10% 20 years PV
9.96%	100.1546	100.3441
9.97%	100.1159	100.2579
9.98%	100.0773	100.1718
9.99%	100.0386	100.0859
10.00%	100.00	100.00
10.01%	99.9614	99.9143
10.02%	99.9228	99.8286
10.03%	99.8843	99.7431
10.04%	99.8457	99.6578

Pricing a Bond

Equal to the present value of the expected cash flows from the financial instrument. Determining the price requires:

- An estimate of the expected cash flows
- An estimate of the appropriate required yield

The price of the bond is the present value of the cash flows, it is determined by adding these two present values:

i) The present value of the semi-annual coupon payments

ii) The present value of the par or maturity value at the maturity date

$$P=\frac{C}{(1+r)}+\frac{C}{(1+r)^2}+\frac{C}{(1+r)^3}+\ldots+\frac{C}{(1+r)^n}+\frac{M}{(1+r)^n}$$

$$P=\sum_{i=1}^{n} \frac{C}{(1+r)^{t}}+\frac{M}{(1+r)^{n}}$$

P = Price

n =number of periods (nr of years times 2)

C = semi-annual coupon payment

r = periodic interest rate (required annual yield divided by 2)

t = time period when payment is to be received

Because the semi-annual coupon payments are equivalent to an ordinary annuity, applying the equation for the present value of an ordinary annuity gives the present value of the coupon payments:

$$C\left[\frac{1-\frac{1}{(1+r)^{n}}}{r}\right]$$

Consider a 20 year 10% coupon bond with a par value of $1,000. The required yield on this bound is 11%.

$$\$50\left[\frac{1-\frac{1}{(1+0.055)^{40}}}{0.055}\right]=\$802.31$$

The PV of the par or maturity value of $1,000 received 40 six-month periods from now, discounted at 5.5%, is $117.46, as follows:

$$\frac{\$1{,}000}{(1.055)^{40}}=\frac{\$1{,}000}{8.51332}=\$117.46$$

Price = PV coupon payments + PV of par (maturity value)

$802.31 + $117.46 = $919.77

Suppose that, instead of an 11% required yield, the required yield is 6.8%.

Price of the bond: $1,347.04

Pricing Zero-Coupon Bonds

Vn= market Price of the Bond in period n

F = Face Value;

R = Annual percentage rate

m = compounding period (annual, m=1, semi-annual m=2…)

i = Effective periodic interest rate: i = R/m

T = Maturity (years)

N = Nr of compounding periods: N = T*m

Two cash flows to purchase a bond (V0 at time 0 and F at time T

What is the price of the bond?

$$V_0 = \frac{F}{(1+i)^N}$$

Yield to Maturity

Interest rate that equates the present value of cash flows received from a debt instrument with is value today.

Is the yield promised by the bondholder on the assumption that the bond will be held to maturity, that all coupon and principal payments will be made and coupon payments are reinvested at the bond's promised yield at the same rate as invested. It is a measurement of the return of the bond. This technique in theory allows investors to calculate the fair value of different financial instruments. The YTM is almost always given in terms of annual effective rate.

The calculation of YTM is identical to the calculation of internal rate of return.

If a bond's current yield is less than its YTM, then the bond is selling at a discount.
If a bond's current yield is more than its YTM, then the bond is selling at a premium.
If a bond's current yield is equal to its YTM, then the bond is selling at par.

Yields to Maturity on a 10% Coupon rate Bond Maturing in 10 years (Face Value = $ 1,000)

Price of Bond (4)	Yield to Maturity
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

1. When the coupon bond is priced at is face value, the YTM equals the coupon rate
2. The price of a coupon bond and the YTM are negatively related; that is, as the YTM rises, the price of the bond falls. If the YTM falls, the price of the bond rises
3. The YTM is greater than the coupon rate when the bond price is below its face value

Variants of Yield to Maturity

Given that many bonds have different characteristics, there are some variants of YTM:

• Yield to Call: when a bond is callable (can be repurchased by the issuer before the maturity), the market looks also to the Yield to Call, which is the same calculation of the YTM, but assumes that the bond will be called, so the cash flow is shortened.

• Yield to Put: same as Yield to Call, but when the bond holder has the option to sell the bond back to the issuer at a fixed price on specified date.

• Yield to Worst: when a bond is callable, "*puttable*" or has other features, the yield to worst is the lowest yield of Yield to Maturity, Yield to Call, Yield to Put, and others.

Example

Consider a 30-year zero coupon bond with a face value of $100. If the bond is priced at a yield-to-maturity of 10%, it will cost $5.73 today (the present value of this cash flow). Over the coming 30 years, the price will advance to $100, and the annualized return will be 10%.

Suppose that over the first 10 years of the holding period, interest rates decline, and the yield-to-maturity on the bond falls to 7%.

With 20 years remaining to maturity, the price of the bond will be $25.84.

Even though the yield-to-maturity for the remaining life of the bond is just 7%, and the yield-to-maturity bargained for when the bond was purchased was only 10%, the return earned over the first 10 years is 16.26%. This can be found by evaluating:

$$(1+i) = \left(25.84/5.73\right)^{0.1} = 1.1626$$

Over the remaining 20 years of the bond, the annual rate earned is not 16.26%, but 7%

This can be found by evaluating:

$$25.84 = 100/(1+i)^{20}$$

$$(1+i) = \left(100/25.84\right)^{0.05} = 1.07$$

Over the entire 30 year holding period, the original $5.73 invested matured to $100, so 10% annually was made, irrespective of interest rate changes in between

Real and Nominal Interest rates

Inflation - Rate at which prices as a whole are increasing.

Nominal Interest Rate - Rate at which money invested grows.

Real Interest Rate - Rate at which the purchasing power of an investment increases

$$1+\text{real interest rate}=\frac{1+\text{nominal interest rate}}{1+\text{inflation rate}}$$

Approximation formula

$$\text{Real int. rate»nominal int. rate - inflation rate}$$

The Fisher Effect And Expected Inflation

The relationship between nominal and real (inflation-adjusted) interest rates and expected inflation called the Fisher Effect (or Fisher Equation).

Nominal rate (*r*) is approximately equal to real rate of interest (*a*) plus a premium for expected inflation (*i*).

If real rate equals 3% (a = 0.03) and expected inflation equals 2% (i = 0.02):

$$r \cong a + i \cong 0.03 + 0.02 \cong 0.05 \cong 5\%$$

True Fisher Effect multiplicative, rather than additive:

$$(1+r) = (1+a)(1+i) = (1.03)(1.02) = 1.0506; \quad \text{so } r = 5.06\%$$

Interest Rates and Returns

Rate of return: payments to the owner plus the change in its value, expressed as a fraction of its purchase price

What would the rate of return be on a bond bought for $1,000 and sold one year later for $800. The bond has a face value of $1,000 and a coupon rate of 8%

$$R=\frac{C+P_{t+1}-P_t}{P_t}$$

Where: R is the rate of return, C the coupon payment, P_{t+1} price of the bond one year later and P_t price of the bond today

$$R = \frac{\$80+(\$800-\$1,000)}{\$1,000} \Leftrightarrow R = -12\%$$

One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%

(1)	(2)	(3)	(4)	(5)	(6)
Years to Maturity when Bond is Purchased	Initial Current Yield (%)	Initial Price ($)	Price Next Year ($)	Rate of Capital Gain (%)	Rate of Return (2+5)
30	10	1000	503	-49.7	-39.7
20	10	1000	516	-48.4	-38.4
10	10	1000	597	-40.3	-30.3
5	10	1000	741	-25.9	-15.9
2	10	1000	917	-8.3	+1.7
1	10	1000	1000	0.0	+10.0

Calculating DURATION to measure Interest-rate risk

When interest rates change, a bond with *longer term to maturity* has a *larger change in its price* and hence *more interest-rate risk* than a bond with *a shorter term to maturity*.

More precise information needed: actual capital gain or loss that occurs when the interest rate changes by a certain amount.

Duration: average lifetime of a debt security's stream of payments

Two bonds with the same term to maturity does not mean that they have the same interest-rate risk

Example: A long-term discount bond with ten years to maturity, a so-called zero-coupon bond, makes all of its payments at the end of the ten years, whereas a 10% coupon bond with ten years to maturity makes substantial cash payments before the maturity date.

Rate of Capital Gain

Calculate the rate of capital gain or loss on a ten-year zero coupon bond for which the interest rate has increased from 10% to 20%. The bond has a face value of $1,000

$$g=\frac{P_{t+1}-P_t}{P_t}$$

P_{t+1}= price of the bond one year from now $$=\frac{\$1{,}000}{(1+0.20)^9}=\$193.81$$

P= price of the bond today $$=\frac{\$1{,}000}{(1+0.10)^{10}}=\$385.54$$

Thus, $$g=\frac{\$193.81-\$385.54}{\$385.54}=-0.497=-49.7\%$$

But as we already calculated (slide 28), the capital gain on the 10% ten-year coupon bond is -40.3%. The interest rate risk for the ten-year coupon bond is *less* than for the ten-year zero coupon bond. The effective maturity on the coupon bond (which measures interest rate risk) is, as expected, *shorter* than the effective maturity on the zero-coupon bond.

Key relationships:

• For small changes in yield (such as one basis point) the percentage change in a bond price is roughly the same whether the yield is increased or decreased.

• For a larger change in yield the percentage rise in the bond price if the yield is decreased is greater than the percentage fall in the bond price if the yield is increased.

Maturity and Price Risk

Zero coupon bonds have well-defined relationship between maturity and interest rate sensitivity.

Coupon Bonds can have different sensitivities for the same maturity

Need concept of “average maturity” of coupon bonds

- DURATION

Macaulay's Duration

Measure the effective maturity of a coupon bond

Realize that a coupon bond is equivalent to a set of *zero-coupon discount bonds*

A ten-year 10% coupon bond is equivalent to a set of zero-coupon discount bonds

A ten-year 10% coupon bond with $1,000 face value has cash payments identical to the following set of a zero-coupon bonds:

- a $100 one year zero-coupon bond (which pays the equivalent of the $100 coupon payment made by the $1,000 ten-year 10% coupon bond at the end of one year;
- a $100 two-year zero-coupon bond (which pays the equivalent of the $100 coupon payment at the end of two years)
- ………………..
- and a $1,000 ten-year zero-coupon bond (which pays back the equivalent of the coupon bond's $1,000 face value)

Calculating Duration on a $1,000 ten-year 10% Coupon Bond when its Interest Rate is 10%

(1)	(2)	(3)	(4)	(5)
Year	Cash Payments (Zero-Coupon Bonds)	Present Value of Cash Payments	Weights (% of total)	Weighted Maturity (1×4)/100 years
1	**100**	**90.91**	**9.091**	**0.09091**
2	**100**	**82.64**	**8.264**	**0.16528**
3	**100**	**75.13**	**7.513**	**0.22539**
4	**100**	**68.30**	**6.830**	**0.27320**
5	**100**	**62.09**	**6.209**	**0.31045**
6	**100**	**56.44**	**5.644**	**0.33864**
7	**100**	**51.32**	**5.132**	**0.35924**
8	**100**	**46.65**	**4.665**	**0.37320**
9	**100**	**42.41**	**4.241**	**0.38169**
10	**100**	**38.55**	**3.855**	**0.38550**
10	**1000**	**385.54**	**38.554**	**3.85500**
Total		1,000.00	100.00	**6.75850**

Duration is a weighted average of the maturities of the cash payments

Can be written as follows:

$$DUR = \sum_{t=1}^{n} t \frac{CP_t}{(1+i)^t} \Bigg/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

Where:

DUR = Duration

t = years until cash payment is made

CP_t = Cash payment (interest plus principal) at time t

i = interest rate

n = years to maturity of the security

All else being equal, the *longer the term to maturity* of a bond, the *longer its duration*.

All else being equal, when *interest rates rise*, the *duration of a coupon bond falls*.

All else being equal, the *higher the coupon rate* on the bond, the *shorter the bond's duration*.

The duration of a portfolio of securities is the *weighted average of the durations* of the individual securities, with the weights reflecting the proportion of the portfolio invested in each.

The *greater the duration* of a security, the *greater the percentage change* in the market value of the security for a *given change in interest rates*. Therefore, the *greater the duration* of a security, the *greater its interest-rate risk*

Key findings

1. The only bond whose return equals the initial YTM is one whose time to maturity is the same as the holding period
2. A rise in interest rates is associated with a fall in bond prices, resulting in capital losses on bonds whose terms to maturity are longer than the holding period
3. The more distant a bond's maturity, the greater the size of the price change associated with an interest-rate change
4. The more distant a bond's maturity, the lower the rate of return that occurs as a result of the increase in the interest rate
5. Even though a bond has a substantial initial interest rate, its return can turn out to be negative if interest rate rises

Questions and Quantitative Problems

1. A ten year, 7% coupon bond with a face value of $1,000 is currently selling for $871.65. Compute your rate of return if you sell the bond next year for $880.10

2. Calculate the duration of a $1,000, 6% coupon bond with three years to maturity. Assume that all market interest rates are 7%.

3. Consider a bond that promises the following cash flows. The required discount rate is 12%.

4. Consider the two bonds described below:

	Bond A	Bond B
Maturity (yr)	15	20
Coupon rate (%) (Paid semi-annually)	10	6
Par value	$1,000	$1,000

a) If both bonds had a required return of 8%, what would the bond's prices be?

b) Describe what it means if a bond sells at a discount, a premium, and at its face amount (par value). Are these two bonds selling at discount, premium, or par?

c) If the required return on the two bonds rose to 10%, what would the bond's prices be?

5) The yield on a corporate bond is 10%, and it is currently selling at par. The marginal tax rate is 20%. A par value municipal bond with a coupon rate of 8.50% is available. Which security is a better buy?

6) If the municipal bond rate is 4.25% and the corporate bond rate is 6.25%, what is the marginal tax rate, assuming investors are indifferent between the two bonds.

7)A $1,000 par bond with an annual coupon has only one year until maturity. Its current yield is 6.713% and its yield to maturity is 10%. What is the price of the bond?