Portfolio Management
2010-2011

Question 1

Assume an economy under certainty in which the annualized spot interest rates for investments at 1 and 2 years are the following:

$$r_1 = 3\% \qquad r_2 = 4\%$$

a) Calculate the prices of the 1 and 2 year basic bonds and the forward interest rate f_2.

b) Suppose that we observe that a coupon bond is trading in the market at € 103.45. The nominal value of the bond is €100, the coupon rate is 5% and matures in three years. Is the price of this bond consistent with the absence of arbitrage?

c) Suppose now that there is a 6% coupon bond with 10 years to maturity and a nominal value of €1000. The yield to maturity is 4%.
   1) Compute the *Duration*.
   2) If the interest rate goes up from 4 to 4.1% what is the change in the bond price?

d) What are the limitations of using *DURATION* as a measure of interest rate sensitivity?

Question 2

Calculate the fair price of a 5% coupon bond maturing in 10 years if the yield is 5%. What is the fair price if the yield is 6% instead and its face value is 100? Assume coupons are paid semi-annually.

Question 3

Consider a 4% coupon bond maturing in 8 years. It is currently trading at 90 with a face value of 100. What would you do if the yield is 5%?

Question 4

Consider a 10% coupon selling at par (face value = 100); coupons are paid semi-annually and the yields are 10%. Graph the changes in price for different yields. What do you notice?

Question 5

Assume that there are 2 riskless coupon bonds. Their face values are both equal to €100 and their maturities are one and two years. Their corresponding coupons are 6% and 10%, respectively, and their market prices are €100.95 and €101.605.

a) Compute the prices of the basic bonds 1 and 2 and the spot interest rates for one and two years.

b) What is the yield to maturity (or internal rate of return) of the two-year bond and the forward rate f_2.

c) Above you have calculated the implied forward rate f_2. Assume that the prevailing forward rate in the market was 15%? Is there an arbitrage opportunity and if so, how would you take advantage of it? Suppose that you make an investment of €1,000 for two years.

d) Consider a new bond with maturity of two years, a face value of €1000 and a coupon of 15%. How much should it cost?

e) Design a way to exploit an arbitrage opportunity if the price of the previous bond was €1000. (Specify your positions in all three bonds, calculate the immediate profit from the arbitrage, and show that the strategy does not generate any future payments or liabilities).

SOLUTIONS

Question 1

a) $b_1 = 1/(1+r_1) = 0,9709$ $b_2 = 1/(1+r_2)^2 = 0,9246$

$r_2 = (1+r_2)^2 = (1+f_2) \times (1+r_1)$
$(1+f_2) = 1.05$
$f_2 = 5\%$

b)
No arbitrage bond price

$B^{NA} = 5/(1.03) + 5/(1.04)^2 + 105/(1.0416)^3 = 102.39 < 103.45$, therefore overvalued bond

The market price of the bond is not consistent with no arbitrage.

c)

1)

Calculating DURATION

Time	Payoffs	Present Value	Weights	T × Weights
1	60	57.69231	0.04964	0.04964
2	60	55.47337	0.047731	0.095461
3	60	53.33978	0.045895	0.137684
4	60	51.28825	0.04413	0.176519
5	60	49.31563	0.042432	0.212162
6	60	47.41887	0.0408	0.244802
7	60	45.59507	0.039231	0.274618
8	60	43.84141	0.037722	0.301778
9	60	42.1552	0.036271	0.326442
10	1060	716.098	0.616148	6.161478
Totals	1600	1162.218	1	7.980583

Duration: 7.980583 years

2)
Modified Duration: MD = D/1+ YTM = 7.673

If the interest rate goes up from 4 to 4.1%:
$dB/B = - (DM)\ di = - (7.673) \times (0.001) = 0.007673 = 0.7673\%$

d)
Duration, used as a measure of interest rate sensitivity has limitations. The statistic calculates a linear relationship between price and yield changes in bonds. In reality, the relationship between the changes in price and yield is convex.

Convexity, which is a measure of the curvature of the changes in the price of a bond in relation to changes in interest rates, is used to address this error.

Question 5
Bond 1: N = €100, coupon = 6,0%, T = 1, B_1 = €115
Bond 2: N = €100, coupon = 10%, T = 2, B_2 = €110,55

a) Compute the prices of the basic bonds 1 and 2 and the spot interest rates for one and two years.

	(Price) t = 0	(t = 1)	(t = 2)
Bond 1	100,95	106,00	
Bond 2	110,55	10,00	110,00

Bond 1 = 106,00 u. bb1 $\Rightarrow 100{,}95 = 106{,}00 \cdot b_1 \Rightarrow b_1 = \dfrac{100{,}95}{106{,}00} = 0{,}9524$

$$r_1 = \frac{1}{0{,}9524} - 1 = 0{,}05 = 5\%$$

Bond 2 = 15 u. bb1 and 115 u. bb2

$$\Rightarrow 101{,}605 = 10 \cdot b_1 + 110 \cdot b_2 \Rightarrow b_2 = \frac{101{,}605 - 10 \cdot b_1}{110} = \frac{101{,}605 - 10 \cdot (0{,}9524)}{110} = 0{,}8371$$

$$b_t = \frac{1}{(1+r_t)^t} \Rightarrow r_t = \left(\frac{1}{b_t}\right)^{1/t} - 1$$

$$r_2 = \left(\frac{1}{0{,}8371}\right)^{1/2} - 1 = 0{,}093 = 9{,}3\%$$

b) IRR and forward rate

IRR $\equiv$ annualized effective compounded return rate:

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+i)^t} + \frac{N}{(1+i)^T}$$

Therefore, one should calculate i in order to solve the equation,

$$101{,}605 = \frac{10}{(1+i)} + \frac{110}{(1+i)^2}$$

One can define a "new variable" $X = \dfrac{1}{(1+i)}$. The new equation to be solved is;

$$110X^2 + 10X - 101{,}605 = 0$$

$$\Rightarrow X = \frac{-10 \pm \sqrt{10^2 - (4 \times 110 \times (-101{,}605))}}{2 \times 110} = \frac{-10 \pm 211{,}6748}{220} = 0{,}9167$$

$$X = 0{,}9167 = \frac{1}{1+i} \Rightarrow i = \frac{1}{0{,}9167} - 1 = 9\% \Rightarrow \textbf{IRR = 9,1\%}$$

<u>Forward rate</u>

$$(1+r_t)^t = (1+r_{t-1})^{t-1}(1+f_t) \Rightarrow (1+f_t) = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}}$$

f_2 is given by the following equation:

$$(1+f_2) = \frac{(1+r_2)^2}{(1+r_1)} = \frac{1{,}093^2}{1{,}05} = 1{,}1378 \Rightarrow f_2 = 13{,}78\%$$

c)
Direct strategy (only *spot* market) = Invest €1,000 in bond 2
Payoff at t=2: €1,000 × $(1+r_2)^2$ = €1,000 × $(1{,}093)^2$ = €1,194.65

Indirect Strategy (*spot* + *forward* market) = Invest €1,000 in bond 1 and reinvest that amount at the forward rate.
Payoff at t=2: €1,000× $(1+r_1)$ × $(1+f_2)$ = €1,000 × 1,05 × 1,15 = €1,207.5

Or,

$$(1+f_2) \neq \frac{(1+r_2)^2}{(1+r_1)} \Leftrightarrow 1{,}15 \neq \frac{1{,}093^2}{1{,}05} \Leftrightarrow 1{,}15 \neq 1{,}1378$$

Obviously, there exists arbitrage

Arbitrage strategy:
Borrow € 1000 for two years at 9.3%
Invest € 1000 for one year, and invest the proceeds at the one year forward rate one year hence.

Direct Strategy		
t=0	t=1	t=2
€ 1000		€-1.194,65
Indirect Strategy		
€ 1000	€ 1.050,00	
	€ 1.050,00	€1.207,50

Arbitrage Profit: €1.207,50 - € 1.194,65 = € 12,85

<u>d) New bond price</u>
Bond 3: N = €1000, coupon = 15%, T = 2.
Bond 3 = portfolio 150 bb1 and 1150 bb2

$$Bond\ 3 = 150\ b1 + 1150\ b2 = 150(0.9524) + 1150(0.8371) = € \ 1105{,}53$$

e) Arbitrage Opportunities

If the bond's market price was €1000 arbitrage opportunities will exist since the price of the previous bond is not coincident with the no-arbitrage price, € 1105.53. The arbitrage strategy will consist in *long* bond 3 and *short* a replicate portfolio of bonds 1 and 2.

How will found the replicated portfolio? Solving the following equation system:

$$\begin{cases} 150 = 106 \cdot n_1 + 10 \cdot n_2 \\ 1150 = 0 \cdot n_1 + 110 \cdot n_2 \end{cases}$$

The solution is: $n_1 = 0{,}43; n_2 = 10{,}45$. To replicate Bond 3 we should buy (long position) 0,43 units of bond 1 and 10,45 units of bond 2. We will define this new portfolio as Z.

Therefore, the arbitrage strategy will be:

- Long one unit of Bond 3
- Short (sell) one unit of portfolio Z = (n_1 , n_2)

	t = 0	t = 1	t = 2
Long Position Bond 3	-1000,00	+150	+1150
Short Position Portfolio Z			
- short 0,43 units bond 1	43,4	- 45,5	0
- short 10,45 units bond 2	1155.2	-104,5	-1150
TOTAL	+198,66	0	0