Portfolio Management
2010-2011

1.

Consider the following prices (calculated under the assumption of absence of arbitrage) corresponding to three sets of options on the Dow Jones index. Each point of the index is worth $1.00. All options mature on December 20.

	Exercise price (K)	Call price (c_0)	Put price (p_0)
Option 1	8000	810	220
Option 2	8500	520	420
Option 3	9000	310	710

a) Derive the payment table and the profit diagram of a *bull spread* made with call options 1 and 3. That is, buy 1 unit of call 1 and sell 1 unit of call 3. What expectations should an investor have in order to be interested in this strategy?

b) Derive the payment table and the profit diagram of a *bull spread* made with put options 1 and 3. That is, buy 1 unit of put 1 and sell 1 unit of put 3. What expectations should an investor have in order to be interested in this strategy?

c) Derive the payment table and the profit diagram of a portfolio that consists of two long positions in call options 1 and 3, and four short positions in call option 2. What expectations should an investor have in order to be interested in this strategy?

d) Assume that the Dow Jones index can only take discrete values with a step of 500 points (that is, values like ..., 7000, 7500, 8000, 8500, 9000, 9500, ...) and that the prices in the previous table are still correct. Calculate the price of an asset that pays $3000 if the index is at 8500 on December 20 and $0 otherwise.

2.

Imagine a market where a stock and a riskless bond are traded. The price of the stock is €10 and its annualized volatility is 20%. The continuously compounded risk-free rate is 4%.

a) Using the Black-Scholes model find the price of a European call on the stock with exercise price of €10 and 6 months to expiration.

b) Describe the possible profits and losses from the following strategy: buy one call with exercise price K_1 = €8, sell two calls with exercise price K_2 = €10 and buy one call with exercise price K_3 = €12. All calls are written on the previous stock and have 6 months to expiration. The price of the first call is €2.20, the price of the second call is obtained in section a), and the price of the third call is €0.10. Discuss the role of this type of strategy in pricing an asset under the absence of arbitrage.

3.

a. Critically discuss the mean-variance approach of portfolio theory

b. According to Markowitz portfolio theory, can we find a single risky optimal portfolio which is suitable to the demands of all investors? Explain why.

c. Critically explain in situation (b), how the fund managers' portfolio selection work is simplified when facing with different demands of investors, given their different risk attitudes.

d. A portfolio management organisation analyses 60 stocks and construct a mean-variance efficient portfolio using only these 60 securities.

   i. How many estimates of expected returns, variances, and covariances are needed to optimize this portfolio?

   ii. If one could safely assume that stock market returns closely resemble a single-index structure, how many estimates would be needed?

4.

a. Define and critically discuss the single index model.

b. The below situation describes a three-stock financial market that satisfies the single index model.

Stock	Capitalisation	Beta	Mean excess return	Standard deviation
A	£3,000	1.0	10%	40%
B	£1,940	0.2	2%	30%
C	£1,360	1.7	17%	50%

The single factor in this economy is perfectly correlated with the value-weighted index of the stock market. The standard deviation of the market index portfolio is 25%.

   i. What is the mean excess return of the index portfolio?
   ii. What is the covariance of stock B with the index?
   iii. Break down the variance of stock B into its systematic and firm specific components

5.

a. Critically discuss the major differences between the single-index model over Markowitz portfolio selection procedure.

b. The following are estimates for two of the stocks:

Stock	Expected return	Beta	Firm-specific standard deviation
A	13	0.8	30
B	18	1.2	40

The market index has a standard deviation of 22% and the risk-free rate is 8%.

i. What is the standard deviation of stock A and B?

ii. Suppose that we were to construct a portfolio with proportions:

Stock A:	30%
Stock B:	45%
T-bills:	25%

Compute the expected return, standard deviation, beta and non-systematic standard deviation of the portfolio.

6.

Philip Morris has issued bonds that pay coupons annually with the following characteristics:

Annual coupon rate	Yield to Maturity per annum	Maturity	Macaulay Duration
4%	7%	15 years	10.62 years

a. Calculate modified duration using the information above.

b. Explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates.

c. Using the case of a bank to illustrate how the economic value of assets could mismatch that of liabilities when there is a change of interest rate.

d. Define convexity and explain how modified duration and convexity are used to approximate the bond's percentage change in price, given a change in interest rates.

7.

a. An investor purchased 500 half year put options with a strike price equal to \$54 at a premium of \$2.75. The investor purchased 500 shares of the stock at \$57.8. The short-term risk-free interest rate is 5% annum.

   1) State the name of this strategy

   2) Calculate the maximum loss and profit

   3) Determine the breakeven point

8.

a. Identify the fundamental distinction between a futures contract and an option contract and briefly explain the difference in the manner that futures and options modify portfolio risk.

b. Suppose the value of S&P 500 stock index is currently 1,200. If the one-year T-bill rate is 6.5% and the expected dividend yield on the S&P 500 is 2%, what should the one year maturity futures price be?

c. Consider a stock that pays no dividends on which a futures contract, a call option and a put option trade. The maturity date for all three contracts is T, the exercise price of the put and the call are both X, and the futures price is F. Show that if $X = F$, then the call price equals the put price. Use parity conditions to guide your demonstration.

9.

a. Briefly define the concept of time value of a call option, and explain why the time value of a call option is always greater than zero?

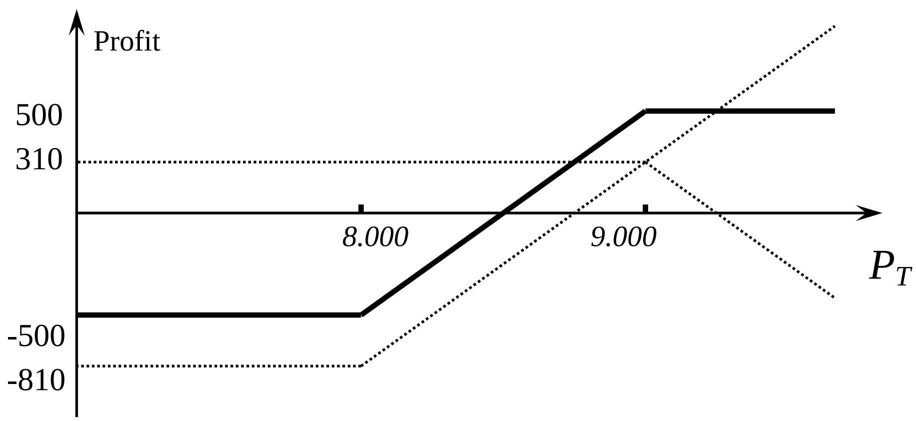
b. Use example to show the strategy using call and put options to replicate a long stock position without actually holding the stock.

c. If you are hold a stock and fear the downside risk of the stock price, what option you can buy to hedge? Draw a payoff diagram showing your hedged position.

SOLUTIONS

1.

a) ***Bull Spread* with *Calls***

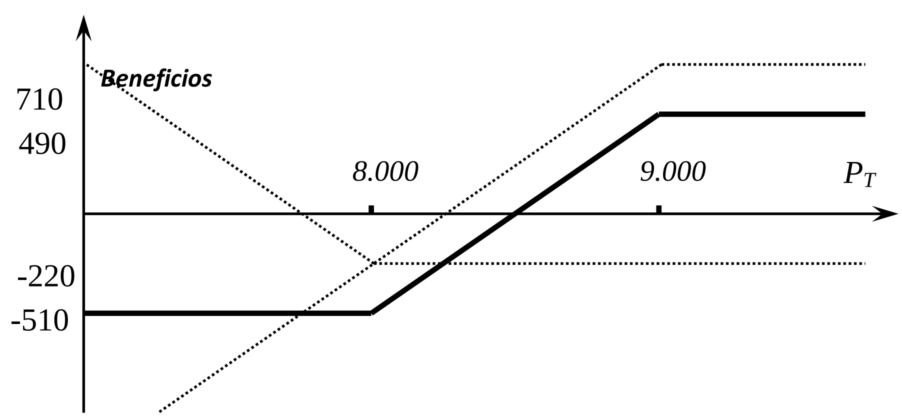


Payment Table

	$t = 0$	$t = T$		
		$P_T \leq 8.000$	$8.000 < P_T \leq 9.000$	$P_T > 9.000$
Long 1 *call* de X = 8.000	-810	0	$(P_T - 8.000)$	$(P_T - 8.000)$
Short 1 *call* de X = 9.000	+310	0	0	$-(P_T - 9.000)$
Premiums + payoffs	-500	0	$P_T - 8.000$	1.000
Profits		- 500	$P_T - 8.500$	500

Investor with expectation of increase in the Dow Jones Index

b) *Bull Spread* with Puts

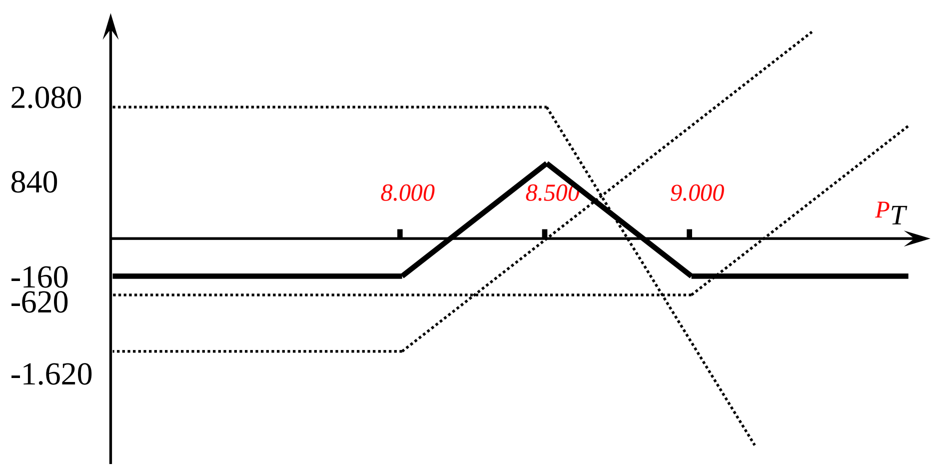


Payment Table

	t = 0	$t = T$		
		$P_T \leq 8.000$	$8.000 < P_T \leq 9.000$	$P_T > 9.000$
Long 1 *put* de X = 8,000	-220	$(8.000 – P_T)$	0	0
Short 1 *put* de X = 9,000	+710	$-(9.000 – P_T)$	$-(9.000 – P_T)$	0
Premiums +Payoffs	+490	-1.000	-(9.000 – PT)	0
Profits		-510	$-(8.510 – P_T)$	490

Investor with expectation of increase in the Dow Jones Index

c)



Payment Table

	$t = 0$	$t = T$			
		$P_T \leq$ 8.000	8.000< $P_T \leq$ 8.500	8.500< $P_T \leq$ 9.000	$P_T >$ 9.000
Long 2 *calls* de X = 8.000	-1.620	0	2(P_T – 8.000)	2(P_T – 8.000)	2(P_T – 8.000)
Long 2 *calls* de X = 9.000	-620	0	0	0	2(S_T – 9.000)
Short 4 *calls* de X = 8.500	+2.080	0	0	-4(P_T – 8.500)	-4(P_T – 8500)
Premiums +Payoffs	-160	0	2 P_T – 16.000	-2 P_T + 18.000	0
Profits		-160	2 P_T – 16.160	-2 P_T + 17.840	-160

Investor with expectations of lower volatility

d)
Since for the interval (8.000, 9.000) the only value that Dow Jones can take is 8.500, the previous strategy offer a profit of $1.000 when Dow Jones takes the value of $8.500 with a cost (premium) of $160. Given the underlying asset which one wants to valuate is similar to a portfolio of the three previous strategies one obtain:

$$\text{Asset Price} = 3 \times \text{portfolio cost} = 3 \times 160 = 480$$

2.

a) European call option

$$c = P_0 * N(d_1) - K * e^{-r*T} * N(d_2)$$

where,

$$d_1 = \frac{\ln(P_0/K) + (r + \sigma^2/2) * T}{\sigma * \sqrt{T}}$$

$$d_2 = d_1 - \sigma * \sqrt{T}$$

$$d_1 = \frac{\ln(10/10) + (0{,}04 + 0{,}2^2/2) * 0{,}5}{0{,}2 * \sqrt{0{,}5}} = 0{,}2121$$

$$d_2 = 0{,}2121 - 0{,}2 * \sqrt{0{,}5} = 0{,}0707$$

Using the cumulative density function

$$N(0{,}2121) \approx 0{,}5832$$
$$N(0{,}0707) \approx 0{,}5279$$

Call price:

$$c = 10 * 0{,}5832 - 10 * e^{-0{,}04*0{,}5} * 0{,}5279 = 0{,}663$$

European Put. Use the put-call parity

$$p = c + Ke^{-rT} - S = 0{,}663 + 10e^{-0{,}04*0{,}5} - 10 = 0{,}465$$

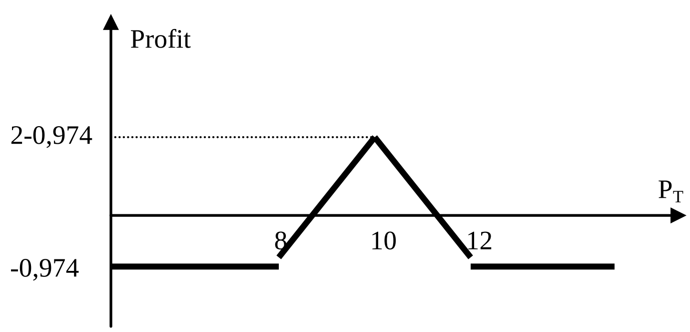
b)

The prices are derived from the Black-Scholes to each option

***Butterfly Spread*:**
Payoff

Is 0 para $P_T<8$, slightly increase to a máximum of 2 in $P_T=10$, and decrease to $10<P_T<12$ and is 0 for $P_T>12$.

Premium
2,2-2*0,663+0,1=0,974.



A *Butterfly Spread* is an aproximation of a replicate portofolio of an Arrow-Debreu Security for the state $P_T \cong K_2$. Therefore, is cost allows us to calcultae the Price of this ADS

3.

a. Mean-variance approach is basic portfolio selection, which assume the mean and variances and covariance between assets in the portfolio are informationally sufficient to identify the asset needed.

The criterion of choosing one asset is mean-variance: for a given mean return, an asset with lower variance is preferred.

But the approach is under some criticisms arguing that the distribution of return is not normal and stress the need for attention paid on shortfall impacts on portfolio selection.

b. We can find a portfolio which suitable for all because different risk attitudes of different investors, that is, tangent portfolio.

c. The work of fund manager can be simplified by choosing a proportion of tangent portfolio and risk-free asset according to different risk attitudes of investors. When investors are less risk averse, they can borrow risk-free asset to invest into more tangent portfolio to achieve higher returns (but face higher risk too). The fund managers' work is simplified since with risk-free asset, they only need to estimate expected returns, variances and particularly covariances of risky assets for once.

d.

i. To optimize this portfolio one would need:

n = 60 estimates of means

n = 60 estimates of variances

$\frac{n^2 - n}{2} = 1{,}770$ estimates of covariances

Therefore, in total: $\frac{n^2 + 3n}{2} = 1{,}890$ estimates

ii. In a single index model: $r_i - r_f = \alpha_i + \beta_i (r_M - r_f) + e_i$

Equivalently, using excess returns: $R_i = \alpha_i + \beta_i R_M + e_i$

The variance of the rate of return on each stock can be decomposed into the components:

(l) The variance due to the common market factor $\beta_i^2 \sigma_M^2$

(2) The variance due to firm specific unanticipated events: $\sigma^2(e_i)$

In this model: $Cov(r_i, r_j) = \beta_i \beta_j \sigma$

The number of parameter estimates is:

n = 60 estimates of the mean $E(r_i)$

n = 60 estimates of the sensitivity coefficient β_i

n = 60 estimates of the firm-specific variance $\sigma^2(e_i)$

1 estimate of the market mean $E(r_M)$

1 estimate of the market variance σ_M^2

Therefore, in total, 182 estimates.
Thus, the single index model reduces the total number of required parameter estimates from 1,890 to 182. In general, the number of parameter estimates is reduced from:

$$\left(\frac{n^2 + 3n}{2}\right) \text{to} \, (3n + 2)$$

4.

a. the students should define the single index model first. Then point out the systematic risk and firm risk. Particularly, the students should discuss the advantages of the model over Markowitz portfolio selection procedure.

b.

i. total market capitalisation is

3000+1940+1360 = 6300

Therefore, the mean excess return of the index portfolio is
3000/6000 * 10 + 1940/6300*2 + 1360/6300 * 17 = 10

ii. the covariance between stock B and the index portfolio equals

$$Cov(R_B, R_M) = \beta_B \sigma_M^2 = 0.2 \times 25^2 = 125$$

iii.
The total variance of B equals

$$\sigma_B^2 = Var(\beta_{B,} R_M + e_B) = \beta_B^2 \sigma_M^2 + \sigma^2(e_B)$$

Systematic risk equals $\beta_B^2 \sigma_M^2 = 0.2^2 \times 25^2 = 25$

Thus the firm-specific variance of B equals

$$\sigma^2(e_B) = \sigma_B^2 - \beta_B^2 \sigma_M^2 = 30^2 - 0.2^2 \times 25^2 = 875$$

5.

a. The students should list and discuss the main different between the two methods, including simplification of portfolio selection, expected return vs. realized return.

b.

i. The standard deviation of each individual stock is given by:

$$\sigma_i = [\beta_i^2 \sigma_M^2 + \sigma^2(e_i)]^{1/2}$$

Since $\square_A = 0.8$, $\square_B = 1.2$, $\square(e_A) = 30\%$, $\square(e_B) = 40\%$, and $\square_M = 22\%$, we get:

$\sigma_A = (0.8^2 \times 22^2 + 30^2)^{1/2} = 34.78\%$

$\sigma_B = (1.2^2 \times 22^2 + 40^2)^{1/2} = 47.93\%$

ii.

The expected rate of return on a portfolio is the weighted average of the expected returns of the individual securities:

$$E(r_P) = w_A E(r_A) + w_B E(r_B) + w_f r_f$$

where w_A, w_B, and w_f are the portfolio weights for Stock A, Stock B, and T-bills, respectively.

Substituting in the formula we get:

$$E(r_P) = (0.30 \times 13) + (0.45 \times 18) + (0.25 \times 8) = 14\%$$

The beta of a portfolio is similarly a weighted average of the betas of the individual securities:

$$\beta_P = w_A\beta_A + w_B\beta_B + w_f\beta_f$$

The beta for T-bills (β_f) is zero. The beta for the portfolio is therefore:

$$\beta_P = (0.30 \times 0.8) + (0.45 \times 1.2) + 0 = 0.78$$

The variance of this portfolio is:

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma^2(e_P)$$

where $\beta_P^2 \sigma_M^2$ is the systematic component and $\sigma^2(e_P)$ is the nonsystematic component. Since the residuals (e_i) are uncorrelated, the non-systematic variance is:

$$\sigma^2(e_P) = w_A^2 \sigma^2(e_A) + w_B^2 \sigma^2(e_B) + w_f^2 \sigma^2(e_f)$$
$$= (0.30^2 \times 30^2) + (0.45^2 \times 40^2) + (0.25^2 \times 0) = 405$$

where $\sigma^2(e_A)$ and $\sigma^2(e_B)$ are the firm-specific (nonsystematic) variances of Stocks A and B, and $\sigma^2(e_f)$, the nonsystematic variance of T-bills, is zero. The residual standard deviation of the portfolio is thus:

$$\sigma(e_P) = (405)^{1/2} = 20.12\%$$

The total variance of the portfolio is then:

$$\sigma_P^2 = (0.78^2 \times 22^2) + 405 = 699.47$$

The standard deviation is 26.45%.

6.

a. Modified duration $= \frac{\text{Macaulay duration}}{1+YTM} = \frac{10.62}{1.07} = 9.83$ years

b. For option-free coupon bonds, modified duration is a better measure of the bond's sensitivity to changes in interest rates. Maturity considers only the final cash flow, while modified duration includes other factors, such as the size and timing of coupon payments, and the level of interest rates (yield to maturity). Modified duration, unlike maturity, indicates the approximate percentage change in the bond price for a given change in yield to maturity.

c. When there is a change of interest, the economic value of the assets will reduce more as its longer duration than that of liabilities. This potentially leads to economic bankruptcy of a bank.

d. Convexity measures the curvature of the bond's price-yield curve. Such curvature means that the duration rule for bond price change (which is based only on the slope of the curve at the original yield) is only an approximation. Adding a term to account for the convexity of the bond increases the accuracy of the approximation.

7.

1) Protective put strategy.

2) The maximum profit is infinite.

The maximum loss = 500 ((54-57.8)-2.75) = $3,275

3) The breakeven point = 54 - 2.75 = $51.25

8.

a. The important distinction between a futures contract and an options contract is that the futures contract is an obligation. When an investor purchases or sells a futures contract, the investor has an obligation to accept or deliver, respectively, the underlying commodity on the expiration date. In contrast, the buyer of an option contract is not obligated to accept or deliver the underlying commodity but instead has the right, or choice, to accept delivery (for call holders) or make delivery (for put holders) of the underlying commodity anytime during the life of the contract.

Futures and options modify a portfolio's risk in different ways. Buying or selling a futures contract affects a portfolio's upside risk and downside risk by a similar magnitude. This is commonly referred to as symmetrical impact. On the other hand, the addition of a call or put option to a portfolio does not affect a portfolio's upside risk and downside risk to a similar magnitude. Unlike futures contracts, the impact of options on the risk profile of a portfolio is asymmetrical.

b. $F_0 = S_0(l + r_f - d) = 1{,}200\ (1 + .065 - .02) = 1254$

c. The put-call parity relation states that

$P = C - S_0 + X / (1 + r_f)^T$

If F = X, then $P = C - S_0 + F / (1 + r_f)^T$

But spot-futures parity tells us that $F = S_0\ (1 + r_f)^T$. Substituting, we find that:

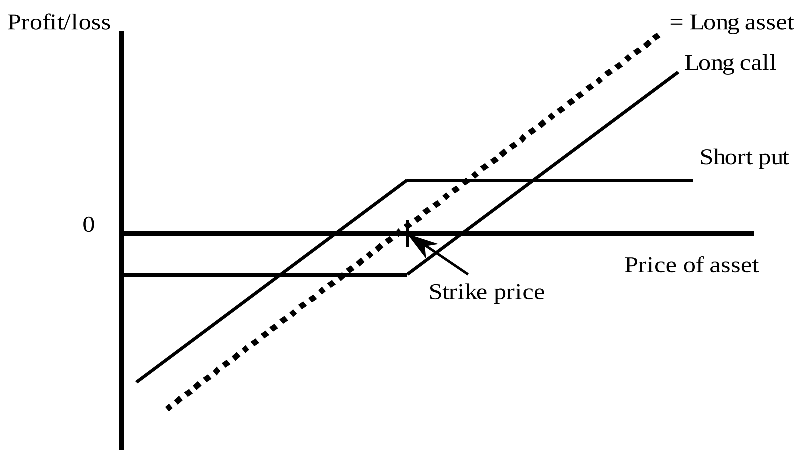
$P = C - S_0 + [S_0\ (1 + r_f)^T]/(1 + r_f)^T = C - S_0 + S_0$, which implies that P = C.

9.

a. The part of the value of an option that is due to its positive time to expiration.

The time value of a call option is always greater than zero in that it gives the buyer the chance to reach higher return in the future, even though in the case of out-of-the money situation.

b. By long call and short put at same strike price, a replicated long stock position can be shown as follow:



c. The long stock position can be hedged by holding a put option. Payoff diagram: