

## Seminar 2 Solutions

### A. Multiple Choice Questions

**A1.** C is correct. We need the proceeds received by the firm net the acceptance fee, which is 978,189.81

**A2.** D is correct. We need the yield to maturity on a one-year bond which is 15.0%.

**A3.** C is correct. Since the bond is selling at the discount we know that its yield maturity must be greater than the coupon rate of 12%. So, the only two possible answers are 15% and 16% of which 15% results in the right hand side of the pricing relationship being equal to the current price.

**A4.** C is correct. The percent price decline for longer maturity bond will be higher than for the shorter maturity bond, all else being the same.

### B. Problems

**B1.** Using the relationship for the price of zero-coupon bond, we get the yield to maturity as 10.0%.

**B2.** If interest is paid semiannually:

$$\text{Value} = \sum_{t=1}^{14} \frac{\$40}{(1.05)^t} + \frac{\$1,000}{(1.05)^{14}}$$

Thus,

$$\$40 (9.899) = \$ 395.96$$

$$\$1,000 (0.505) = \$ 505.00$$

$$\text{Value} = \$ 900.96$$

If interest is paid annually:

$$\text{Value} = \sum_{t=1}^7 \frac{\$80}{(1.10)^t} + \frac{\$1,000}{(1.10)^7}$$

$$\$80 (4.868) + \$1,000 (0.513)$$

$$\text{Value} = \$ 902.44$$

**B3.** 
$$\text{Value} = \sum_{t=1}^{20} \frac{\$80}{(1.07)^t} + \frac{\$1,000}{(1.07)^{20}}$$

Thus,

$$\begin{aligned}
 \text{Present value of interest:} & \quad \$80 (10.594) = \$ 847.52 \\
 \text{Present value of par value:} & \quad \$1,000 (0.258) = \$ 258.00 \\
 & \quad \text{Value} = \$1,105.52
 \end{aligned}$$

If you pay more for the bond, your required rate of return will not be satisfied. In other words, by paying an amount for the bond that exceeds \$1,105.52, the expected rate of return for the bond is less than the required rate of return. If you have the opportunity to pay less for the bond, the expected rate of return exceeds the 7-percent required rate of return.

**B4.** Because the bond has an 8 percent coupon yield and investors require a 10 percent return, we know that the bond must sell a discount. Notice that, because the bond pays interest semi-annually, the coupons amount to  $\$80/2 = \$40$  every six months. The required yield is  $10\%/2=5\%$ . Finally, the bond matures in 10 years, so there are a total of 20 six-month periods. The bond's value is thus equal to the present value of \$40 every six months for the next 20 six-month periods plus the present value of the \$1,000 face amount.

$$\begin{aligned}
 \text{Bond value} &= \$40 \times \left(1 - 1/1.05^{20}\right) / 0.05 + 1,000 / 1.05^{20} \\
 &= \$40 \times 12.462 + 1,000 / 2.6533 \\
 &= \mathbf{\$875.38}
 \end{aligned}$$

**B5.** Because the bond has a 10 percent coupon yield and investors require a 12 percent return, we know that the bond must sell a discount. Notice that, because the bond pays interest semi-annually, the coupons amount to  $\$100/2 = \$50$  every six months. The required yield is  $12\%/2=6\%$ . Finally, the bond matures in 20 years, so there are a total of 40 six-month periods. The bond's value is thus equal to the present value of \$50 every six months for the next 40 six-month periods plus the present value of the \$1,000 face amount.

$$\begin{aligned}
 \text{Bond value} &= \$50 \times \left(1 - 1/1.06^{40}\right) / 0.06 + 1,000 / 1.06^{40} \\
 &= \$50 \times 15.04630 + 1,000 / 10.2857 \\
 &= \mathbf{\$849.54}
 \end{aligned}$$

Notice that we discounted the \$1,000 back 40 periods at 6 percent per period, rather than 20 years at 12 percent. The reason is that the effective annual yield on the bond is  $1.06^2 - 1 = 12.36\%$ , not 12%. We thus could have used 12.36 percent per year for 20 years when we calculated the present value of the \$1,000 face amount, and the answer would have been the same.

**B6.** The present value of the bond's cash flows is its current price \$911.37. The coupon is \$40 every six months for 12 periods. The face value is \$1,000. So the bond's yield is the unknown discount rate in the following:

$$\$911.37 = \$40 \times \left[1 - 1/(1+r)^{12}\right] / r + 1,000 / (1+r)^{12}$$

The bond sells at a discount. Because the coupon rate is 8 percent, the yield must be something in excess of that.

If we were to solve it by trial and error, we might try 12 percent (or 6 percent per six months).

$$\text{Bond value} = \$40 \times \left[ 1 - 1/(1.06)^{12} \right] / .06 + 1,000 / (1.06)^{12} = \$832.32$$

This is less than the actual value, so our discount rate is too high. We now know that the yield is somewhere between 8 and 12 percent. With further trial and error (or a little machine assistance), the yield works out to be 10 percent, or 5 percent every six months.

By convention, the bond's yield to maturity would be quoted as  $2 \times 5\% = 10\%$ . The effective yield is thus  $1.05^2 - 1 = 10.25\%$ .

### B7.

#### **Interest-rate risk or market risk:**

As interest rates rise, the price of a bond falls (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price)

#### **Call Risk:**

**Issuer** can retire or "call" all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate)

**Investor** perspective: i) the CF pattern is not known with certainty, ii) exposed to reinvestment risk (issuers will call the bonds when interests have dropped) and iii) capital appreciation of a bond will be reduced

#### **Liquidity Risk:**

Size and spread between the bid and ask price. The wider the dealer spread, the more the liquidity risk

### B8.

The present value formula for zero-coupon is:

$$B_0 = \frac{F}{(1 + R)^T}$$

Solving for R gives:

$$R = \sqrt[T]{F / B_0} - 1$$

Applying this formula to the above data gives.

Maturity	Yield
1	5.00%
2	6.00%
3	6.50%

**B9.**

**a.**

$$C \left[ \frac{1 - \frac{1}{(1+r)^n}}{r} \right] + \frac{FV}{(1+r)^n} = 4 \left[ \frac{1 - \frac{1}{(1+0.03)^5}}{0.03} \right] + \frac{100}{(1+0.03)^5} = \$104,58$$

**b.**

$$4 \left[ \frac{1 - \frac{1}{(1+0.03)^4}}{0.03} \right] + \frac{100}{(1+0.03)^4} = \$103,72$$

**c.**

$$4 \left[ \frac{1 - \frac{1}{(1+0.035)^4}}{0.035} \right] + \frac{100}{(1+0.035)^4} = \$101,84$$

**d.** As a bond approaches maturity its price approaches the bond's par value.

To compute the value change attributable to the passage of time, compare the value of the bond at time t with the bond value in a prior period.

**e.** The higher the yield at which a bond trades, the lower its price sensitivity for a given basis point change in interest rates will be, all other factors being equal.