Seminar 3
Solutions

A. Multiple Choice Questions

A1. Choice "b" is correct.

$$Duration = \frac{P_- - P_+}{2P_0\Delta r} = \frac{99.75 - 99.30}{2(99.50)(0.001)} = 2.26$$

Choice "a" is incorrect. It would be correct if $\Delta r = .01$ not .001
Choice "c" is incorrect. 4.5 is simply the $\Delta P \times 10$

A2. Choice "c" is correct. Duration is a measure of the sensitivity of a bond's price to changes in the bond's yield.
Choice "a" is incorrect. It measures the effect of changes in the level of interest rates, not yield curve.
Choice "b" is incorrect. Bid-ask spreads have nothing to do with duration.

A3. Choice "b" is correct.
$\%\Delta P = -D_E\Delta r$
4.2% - 4% = .002 = Δr
$\%\Delta P = -2.5\ (.002) = -.005 = -0.5\%$
Remember: rates up, price down

Choice "a" is incorrect. This is the correct answer off by one decimal place.
Choice "c" is incorrect. To have this percentage change, the change in rates would have to be 160 basis points, or an increase from 4.0% to 5.6%.

A4. Choice "c" is correct. When long and short-term rates change by differing amounts, this result in a nonparallel yield curve shift. There is no single basis point change that impacts all maturities in the same way. Therefore, the effective duration for the bond portfolio is not useful.

A5. Choice "a" is correct. It is calculated as:
Formula:
% change in bond price = $-D_E\Delta r = -6.7\ (.90) = -6.03\%$
Determine previous price of bond:
$P_{BEG} \times (1 + \%\Delta P) = P_{END}$
$P_{BEG} \times (1 - 0.0603) = 101.28$
$P_{BEG}\ (0.9397) = 101.28$
$P_{BEG} = 107.78$

A6. Choice "b" is correct. The data table indicates the price distribution for rate changes of 0.2% lower and 0.2% higher than the original rate of 4.2%, where the price is 94.0.

$$DE = \frac{P_- - P_+}{2P_0\Delta r} = \frac{95.0\text{-}93.5}{2(94.0)(0.002)} = 4.0$$

Choice "a" is incorrect. This choice incorrectly uses Δr of 0.004 rather than 0.002.
Choice "c" is incorrect. This choice incorrectly omits the multiplier 2 in the denominator.
Choice "d" is incorrect. This choice incorrectly divides the high and low prices differences by the rate change (1.5/0.002), and then divides the result by 100.

A7. Choice 'c' is correct. All other things being equal, bonds with higher coupons and shorter maturities have shorter durations than bonds with lower coupons and longer maturities. Lower durations exhibit less price risk. Bond "c" has the combination of higher coupon and shorter maturity.

Choice "a" is incorrect. Since bond "c" has a higher coupon, bond "c" will have lower duration.
Choice "b" is incorrect. Since bond "c" has a higher coupon and a shorter maturity than bond "b", bond "c" will have lower duration.
Choice "d" is incorrect. Since bond "c" has shorter maturity than bond "c" will have lower duration.

A8. Choice "c" is correct. It is accurate that the difference in durations is small between two bonds maturing in more than 15 years. This is because the present value of principal is less important for bonds with long maturities.

Choice "a" is incorrect. Duration shortens as yields rise, because at higher discount rates, the present values of further-out cash flows become less significant. Therefore, a greater proportion of the bond's value will be attributed to earlier cash flows, which shortens the duration.
Choice "b" is incorrect. When coupons are higher, they become more significant relative to the value of principal to be received at maturity. This shortens duration.
Choice "d" is incorrect. This is accurate only for zero-coupon bonds.

A9. Choice "b" is correct.

$$\frac{\Delta P}{P_0} = -D\Delta r + C(\Delta r)^2$$

The 2nd term is the percentage change due to convexity.

$$C\times(\Delta r)^2 = 60\times 0.015^2 = 0.0135 = 1.35\%$$

Choice "a" is incorrect. This choice incorrectly uses convexity of 48 (80% of 60) rather than 60.
Choice "c" is incorrect. This choice incorrectly uses Δr of 0.02 rather than 0.015.
Choice "d" is incorrect. This choice incorrectly uses Δr of 0.035 rather than 0.015.

A10. Choice "b" is correct. When a bond contains embedded options, its price risk must be modeled using effective duration because cash flows of the bonds may change when interest rates change.

Choice "a" is incorrect. Modified convexity ignores the effect of any embedded options on a bond's cash flows.
Question "c" is incorrect. Modified duration ignores the effect of any embedded options on a bond's cash flows.
Question "d" is incorrect. Macaulay duration ignores the effect of any embedded options on a bond's cash flows.

A11. Choice "a" is correct. 75 basis points = 0.75%
$\%\Delta P = -D(\Delta r) = (-3.65)\times(-0.0075) = 0.0274 = 2.74\%$

Choice "b" is incorrect. This choice incorrectly states the price to be negative, and uses duration of 3.376 (92.5% of 3.65).
Choice "c" is incorrect. This choice incorrectly assumes duration carries a minus sign.
Choice "d" is incorrect. This choice incorrectly uses duration of 3.376 992.5% of 3.65).

A12. Choice "b" is correct.
The formula for effective duration is:

$$D_E = \frac{P_- - P_+}{2P_0\Delta r} = \frac{1{,}027-975}{2\times 1{,}000\times 0.002} = 13.0$$

Choice "a" is incorrect. This choice incorrectly ignores the 2 in the denominator and uses Δr of 0.02.
Choice "c" is incorrect. This choice incorrectly uses P_+ of $1,000.
Choice "d" is incorrect. This choice incorrectly uses Δr of 0.004.

A13. Choice "b" is correct.

$$\frac{\Delta P}{P_0} = -D_E\Delta r = -4.5\ (0.004) = -0.0180 = -1.80\%$$

The new price of the bond is the original price minus 1.8% of the original price.
102.5-(102.5 × 1.80%) = 102.5 – 1.845 = 100.66

Choice "a" is incorrect. This choice incorrectly computes the price change to be +1.8%, based on $100 as the existing price.
Choice "c" is incorrect. This choice incorrectly uses $100 as the existing price in the computation, and subtracts 1.8% of $100.
Choice "d" is incorrect. This choice incorrectly computes the price change to be + 1.8% rather than – 1.8%.

A14. Choice "c" is correct. Percentage price change formula:
% change in price = $- D_E\Delta r + C_E\Delta r^2$ = -4.68 (-.0065) + 16.35 $(-0.065)^2$ .0304 + 0.007 = **3.11%**

Choice "a" is incorrect. This choice fails to calculate price change using negative sign for duration.
Choice "b" is incorrect. This choice incorrectly uses 16.35 as duration and 4.68 as convexity, rather than the reverse.
Choice "d" is incorrect. This choice incorrectly uses Δr of +0.0065 rather than -0.0065.

A15. Choice "a" is correct. This statement is accurate. Duration and convexity are a decreasing function of the bond's yield.
Choice "b" is incorrect. For option-free bonds, the price-yield relationship is negative; that is, price and yield move in opposite directions.
Choice "c" is incorrect. All option-free bonds exhibit positive convexity.
Choice "d" is incorrect. The upside price movement will be greater than the downside price movement on a percentage basis.

A16. Choice "b" is correct. When the options are out of the money, modified convexity and effective convexity will be the same.
Choice "a" is incorrect. Effective convexity must be used when embedded options are in or near the money.
Choice "c" is incorrect. Modified convexity ignores the effect of embedded options, while effective convexity takes them into account.
Choice "d" is incorrect. Effective convexity must be used when embedded options are in or near the money.

B. Excel Application

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