

## Seminar 6 Solutions

### A. Multiple Choice Questions

**A1. Correct answer “a”.** The current portfolio has an equal amount invested in each of the four securities. The expected return on the current portfolio is the simple average of the individual securities:  $(0.10 + 0.12 + 0.16 + 0.22)/4 = 0.15$  or 15 percent. Replacing a security with a 16 percent return with a security having a 15 percent return will lower the portfolio's expected return. Correlations have no effect on the return calculation.

**A2. Correct answer “c”.** MPT assumes investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.

**A3. Correct answer “b”.** Replacing a security with a 14 percent return with a security having only a 13 percent return will lower the expected return of the portfolio. The expected return on a portfolio is simply a weighted average of the expected returns for each of the individual securities in the portfolio.

### B. Problems

**B1.**

Month	Madison ( $R_i$ )	General Electric ( $R_i$ )	$R_i - E(R_i)$	$R_j - E(R_j)$	$[R_i - E(R_i) \times R_j - E(R_j)]$
1	-0.04	0.07	-0.057	0.06	-0.0034
2	0.06	-0.02	0.043	-0.03	-0.0013
3	-0.07	-0.10	-0.087	-0.11	0.0096
4	0.12	0.15	0.103	0.14	0.0144
5	-0.02	-0.06	-0.037	-0.07	0.0026
6	0.05	0.02	0.033	0.01	-0.003
Sum	<b>-0.10</b>	<b>0.06</b>			<b>0.0222</b>

a.

$$E(R_{\text{Madison}}) = \frac{0.10}{6} = 0.0167 \text{ and } E(R_{\text{General Electric}}) = \frac{0.06}{6} = 0.01$$

b.

$$\sigma_{\text{Madison}} = \sqrt{0.0257/5} = \sqrt{0.0051} = \mathbf{0.0715}$$

$$\sigma_{\text{General Electric}} = \sqrt{0.04120/5} = \sqrt{0.0082} = \mathbf{0.0908}$$

c.

$$COV_{ij} = \left(\frac{1}{5}\right) \times (0.0222) = \mathbf{0.0044}$$

d.

$$r_{ij} = \frac{0.0044}{(0.0715) \times (0.0908)} = \frac{0.0044}{0.006512} = \mathbf{0.676}$$

Once should have expected a positive correlation between the two stocks, since they tend to move in the same direction(s), risk can be reduced by combining assets that have low positive or negative correlations, which is not the case for Madison and General Electric.

**B2.**

$$E(R_1) = 0.15 \quad E(\sigma_1) = 0.10 \quad w_1 = 0.5$$

$$E(R_2) = 0.20 \quad E(\sigma_2) = 0.20 \quad w_2 = 0.5$$

$$E(R_{portfolio}) = 0.5 \times 0.15 + 0.5 \times 0.20 = 0.175$$

$$\text{If } \rho_{1,2} = 0.40$$

$$\sigma_p = \sqrt{(0.5)^2(0.10)^2 + (0.5)^2(0.20)^2 + 2(0.5)(0.5)(0.10)(0.20)(0.40)}$$

$$\sigma_p = \sqrt{0.0025 + 0.10 + 0.004} = \sqrt{0.0165} = 0.12845$$

$$\text{If } \rho_{1,2} = -0.60$$

$$\sigma_p = \sqrt{(0.5)^2(0.10)^2 + (0.5)^2(0.20)^2 + 2(0.5)(0.5)(0.10)(0.20)(-0.60)}$$

$$\sigma_p = \sqrt{0.0025 + 0.10 + (-0.006)} = \sqrt{0.0065} = 0.08062$$

The negative correlation coefficient reduces risk without sacrificing return.

**B3.**

For all values of  $\rho_{1,2}$

$$E(R_{portfolio}) = 0.6 \times 0.10 + 0.4 \times 0.15 = 0.12$$

$$\sigma_p = \sqrt{(0.6)^2(0.03)^2 + (0.4)^2(0.05)^2 + 2(0.6)(0.4)(0.03)(0.05)\rho_{1,2}}$$

$$\sigma_p = \sqrt{0.00324 + 0.0004 + 0.00072(\rho_{1,2})} = \sqrt{0.000724 + 0.00072(\rho_{1,2})}$$

- a.  $\sqrt{0.000724 + 0.00072(1.00)} = \sqrt{0.001444} = \mathbf{0.0380}$   
 b.  $\sqrt{0.000724 + 0.00072(0.75)} = \sqrt{0.001264} = \mathbf{0.0356}$   
 c.  $\sqrt{0.000724 + 0.00072(0.25)} = \sqrt{0.000904} = \mathbf{0.0301}$   
 d.  $\sqrt{0.000724 + 0.00072(0.00)} = \sqrt{0.000724} = \mathbf{0.0269}$   
 e.  $\sqrt{0.000724 + 0.00072(-0.25)} = \sqrt{0.000544} = \mathbf{0.0233}$   
 f.  $\sqrt{0.000724 + 0.00072(-0.75)} = \sqrt{0.000184} = \mathbf{0.0136}$   
 g.  $\sqrt{0.000724 + 0.00072(-1.00)} = \sqrt{0.000004} = \mathbf{0.0020}$

### C. Long Answer Questions

**C1.** In mean-variance analysis, we use the expected returns, variances, and covariances of individual investment returns to analyze the risk-return trade-off of combinations (portfolios) of individual investments.

Key assumptions made in mean-variance analysis include: investors are risk averse; statistical inputs (means, variances, covariances) are known, investors make all portfolio decisions based solely on means, variances, and covariances; and investors face no taxes or transaction costs.

The expected return on a portfolio of two assets is the weighted average of the returns on the individual assets:

$$E(R_p) = W_1 E(R_1) + W_2 E(R_2)$$

Similarly, the expected return for a three-asset portfolio equals:

$$E(R_p) = W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3)$$

The variance of a portfolio of two assets equals:

$$\sigma_p^2 = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2W_1 W_2 \rho_{1,2} \sigma_1 \sigma_2$$

And, the variance of a portfolio of three assets equals:

$$\sigma_p^2 = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + W_3^2 \sigma_3^2 + 2W_1 W_2 \rho_{1,2} \sigma_1 \sigma_2 + 2W_1 W_3 \rho_{1,3} \sigma_1 \sigma_3 + 2W_2 W_3 \rho_{2,3} \sigma_2 \sigma_3$$

The standard deviation equals the square root of the variance

**C2.** The minimum-variance frontier is a graph drawn in risk-return space of the set of portfolios that have the lowest variance at each level of expected return. To derive the minimum-variance frontier, we must: estimate the risk-return attributes of all assets, run an optimizer that selects portfolio weightings that minimize portfolio variance subject to expected return constraints, and calculate and graph the risk and return for the minimum-variance portfolios.

The efficient frontier is the positively sloped portion of the minimum-variance frontier. Portfolios on the efficient frontier have the highest expected return at each given level of risk.

The minimum-variance frontier and efficient frontier are unstable because expected returns, variances, and covariances change over time. This is problematic and may lead to large portfolio weighting errors.

**C3.** Portfolio diversification refers to the strategy of reducing risk combining different types of assets into a portfolio. Diversification benefits increase as the correlation among assets decrease, and as the number of assets included in the portfolio increase (but portfolio risk falls at a decreasing rate as the size of the portfolio increases).