Seminar 7
Solutions

A. Multiple Choice Questions

A1.

a. FALSE

b. FALSE

c. TRUE

d. TRUE

e. TRUE. Securities can have a high correlation with the market portfolio but have a high or low level of systematic risk or Beta (relative to the market portfolio0.

f. FALSE. CAPM do not predict anomalies such as:

- Returns lower on Mondays than on other days
- Returns higher in January compared to other months (especially for small firms)
- Returns higher the day before a holiday
- Returns higher at the beginning and end of the trading day
- Returns higher for firms with "low" price-earnings ratios
- Returns higher for smaller firms compared to larger firms
- Returns higher for firms with higher book-to-market value of equity ratios

g. FALSE. Only efficient portfolios lie along the CML.

A2. B is correct. We need the market risk using the CAPM relationship, which is 10%.

A3. A is correct. We have two SML equations and two unknown

A4. B is correct. Using the relationship for Beta we get $\beta = 0.8$

A5. A is correct. Using the SML, the require return is 13%. So, the current stock price using the constant dividends growth model is $10.00.

A6. B is correct. A plot of the expected return and standard deviation of returns for each portfolio shows that only portfolio 2 does not lie on the efficient frontier.

A7.

i. Correct answer is **"b".** The CML is the line connecting T-bills and Portfolio P. The market price of risk is the slope of the CML, which has the same interpretation as the slope coefficient of the CAL. Had risk been measured on the graph withy beta, the graph would represent the SML. The market price of risk would still be the slope of the line.

ii. Correct answer is **"a".** Since risk in the graph is represented by standard deviation and P is the market portfolio, the line represents the CML. The CML is the CAL with the market portfolio as the tangency portfolio. Had risk been measured by beta, the line would be the SML

A8.

i. Correct answer is **"b".** The equation for the CML is:

$E(R_C) = R_F + \left[\frac{E(R_M)-R_F}{\sigma_M}\right]\sigma_C$ where $R_F = 0.04$

$$\left[\frac{E(R_M)-R_F}{\sigma_M}\right] = \left[\frac{0.12-0.04}{0.20}\right] = 0.40$$

Therefore, the equation of the CML is:
$E(R_C) = 0.04 + 0.40\sigma_C$

Setting the standard deviation equal to 0.10:
$E(R_C) = 0.04 + 0.40(0.10) = 0.08 = 8\%$

ii.
Correct answer is **"a".** The equation for the CML is $E(R_C) = 0.04 + 0.40\sigma_C$. Setting the standard deviation equal to 0.15.

$$E(R_C) = 0.04 + 0.40(0.15) = 0.10 = 10\%.$$

Therefore, John can improve the client's expected return by two percentage points:
New expected return = 10
old expected return = 8%

iii. Correct answer is **"c".** The equation for the CML is $E(R_C) = 0.04 + 0.40\sigma_C$. Setting the expected return to 0.10:

$$E(R_C) = 0.10 = 0.04 + 0.40\sigma_C.$$

Solving for σ_C:

$0.40\sigma_C = 0.10 - 0.04 = 0.06$
$\sigma_C = 0.06/0.40 = 0.15 = 15\%$

iv. Correct answer is **"c".** The standard deviation for the investment combination (of treasury bills and the market portfolio) equals:

$\sigma_C = W_M \sigma_M$

The client wants $\sigma_C = 0.10$
Therefore, $0.10 = W_M(0.20)$
$W_M = 0.10/0.20 = 0.50 = 50\%$

Therefore, the portfolio should be allocated 50% to Treasury bills and 50% to the market portfolio.

v.

Correct answer is **"b".** The equation for the CML is:

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_C$$

Where the intercept is the risk-free rate, $R_F = 0.04$, or 4%, and the slope equals the market risk premium $[E(R_M) - R_F]$ per unit of market risk, σ_M.

$$\left[\frac{E(R_M) - R_F}{\sigma_M}\right] = \left[\frac{0.12 - 0.04}{0.20}\right] = 0.40, or\ 40\%$$

A8.

The market model makes a number of assumptions which lead to the prediction that the covariance between two assets equals the product of the asset betas times the amrket variance. Therefore Ken is correct. One of the assumptions of the market model is that firm-specific events are uncorrelated across firms. If this assumption is violated, then the market model predictions of the covariances will be wrong. George's statement is correct.

A9.

Correct answer is **"c".** The formula for the variance of an equally-weighted portfolio is:

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{Cov} = \overline{\sigma_i^2}\left[\frac{1-\rho}{n} + \rho\right]$$

where $\overline{\sigma_i^2}$ is the average variance of all assets in the portfolio and $\overline{Cov}$ is the average covariance of all pairings of assets in the portfolio. As *n* increases, the first term in the formula approaches zero, and the second term approaches the average covariance. That is:

$\frac{1}{n}\overline{\sigma_i^2}$,gets closer to zero as *n* gets large

$\frac{n-1}{n}\overline{Cov}$,gets closer to the average covariance as n gets large.

Therefore, as *n* increases, the variance of the equally-weighted portfolio approaches the average covariance (5%). There are limits to the benefits of diversification benefits. As more assets are added to the portfolio, the portfolio variance approaches that of the broad market variance. Therefore, as the portfolio size increases, the portfolio risk falls at a decreasing rate.

B. Long Answer Questions

B1.

The variance of an equally-weighted portfolio is:

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{Cov} = \overline{\sigma_i^2}\left[\frac{1-\rho}{n} + \rho\right]$$

If a risk-free investment is part of the investment opportunity set, the efficient frontier is a straight line called the CAL.

The CAL is developed using the investor's unique set of expectations.

The y-intercept of the CAL is the risk-free rate and the slope in the Sharpe ratio for the optimal risky portfolio. The CAL lies tangent to the efficient frontier of risky assets. The CAL equation is:

$$E(R_C) = R_F + \left[\frac{E(R_T) - R_F}{\sigma_T}\right]\sigma_C$$

Where $E(R_C) =$expected return on the portfolio combination of the risk-free asset and the optimal risky Portfolio T (the tangency portfolio), and σ_C is the standard deviation on the portfolio combination.

The CML is the CAL developed assuming all investors have identical expectations. All investors agree on the composition of the optimal risky portfolio, now called the market portfolio, which uses market-value weights. The equation of the CML is:

$$E(R_C) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_C$$

B2.

The CAPM is an equilibrium asset pricing model that assumes that market risk, measured by beta, is the only risk priced by investors. The Security Market Line is the graph of the CAPM:

$$E(R_i) = R_F + \beta_i[E(R_M - R_F)]$$

The intercept of the SML is the risk-free rate, R_F, and the slope is the market risk premium, $E(R_M) - R_F$

B3.

The market price of risk is the Sharpe ratio for the market portfolio (the slope of the CML). It equals the market risk premium per unit of market risk.

Beta is a measure of an asset's systematic risk, and equals the sensitivity to movements in the market portfolio. The formula for the beta equals:

$$\beta_i = \frac{Cov_{I,M}}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i\sigma_M}{\sigma_M^2} = \rho_{i,M}\left[\frac{\sigma_i}{\sigma_M}\right]$$

B4. The instability in the minimum-variance frontier is a concern for a number of reasons:

- The statistical inputs (means, variances, covariances) are unknown, and must be forecast; greater uncertainty in the inputs leads to less reliability in the efficient frontier.
- Statistical input forecast derived from historical sample estimates often change over time, causing the estimated efficient frontier to change over time (this is called time instability).
- Small changes in the statistical inputs can cause large changes in the efficient frontier (called the “overfitting” problem), resulting in unreasonably large short positions, and overly frequent rebalancing.