# PhD course in Empirical Finance

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#### Introduction

This short course is designed to bridge the gap between theoretical financial models and the (real) world of applied finance.

The main objective of the course is to expose students to:

- the statistical/econometric methodologies as well as important economic issues in finance.
- provide the students with knowledge of quantitative methods used in finance research.

By the end of the course students should know:

- how to access various sources of financial data,
- design empirical tests of theoretical issues
- apply basic programming skills to analyse the data and,
- arrive at conclusions.

## Topics will include:

- 1) return predictability, systematic risk factors (including Fama-French factors, momentum factor, liquidity factors and macro factors), performance evaluation of mutual funds,
- 2) capital structure
- 3) payout policy and,
- 4) IPOs.

## Recommended Reading

"The Econometrics of Financial Markets" by John Y. Campbell, Andrew W. Lo and A. Craig Mackinlay, Princeton University Press

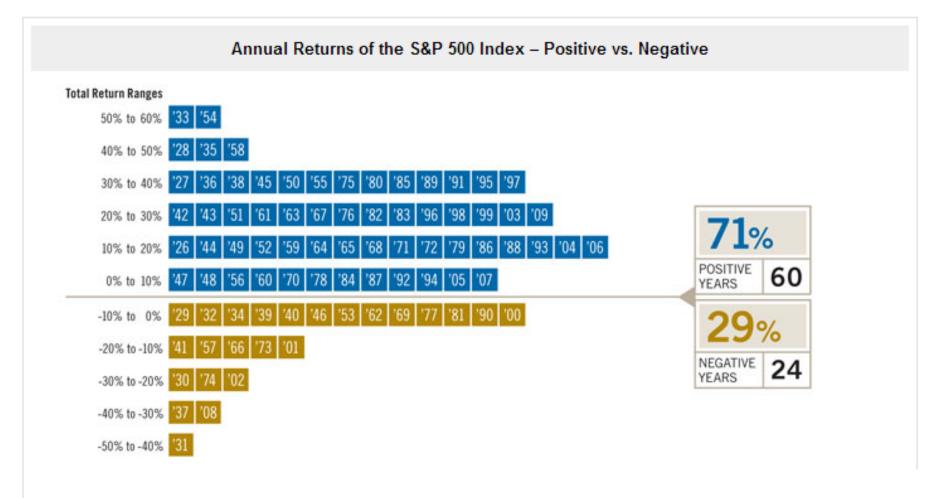
"Asset Pricing" by John H. Cochrane, Princeton University Press

"Quantitative Financial Economics, Stocks Bonds and Foreign Exchange" by Keith Cuthbertson and Dirk Nitzsche, John Wiley & Sons, Ltd

"Time Series Analysis" by James Douglas Hamilton, Princeton University Press

"Econometric Analysis", William H Greene, Pearson Education

### Are Stock Returns Predictable?



# "Real" Returns of the US S&P 500

400/ 500/	100	lie a	1																			
40% to 50%	,33	'54																				
30% to 40%	27	'28	'35	'58	'95																	
20% to 30%	'36	'45	'55	'61	'75	'85	'89	'97	'98	'09												
10% to 20%	'38	'43	'44	'49	'50	'51	'52	'63	'64	'67	72	'76	'80	'83	'86	'91	'96	'99	'03	'10	'12	'13
0% to 10%	'26	'59	'65	'68	71	'82	'88	'92	'93	'04	'05	'06	'07									
-10% to 0%	29	'32	'34	.39	'47	'47	'48	'53	'56	'60	'70	'78	'79	'84	'87	'94	.00	11	J			
-20% to -10%	'40	'57	'62	'66	'69	'77	'81	'90	'01													
-30% to -20%	'30	'41	'46	73	'02																	
-40% to -30%	'31	'37	'74																			
Less than -40%	'08																					



Years

# The Single Index Model

The CAPM is a theory about expected returns

To construct Efficient Frontier: Needed estimates of expected rate of returns, variances and covariances between assets

If you have 25 assets in the investment universe.

How many unique covariances? n(n-1) ÷ 2 = 300 (!)

Estimating many parameters can lead to small errors in each

Can lead to large cumulative error and have a huge impact on the efficient frontier choice

There is a need for a simple model that does not rely on so many estimates of variances/covariances and expected returns

CAPM: behavioural model that provides explanation why securities move together

 Market returns are the source of co-movement between securities in a portfolio

#### Aim of the Single Index Model

Simplifying assumptions regarding the source of association between asset returns

What are the model assumptions?

Which methodology will be use to estimate the parameters of the model using historical data (in particular the parameter used as a proxy for market risk – the beta)?

# **Assumption 1**

Fluctuations in the asset prices are related to the changes in the overall market (which can be proxied by the stock market index).

$$R_{\rm i} = \alpha_{\rm i} + \beta_{\rm i}Rm + \varepsilon_{\rm i}$$
  $R_{\rm i} = \alpha_{\rm i} + \beta_{\rm i}I_{1} + \varepsilon_{\rm i}$  
$$where, \qquad I_{1} = E(R_{\rm m}) + \varepsilon_{\rm m}$$

Where,

 $R_i$  is the return of asset i over a certain period

 $R_m$  is the rate of return on the market index

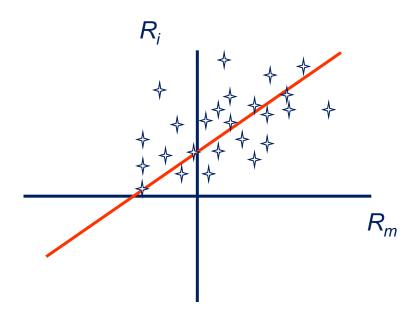
 $\alpha_i$  is the component of security i's return that is independent of the markets performance and,

 $\beta_i$  is a constant beta that measures the sensitivity of asset returns to the market moves.

 $\mathcal{E}_i$  is the random error term representing the deviation of Ri from the return that is predicted by the model

9

# Relationship between market returns and security returns



 $\alpha_i$  and  $\beta_i$  are obtained by regressing historical returns on the assets on the historical returns on the market

Difference between the actual and predicted returns are attributable to the company-specific events and is called the residual

# Assumption 2

Expected (mean) value of residuals is zero

$$E(\varepsilon_{\rm i}) = 0$$

Some residuals are positive and some are negative but their expected value will be equal to zero

## **Assumption 3**

Residuals are uncorrelated with market returns

$$Cov(\varepsilon_i, R_m) = E[(\varepsilon_i - 0)(R_m - E(R_m))] = 0$$

This implies that how well the equation  $R_i = \alpha_i + \beta_i Rm + \varepsilon_i$  describes the return on any security *is independent* of the return of the market.

Extremely large change of the market return will have no impact on the magnitude of the residual

## **Assumption 4**

The residuals of assets are uncorrelated

$$E(\varepsilon_{\rm m}\varepsilon_{\rm i})=0$$
  $E(\varepsilon_{\rm i}\varepsilon_{\rm j})=0$   $\forall {\rm i}=1,...N, {\rm j}=1,...,N, {\rm i}\neq {\rm j}$ 

Only reason why stocks vary together is because of the common comovement with the market (no industry, company, etc, effects).

# The Mean Return of a Security in SIM

From  $R_i = \alpha_i + \beta_i Rm + \varepsilon_i$  the expected return of security is given by:

$$E(R_i) = E(\alpha_i + \beta_i Rm + \varepsilon_i)$$

Which can be rewritten as:

$$E(R_i) = E(\alpha_i) + E(\beta_i Rm) + E(\varepsilon_i)$$

Since  $\alpha_i$  and  $\beta_i$  are constants and known, and by assumption 2,  $E(\varepsilon_i) = 0$ 

Then, the expected return of a security under a single index model is:

$$E(R_i) = \alpha_i + \beta_i E(Rm)$$

#### The Mean Return of a Portfolio in SIM

Return on a portfolio is a weighted average returns of individual securities in that portfolio

$$R_p = \sum_{i=1}^N w_i R_i$$

Combining with equation from Single Index Model

$$R_{p} = \sum w_{i}(\alpha_{i} + \beta_{i}Rm + \varepsilon_{i}) = \sum w_{i}\alpha_{i} + \sum w_{i}\beta_{i}R_{m} + \sum w_{i}\varepsilon_{i}$$

Note that:

$$\sum w_i \alpha_i = \alpha_p \qquad \sum w_i \beta_i = \beta_p \qquad \sum w_i \varepsilon_i = \varepsilon_p$$

Beta of portfolio is the weighted average of betas of individual securities in portfolio.

# The Variance of a Security's return

The variance of a return on any security is:

$$\sigma_i^2 = E(R_i - E(R_i))^2$$

Substituting  $R_i$  and  $E(R_i)$  derived in the previous section in the above expression yields:

$$\sigma_i^2 = E[(\alpha_i + \beta_i Rm + \varepsilon_i) - (\alpha_i + \beta_i E(Rm))]^2 = E[\beta_i (R_m - E(R_m)) + \varepsilon_i]^2$$

By squaring the terms in brackets we obtain:

$$\sigma_i^2 = \beta_i^2 E(R_m - E(R_m))^2 + 2\beta_i E[\varepsilon_i (R_m - E(R_m))] + E(\varepsilon_i)^2$$

From Assumption 3 of the SIM, we know that  $E[\varepsilon_i(R_m - E(R_m))] = 0$  so we can rewrite the above equation as:

$$\sigma_i^2 = \beta_i^2 E(R_m - E(R_m))^2 + E(\varepsilon_i)^2 \quad and$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

Represents the total risk of a security according to the SIM (CAPM). We know that the total risk of a security is decomposed into market (systematic) risk of a security and unique (unsystematic) risk.

Since systematic risk measure is beta:

$$\beta_i^2 \sigma_m^2$$
 Represents the systematic risk

$$\sigma_{ei}^{\ \ 2}$$
 Represents the unique risk

#### The Variance of a portfolio return

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_{ep}^2$$
 The variance of portfolio returns

Also equivalently to the risk of individual security, the

$$\beta_p^2 = \sum w_i^2 \beta_i^2$$
 and systematic risk of a portfolio is  $\beta_p^2 \sigma_m^2$ 

$$\sigma_{ep}^2 = \sum w_i^2 \sigma_{ep}^2$$
 and the unsystematic risk is  $\sigma_{ep}^2$ 

#### The Correlation and covariance in SIM

Estimates of the correlation coefficients of returns (for individual securities) produced by SIM model are different from those produced by a direct estimation using historical data.

The assumption that  $E(\varepsilon_i \varepsilon_i) = 0$  is the only difference between estimation using the SIM model and estimation using historical data (market model).

$$\rho_{ij} = \frac{\text{Cov}(r_i r_j)}{\sigma_i \sigma_j}$$

The correlation coefficient between security *i* and *j* is the  $\rho_{ij} = \frac{\text{Cov}(r_i r_j)}{\sigma_i \sigma_i}$  The correlation coefficient between security *i* and *j* is the covariance divided by the product of the standard deviations of security *i* and security *i*. security *i* and security *j*.

If assumption is not made the correlation is:

$$\rho_{ij} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} + \frac{E(\varepsilon_i \varepsilon_j)}{\sigma_i \sigma_j}$$

After imposing this assumption we have:  $\rho_{ij} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_i}$ 

$$\rho_{ij} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_i}$$

# Single Index Model and Diversification

Diversification into large number of securities will not cause the portfolio beta to increase or decrease significantly (except when deliberately adding extremely high or low beta stocks in a portfolio)

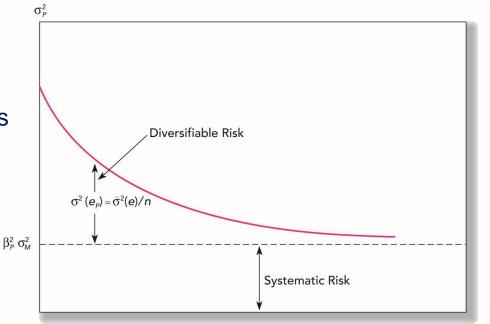
From slide 16 (Variance of portfolio return)

Since  $w_i = \frac{1}{N}$  (well diversified portfolio in N different securities)

$$\sigma_{sp}^2 = \sum w_i^2 \sigma_{si}^2 = \frac{1}{N} \sum \frac{1}{N} \sigma_{si}^2$$

When n gets large,  $\sigma^2(e_p)$  becomes negligible

Risk of portfolio  $\sigma_{\rm p}^2=\beta_{\rm p}^2\sigma_{\rm m}^2$  Standard deviation  $\sigma_{\rm p}=\beta_{\rm p}\sigma_{\rm m}$ 



## Inputs required for the portfolio analysis of N assets

Markowitz: Expected return on a portfolio with N assets and variance is:

$$E(R_p) = \sum_{i=1}^n w_i E(R_i) \qquad \sigma_p^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n w_i w_j \sigma_i \sigma_j \rho_{ij}$$

#### Statistical inputs needed:

- N parameters of expected returns (one for each security)
- N parameters of variance (one for each security)
- N(N-1)/2 parameters of covariance between each pair of risky assets (derived from the variance-covariance matrix)
- In total (N<sup>2</sup>+3N)/2 parameters would need to be estimated

Single Index model: Expected return on a portfolio with N assets and variance is:

$$E(R_p) = \alpha + \beta_p E(R_m) \qquad \sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_{\varepsilon p}^2$$

#### Statistical inputs needed:

- N parameters of vertical intercepts ( $\alpha$ , one for each security)
- N parameters of beta coefficients (one for each security)
- Expected return of the market index
- Variance of the market index
- N parameters of the variance of the random error term
- In total, 3N+2 parameters would need to be estimated.

# **Estimating Betas**

Methodology used to estimate betas from historical data is called least-squares approach

 $R_{\rm i} = \alpha_{\rm i} + \beta_{\rm i} Rm + \varepsilon_{\rm i}$ 

R-squared: Coefficient of determination (measures the proportion of movements in the dependent variable, security returns that is explaining by the independent variable, market returns)

Larger R-squared, the stronger relationship of a portfolio and the market (larger the level of diversification of a portfolio)

Can also be calculated as:  $R^2 = \rho_{im}^2$ 

$$R^{2} = \frac{\beta_{im}^{2} \sigma_{m}^{2}}{\beta_{im}^{2} \sigma_{m}^{2} + \sigma_{ei}^{2}} = \frac{Systematic Risk}{Total Risk}$$

# **Adjusted Betas**

Beta coefficients estimated using historical data and SIM have tendency to move towards 1 over time, i.e. they do not move in a random manner.

If betas were following a random walk over time: then the best predictor of beta in time period t+1 would be the beta estimated in time period t, i.e. historical beta would be the best estimator of future beta.

Betas are mean reverting and move towards 1 over time.

- 1) on the average, the betas of stocks approach the mean market beta (which is one, i.e. which means that the mean-reverting value of a stock's beta is one) and,
- 2) as many companies become more diversified over time, their betas approach unity as more and more of the company-specific risk is eliminated.

$$\beta_{A} = \frac{2}{3} \times \beta_{E} + \frac{1}{3} \times 1$$

$$\beta_{E} = Estimated Beta$$

$$\beta_{E} = Estimated Beta$$

alpha ^	beta	/	cal significa	
S&P/TSX Composite	2.0348	.2547	7.9888	.0000
Intercept	7 .0086	.0099	.8719	.3868
	Coefficients	Standard Error	t-Stat	<i>p</i> -Valu
Total	59	.7162		
Residual	58	.3410	.0059	
Regression	1	.3752	.3752	
	df	SS	MS	
ANOVA	7			
Observations	60			
Standard error	.0767			
Adjusted R-squared	.5157			
R-squared	.5239			
Multiple R	.7238			

Fama-French (1993) Three factor Model

Carhart (1997) Four factor Model

Measuring Performance of Market-Timing Funds

Persistence of Performance

## Fama-French (1993) Three factor Model Alpha

Two groups of stocks consistently tended to outperform the market as a whole:

Small cap stocks and stocks with a high book-value-to price (value vs. growth stocks)

Two factors are added to the CAPM reflecting a portfolio's exposure to these two asset classes:

$$R_{pt} - r_{ft} = \alpha_p + \beta_{p,m}(R_{mt} - r_{ft}) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \varepsilon_{pt}$$

One-factor CAPM: Alpha is the amount by which an active portfolio manager outperforms a broad market index

FF3 factor model defines alpha (for equities) as the return an active manager achieves above the expected return due to all three equity risk

factors.

Fama/French Benchmark Factors	September 2013	Last 3 months	Last 12 months
Rm-Rf	3.76	6.67	22.62
SMB	2.85	5.14	9.60
HML	-1.51	-3.51	7.33
Small Value	5.08	9.53	35.08
Small Neutral	6.71	10.12	32.10
Small Growth	6.42	12.86	33.50
Big Value	2.51	4.42	32.19
Big Neutral	2.95	4.58	20.57
Big Growth	4.20	8.10	19.12

## Carhart (1997) Four factor Model Alpha

Momentum factor added (performance persistence)

$$R_{pt} - r_{ft} = \alpha_p + \beta_{p,m}(R_{mt} - r_{ft}) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{WML_t} WML_t + \varepsilon_{pt}$$

Equally weighted average of top 30% of firms with highest returns in previous 11 months minus equally weighted average of the 30% firms with the lowest returns in previous 11 months

Carhart alpha represents excess returns after market risk, small cap, value and momentum associated performance is taken into account.

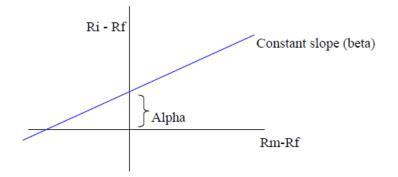
# Measuring Performance of Market-Timing Funds

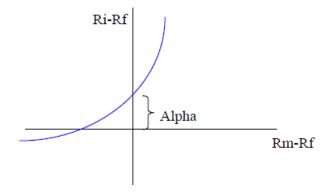
If manager does not engage in market timing, then;

- Portfolio Beta should be constant; and
- Earn excess return (alpha) if there is a stock picking skill.

If manager is engage in market timing, then;

Beta would increase as the market return increases (U-shaped quadratic relation between excess return of the market and excess return of the fund).





$$(R_{it} - R_{ft}) = \alpha_i + b_i (R_{Mt} - R_{ft}) + c_i (R_{Mt} - R_{ft})^2 + \varepsilon_i$$

Positive c<sub>i</sub> = superior market timing ability

No market timing ability, linear relationship between market returns and portfolio returns, c<sub>i</sub> statistically insignificant.

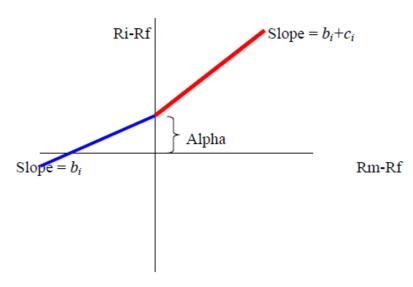
$$(R_{it} - R_{ft}) = a_i + b_i (R_{Mt} - R_{ft}) + c_i D(R_{Mt} - R_{ft}) + \varepsilon_i$$

D (dummy variable): 1 in an up market  $b_i$  is the down market beta,

 $b_i + c_i$  is the up-market beta and

c<sub>i</sub> is their difference or an indicator of the market timing ability.

If  $c_i$  is not significantly different from zero, then the up- and down- market betas are the same and we can conclude that no market timing is exhibited.



#### Fund Performance: Luck or Skill?

(Nitzsche and O'Sullivan, 2008)

- Evaluates performance of individual funds
- 935 open-ended UK equity mutual funds
- Period: April 1975 to December 2002 (surviving and non-surviving funds)
- Use of Carhart (1997) model, conditional alpha and beta model and market timing model.
- Use bootstrapping methodology (see notes)

### Main finding

Evidence of skilful picking ability only for a relatively small number of "top ranked" UK equity mutual funds

#### Persistence of Performance

Try to establish if last year's winners are repeating.

	Rank		
Fund Name	1970 - 1980	1980 - 1990	
Twentieth Century Growth	1	176	
Templeton Growth	2	126	
Quasar Associates	3	186	
44 Wall Street	4	309	
Pioneer II	5	136	
Twentieth Century Select	6	20	
Security Ultra	7	296	
Mutual Shares Corp.	8	35	
Charter Fund	9	119	
Magellan Fund	10	1	
Over-the-Counter Securities	11	242	
American Capital Growth	12	239	
American Capital Venture	13	161	
Putnam Voyager	14	78	
Janus Fund	15	21	
Weingarten Equity	16	36	
Hartwell Leverage Fund	17	259	
Pace Fund	18	60	
Acorn Fund	19	172	
Stein Roe Special Fund	20	57	
Average annual return:			
Top 20 funds	19.00%	11.10%	
All funds	10.40%	11.70%	

1970: 355 equity mutual funds holding broadly diversified portfolios.

More than a half did not survived until 2001

Of the remaining 158, only five produced returns 2% or more in excess of the index fund returns.

Additional studies: see notes

Malkiel (2003)

32

## How to measure persistence in performance?

Contingency tables and Regression

### Contingency tables based persistence

Sort funds into one of four portfolios based on performance in year t and t+1 (WW, WL, LW and LL).

Market adjusted return is looked at and defined as "annual excess return of fund – annual excess return on market index"

- Winner defined as a positive market adjusted return and loser as negative
- Persistence: if there is persistence one would expect to observe more WW and LL

Test for significant persistence

Brown and Goetzmann (1995) log-odds ratio

Log-odds ratio = In[(WW\*LL) / (WL\*LW)] and

Standard error = sqrt [(1/WW) + (1/WL) + (1/LW) + (1/LL)]

Test is standard normally distributed

Null hypothesis is: no persistence in performance

#### Fletcher and Forbes (2002) results based on annual excess returns

Repeat winner tests: annual returns

	WW	WL	LW	LL	Log-odds	Z
82-83	54	63	63	53	-0.33	-1.24
83 - 84	71	46	46	71	0.86	3.24*
84 - 85	80	41	41	79	1.32	4.87*
85 - 86	80	56	56	79	0.71	2.84*
86 - 87	105	46	46	105	1.65	6.60*
87 - 88	93	74	74	93	0.46	2.07*
88 - 89	83	96	96	82	-0.3	-1.43
89 - 90	119	74	74	119	0.95	4.53 *
90 - 91	102	97	97	102	0.1	0.5
91 - 92	98	102	102	98	-0.08	-0.39
92 - 93	95	101	101	94	-0.13	-0.66
93 - 94	119	76	76	119	0.89	4.32*
94 - 95	94	102	102	93	-0.17	-0.86
95-96	134	70	70	134	1.29	6.23 *
All	1327	1044	1044	1321	0.47	8.11 *

<sup>\*</sup> Significant at 5%

Significant persistence in the relative performance rankings using excess returns for both winner and loser portfolios

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#### Fletcher and Forbes (2002) results based on market adjusted returns

	WW	WL	LW	LL	Log-odds	z
82-83	29	35	100	69	- 0.55	- 1.89
83-84	28	105	12	89	0.68	1.82
84-85	31	14	82	114	1.12	3.18 *
85-86	92	41	65	73	0.92	3.64 *
86-87	162	14	85	41	1.72	5.09 *
87-88	52	226	9	47	0.18	0.46
88-89	2	67	11	277	-0.28	-0.36
89-90	7	10	79	290	0.94	1.85
90-91	14	79	51	254	-0.12	-0.38
91-92	18	48	82	252	0.14	0.46
92-93	54	47	152	138	0.04	0.18
93-94	104	105	49	132	0.98	4.52 *
94-95	35	119	37	200	0.46	1.76
95-96	42	34	87	245	1.24	4.75 *
All	670	944	901	2221	1.749	27.27 *

<sup>\*</sup> Significant at 5%

Significant persistence in the performance of the trusts relative to the benchmark index.

Persistence is driven primarily by repeat losers (underperformance). The number of repeat losers is over three times higher (2221) than the number of repeat winners (670).

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## Regression Based Persistence

$$Performance_{pt} = \alpha + \beta Performance_{pt-1} + \varepsilon$$

Where 'performance' can be cumulative total returns, cumulative style adjusted returns or information ratios.

If coefficient **b** is positive, it is considered that period **t**-1 performance contains information for predicting period **t** performance and hence, the evidence of persistence exists

US evidence, Kahn and Rudd (1995), the persistence of performance was not found among 300 equity funds in the early1990s.

This implies that investors, unless they have another basis for choosing winners, should not base their investment decision on the past performance of funds and should invest in equity index funds.