

## Corporate Finance

Dr Cesario MATEUS

[www.cesariomateus.com](http://www.cesariomateus.com)

---

# Session 1 – 06.02.2015

## Module

Introduction to Corporate Finance

The Objective Function in Corporate Finance

Present Value and Related Metrics

Risk and Return – Theory and Practice

Capital Budgeting

Working Capital

Capital Structure

Market Efficiency: Lessons for Corporate Finance

Dividend Policy

Valuation: Basics and Case Studies

Acquisitions and Takeovers

International Finance

Applications of Option Pricing in Corporate Finance

## Assessment

Assignment (4 students): 30%

Examination: 70%

## References

Brealey R., Myers S. and Allen F (2013), Principles of Corporate Finance, McGraw-Hill

# Using CFO Surveys as a Motivational Tool to Teach Corporate Finance

One of the most common questions business school professors hear from students is “Will I use this on the job?”

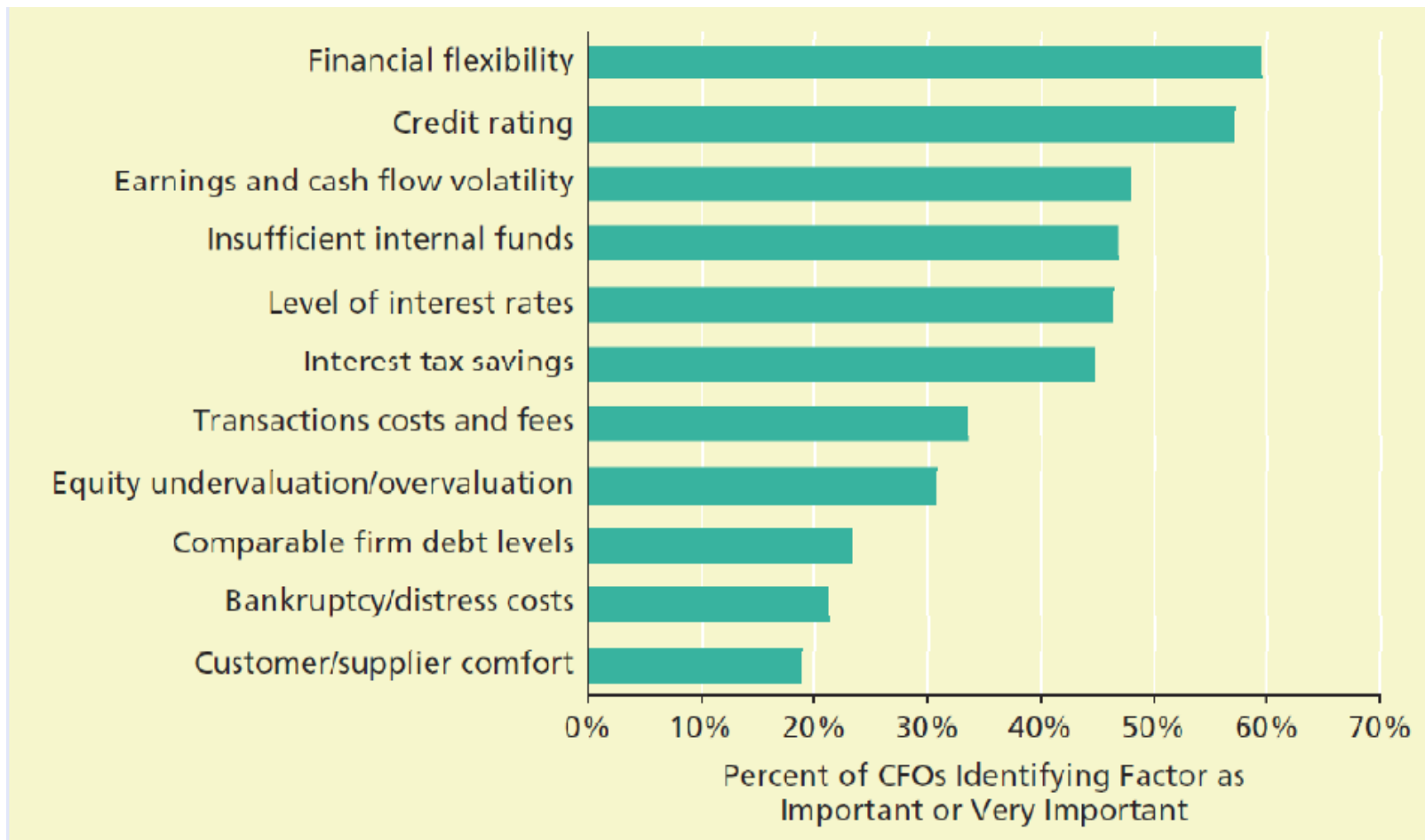
Overlap between the fundamental skills and concepts that a finance practitioner uses on the job and the concepts taught in finance classrooms.

# Which Finance Functions add the most value



Servaes and Tufano  
“CFO views on the  
importance and  
Execution of the  
Finance Function”  
(Deutsche Bank,  
2006)

# What factors Do U.S. Companies Consider When Choosing Debt Policy?



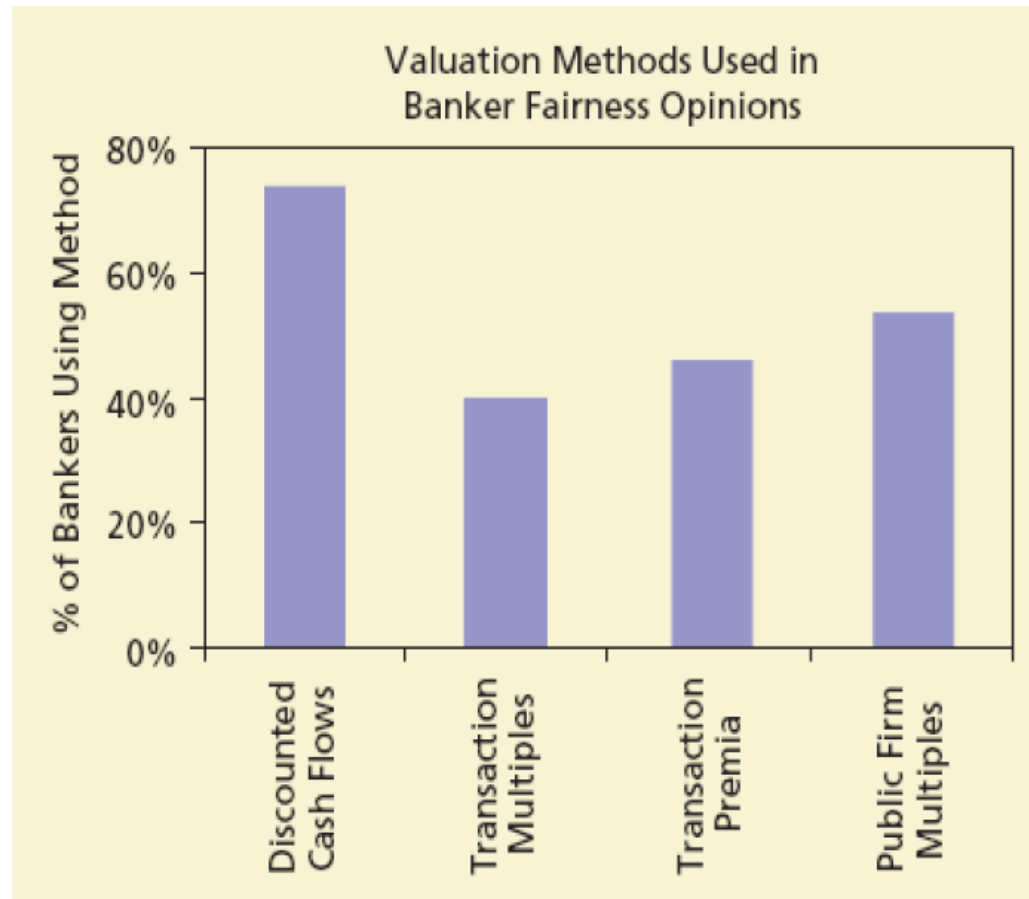
## Popularity of Capital Budgeting Techniques

| TECHNIQUE                 | PERCENT OF CFOs ROUTINELY USING TECHNIQUE <sup>a</sup> |
|---------------------------|--|
| Internal rate of return   | 76%  |
| Net present value         | 75%  |
| Payback                   | 57%  |
| Discounted payback        | 29%  |
| Accounting rate of return | 20%  |
| Profitability index       | 12%  |

<sup>a</sup>Note that these rounded percentages are drawn from the responses of a large number of CFOs and that many respondents use more than one technique.

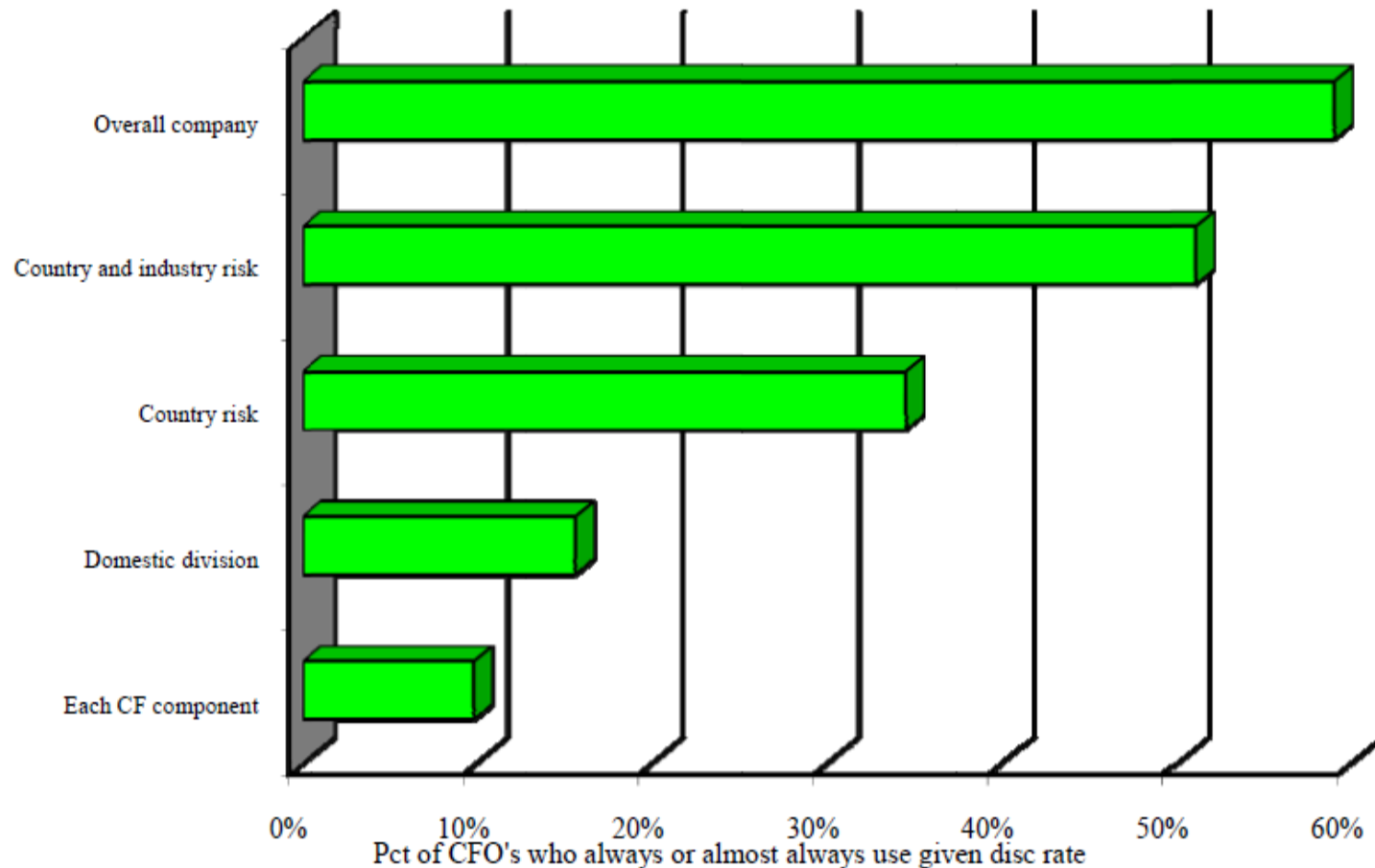
Source: Reprinted from *Journal of Financial Economics*, 60, J.R. Graham and C.R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," pp. 187–243, Copyright 2001, with permission from Elsevier.

## How Investment Bankers Value Companies

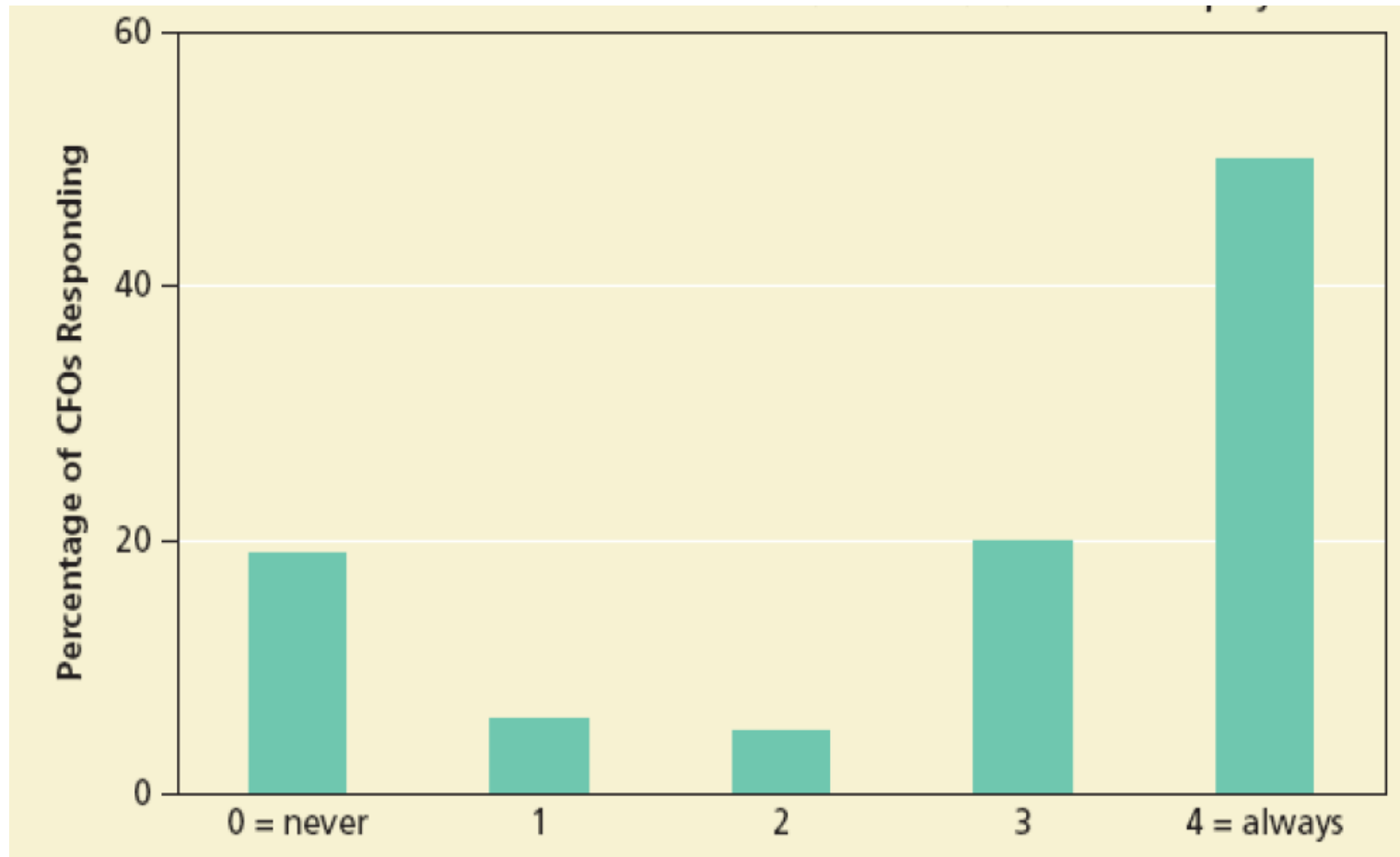




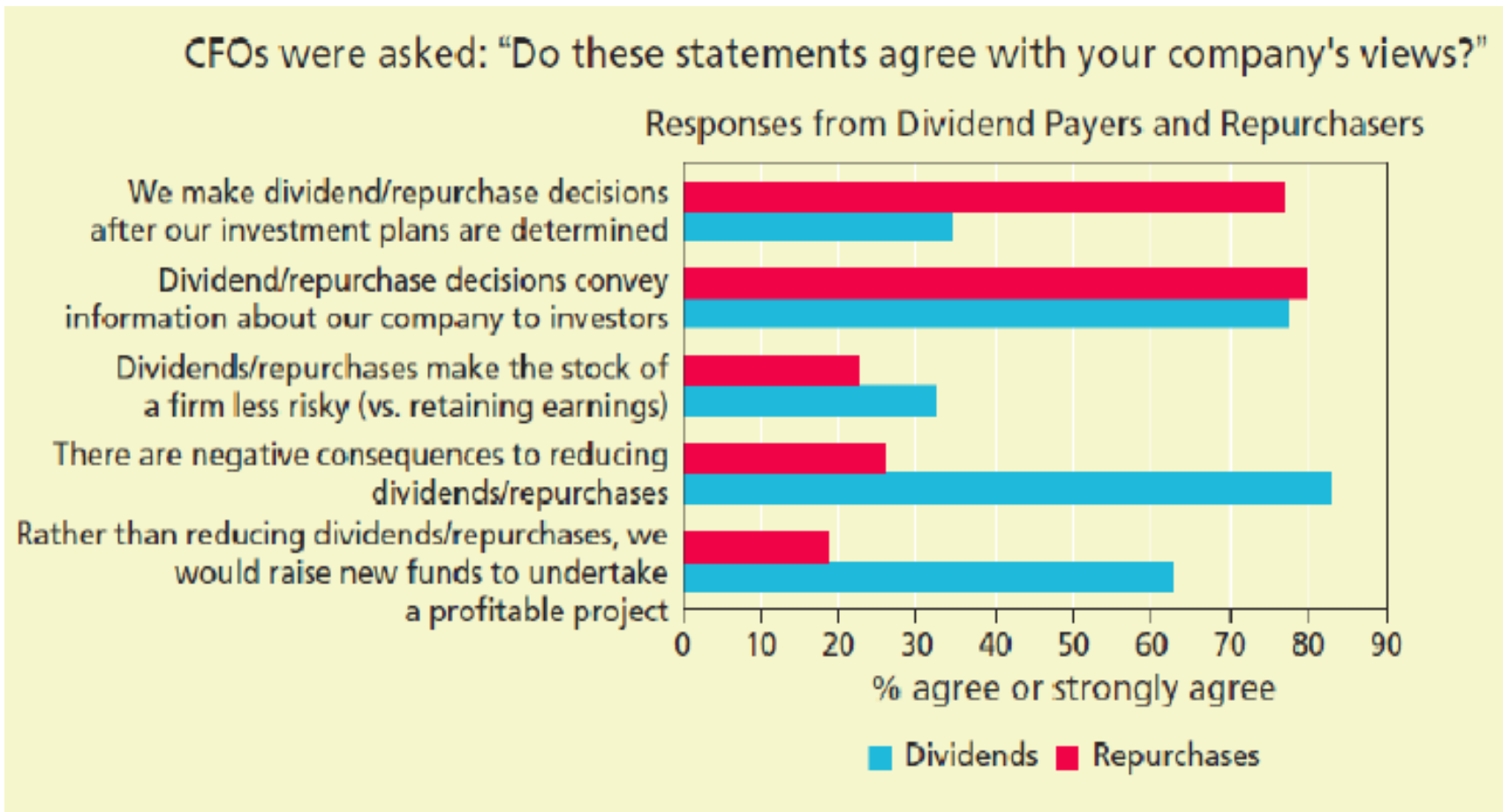
## Which discount rate when evaluating a new project



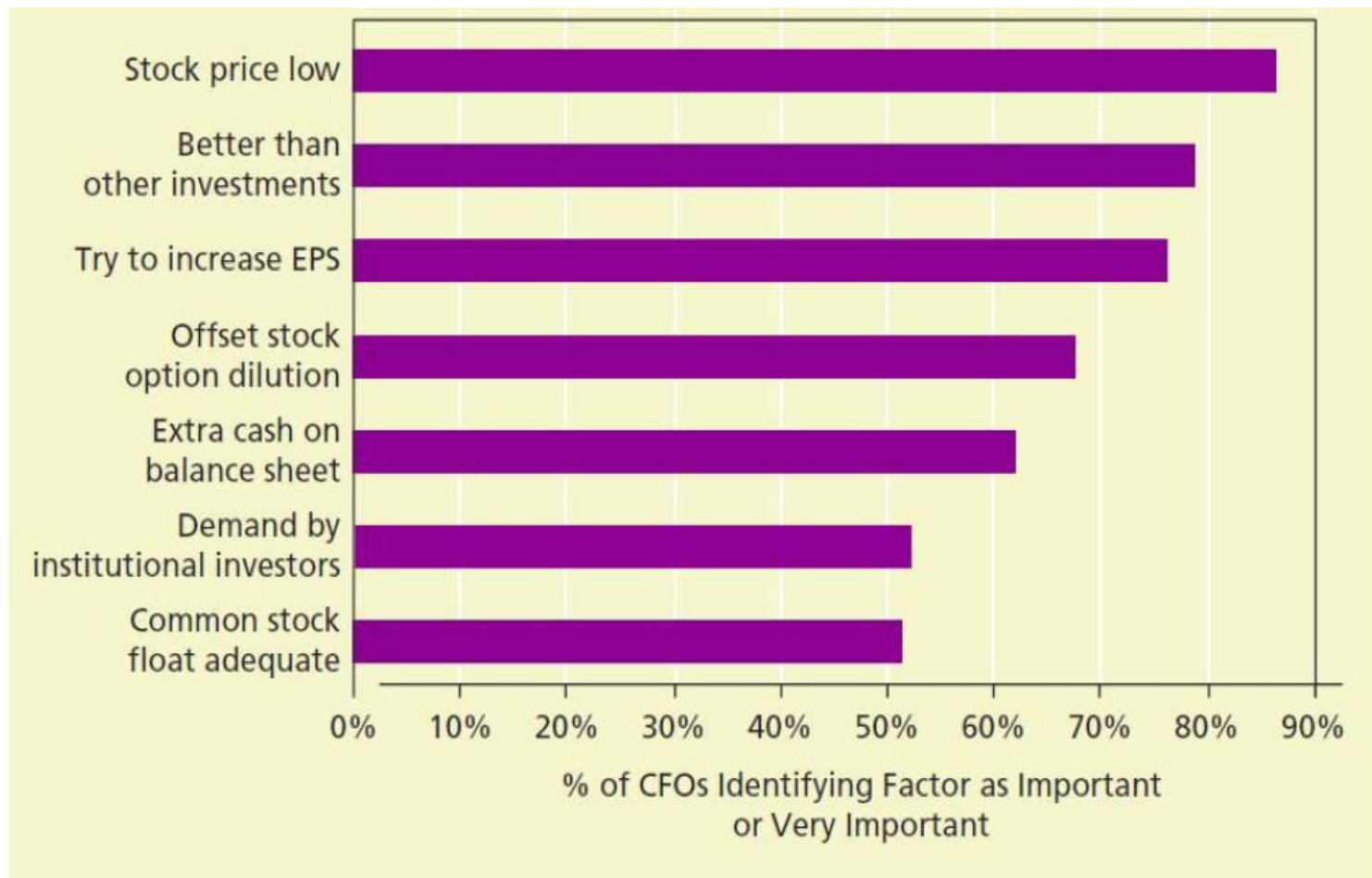
How often to you use the CAPM to calculate the cost of equity?



## CFOs' views on Dividends and Repurchases



## Important Factors in the decision to repurchase Shares



## Motivation to teach Corporate Finance

1. Our firm is thinking of issuing a bond and using the proceeds to buy back shares. What issues should we consider in evaluating this move?
2. What impact would you expect on a firm's earnings if it increases its debt-to-equity ratio?
3. Our cost of debt is 8% and cost of equity 15%. Should we borrow more debt because it is cheaper?
4. If you had an hour and a computer, what would you do to value a company?
5. How does depreciation factor into NPV calculations? Is it a noncash expense, so does it matter?
6. How would you explain NPV calculations to a nonfinancial manager?
7. Would you recommend that our company establish a payout policy by initiating a dividend or buy starting a share repurchase program?

## Motivation to teach Corporate Finance

8. You own a low volatility stock. If you buy shares of a high-volatility stock, will your portfolio volatility go up? Upon what does your answer depend?

9. Under what circumstances might you advise a client to add a stock to his or her portfolio even though you expect the stock to earn a low return?

# What is Finance?

**Finance** is the study of how individuals, businesses and institutions acquire, spend and manage financial resources

- **Major areas of Finance**

- Investment analysis and management
- Corporate Finance
- Capital markets and Financial Institutions
- International Finance
- Personal finance
- Real Estate Finance

# Overview of Business Finance

The study of Finance is related to the corporate objective of **maximizing shareholder wealth**

- Our focus is on financial decision making...
  - **Individuals/Investors**
    - Financial Security Valuation: Earnings and Dividend models
    - Portfolios and Risk Diversification: Portfolio Analysis
    - Determination of Security Prices and Rates of Return: Capital Asset Pricing Model and Arbitrage Pricing Model
    - Using Financial Derivatives: Futures, Forwards and Options
  - **Financial Managers**
    - Investment Decisions: Capital Budgeting Analysis
    - Financing Decisions: Capital Structure and Dividend Policies
  - ....**and the interaction among these decisions**



---

**Investment Analysis** is mainly concerned with **where** and **how** to invest

- Valuation of stocks, bonds and derivatives
- Portfolio Diversification
- Asset Pricing and Market efficiency
- These topics are covered in the first half of this course

**Corporate Finance** is mainly concerned with the **decisions** of **managers**

- Capital Budgeting – **What** investments to make
- Capital Structure – **How** to finance these investments
- Dividend Policy – **What** to payout to Shareholders

# Why Study Finance

- To make informed economic decisions
- To better manage existing financial resources and accumulate wealth over time
- To be successful in the business world you need to have an understanding of finance.

# Corporate Finance

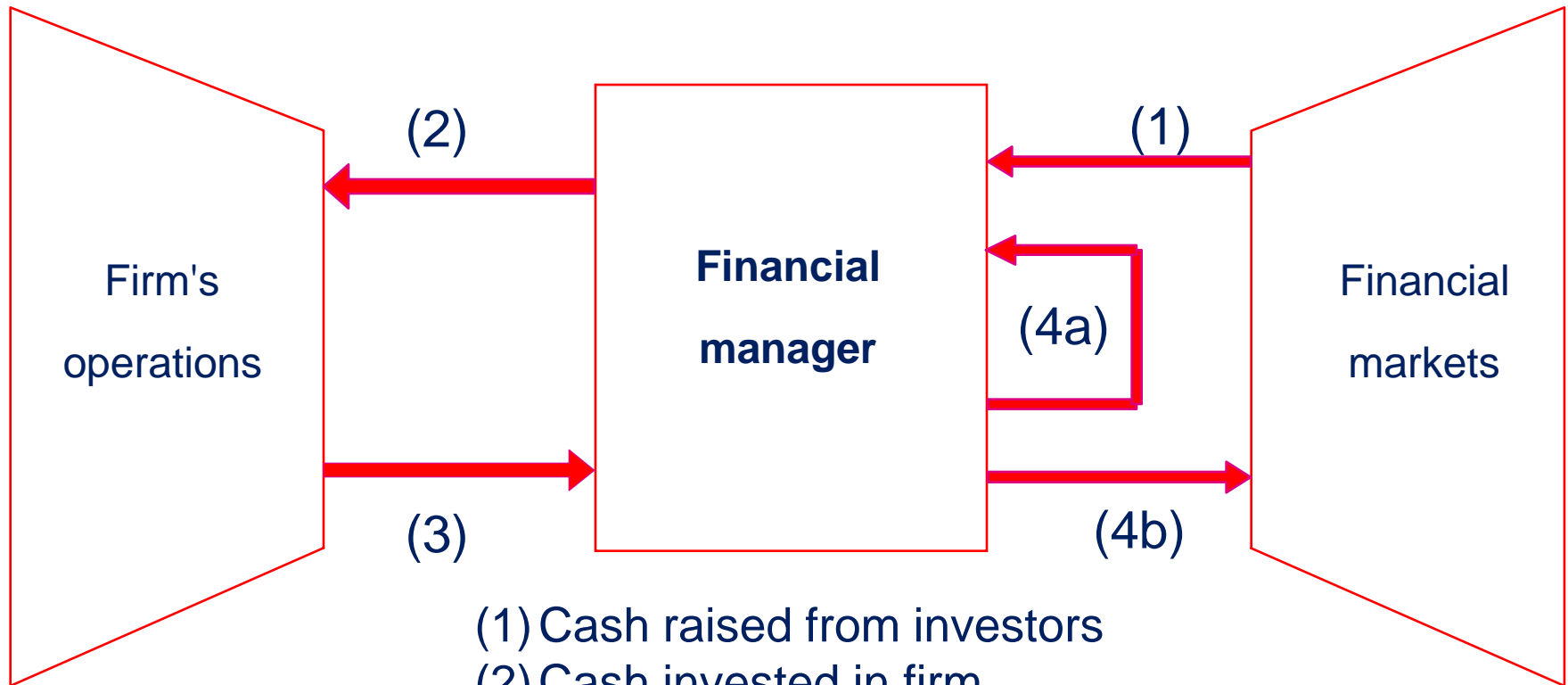
Study of the financing decisions made by firms.

The activities involved in managing money in a business environment

# Corporate Finance Functions

- Capital-Raising (Financing)
- Capital Budgeting
- Financial Management
- Risk Management
- Corporate Governance

# Role of The Financial Manager



- (1) Cash raised from investors
- (2) Cash invested in firm
- (3) Cash generated by operations
- (4a) Cash reinvested
- (4b) Cash returned to investors

# The Dimensions of the Capital Raising Function

- Primary vs. Secondary Market transactions or Offerings
- Funding via Capital Market vs via Financial Intermediary
- Money vs. Capital Markets
- Public vs. Private Capital Markets
- Going Public

# Raising Capital: Key Facts

- Most Financing from **Internal** Rather Than External Sources
- Most External Financing **is Debt**
- Banks Declining **as a Source** of Capital for Large Firms
- **Securities Markets** Growing in Importance

# Capital Budgeting

- The process firms use to **choose** the set of investments that generate the most wealth for shareholders.
- **Selecting the best projects** in which to invest the resources of the firm, based on each project's perceived risk and expected return.



# The Financial Management Function

Managing firms' internal cash flows and their **mix of debt and equity financing**, both to **maximize** the value of the debt and equity claims on firms and to ensure that companies can pay off their obligations when they come due.

Managing Daily **Cash Inflows and Outflows**

**Forecasting** Cash Balances

Building a **Long-Term Financial Plan**

Choosing the **Right Mix** of Debt and Equity

# The Risk Management Function

Managing firms' exposures to all types of risk, both insurable and uninsurable, in order to maintain optimum risk-return trade-offs and thereby maximize shareholder value

Managing the Firm's Exposure to Significant Risks

Interest Rate Risk

Exchange Rate Risk

Commodity Price Risk

# Corporate Governance Function

Developing ownership and corporate governance structures for companies that ensure that managers behave ethically and make decisions that benefit shareholders.

# Corporate goals and wealth maximisation

Maximization of shareholders' wealth is the dominant goal of management in the Anglo-American world.

In the rest of the world, this perspective still holds true (although to a lesser extent in some countries).

In Anglo-American markets, this goal is realistic; in many other countries it is not.

# Shareholder Wealth Maximization

In a Shareholder Wealth Maximization model (SWM), a firm should strive to *maximize the return to shareholders*, as measured by the *sum of capital gains and dividends*, for a given level of risk.

Alternatively, the firm should *minimize the level of risk* to shareholders for a given rate of return.

In a given time period the shareholders are interested in the returns in that time period. The returns are composed of any change in the share price plus the level of dividends

Shareholder Value = Value of Shares + Value of Dividends

Shareholder Return = Capital Gains + Dividends

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$$

In general most companies do not have a large dividends to their share price or indeed their annual capital gain

---

Thus,

- Shareholders are interested that managers operate in such a way as to **rise the price of their shares**,
- The price of shares are influenced by a number of different factors;
  - **Internal policies** of the management
  - **Economic environment** and,
  - **General movements** in share prices in the market

**It is important for companies to carefully set up their internal management systems to ensure the behavior of managers is congruent with the interests of shareholders**

## Agency Costs

Agency Costs are costs that arise when there are conflicts of interest between the firm's stakeholders

Different claimants have different incentives, which can lead firms to undertake actions that hurt one group to benefit another.

- Overinvestment and asset Substitution
- Underinvestment and Debt Overhang

Agency costs are another cost of increasing leverage, just like bankruptcy costs



## The Time Value of Money

Would you prefer to have \$1 million now or \$1 million 10 years from now?

Of course, **we would all prefer the money now!**

This illustrates that there is an inherent monetary value attached to time.

## What is The Time Value of Money?

A dollar received today is worth more than a dollar received tomorrow

- This is because a dollar received today **can be invested to earn interest**
- The amount of interest earned **depends on the rate of return** that can be earned on the investment

Time value of money **quantifies** the value of a dollar through time

# Uses of Time Value of Money

Time Value of Money, or TVM, is a concept that is used in all aspects of finance including:

- Bond valuation
- Stock valuation
- Accept/reject decisions for project management
- Financial analysis of firms
- **And many others!**

# Introduction to Financial Mathematics

## Simple Interest

The value of a cash flow is calculated without including any accrued interest to the principal

**Example:** If you invest \$1,000 at 8% p.a. earning simple interest for 5 years what amount will you have in your account at the end of that time period?

Interest earned in each of the five years =  $1000 \times 0.08 = \$80$

Interest earned in over five years =  $1000 \times (5 \times 0.08) = \$400$

Future value at the end of year 5 =  $1000 + 400 = \$1,400$

Future value at the end of year 5 =  $1000 \times (1 + 5 \times 0.08) = \$1,400$

Future value (Simple interest):  $S_n = P_0 \times (1 + n \times i)$

Present value (Simple interest):  $P_0 = \frac{S_n}{(1+n \times i)}$

# Simple Versus Compounded Interest

## Compound Interest

Interest accrued is added to the principal

The value of a cash flow is calculated based on the principal and interest accrued

**Example:** If you invest \$1,000 at 8% p.a. earning compounded interest for 5 years what amount will you have in your account at the end of that time period?

Future value at the end of year 1  $= 1000 \times (1.08) = \$1,080.00$

Future value at the end of year 2  $= 1080 \times (1.08) = \$1,166.40$

Future value at the end of year 5  $= 1000 \times (1.08)^5 = \$1,469.33$

The difference of **\$69.33** ( $= 1469.33 - 1400.00$ ) is due to the compounding of interest

# Simple Versus Compounded Interest

Amount Invested \$1,000

Interest Rate 8%

| End of year | Simple Interest | Compounded Interest | Difference     |
|-------------|-----------------|---------------------|----------------|
| 1           | \$1,080.00      | \$1,080.00          | \$0.00         |
| 2           | \$1,160.00      | \$1,166.40          | \$6.40         |
| 3           | \$1,240.00      | \$1,259.71          | \$19.71        |
| 4           | \$1,320.00      | \$1,360.49          | \$40.49        |
| 5           | \$1,400.00      | \$1,469.33          | \$69.33        |
| 20          | \$2,600.00      | \$4,660.96          | \$2,060.96     |
| 50          | \$5,000.00      | \$46,901.61         | \$41,901.61    |
| 100         | \$9,000.00      | \$2,199,761.26      | \$2,190,761.26 |

## Future Value of a Single Cash Flow

The future value (or sum) at  $i\%$  p.a. of  $\$P_0$  today is the dollar value to which it grows at the end of time period  $n$

$$S_n = P_0 \times (1 + i)^n$$



*Cash flows occur at the end of the period*

# Future Value of a Single Cash Flow

The future value at  $r\%$  p.a. of  $\$P_0$  today is the dollar value to which it grows at the end of time  $n$

$$FVIF_{r,n} = \$1(1 + r)^n$$

FVIF is short for Future Value Interest Factor



*Cash flows occur at the end of the period*

# Future Value of a Single Cash Flow

**Example:** You decide to invest \$1,000 for different time periods. What is the future value of this \$1,000 in 5, 20 and 100 years at an interest rate of (a) 4% and (b) 6%?

At  $i = 4\%$  p.a.

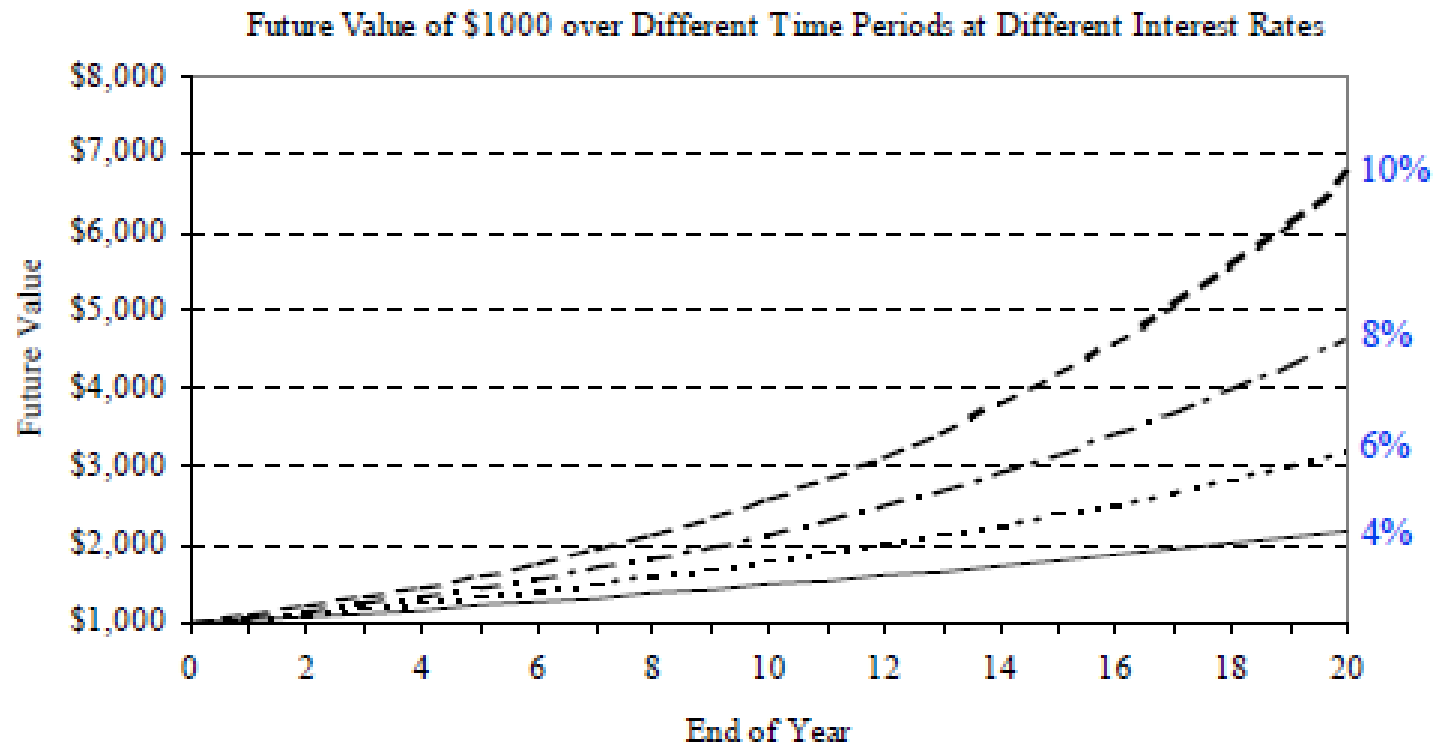
$$\begin{aligned} S_5 &= 1000 \times (1.04)^5 &&= \$1,217 \\ S_{20} &= 1000 \times (1.04)^{20} &&= \$2,191 \\ S_{100} &= 1000 \times (1.04)^{100} &&= \$50,505 \end{aligned}$$

At  $i = 6\%$  p.a.

$$\begin{aligned} S_5 &= 1000 \times (1.06)^5 &&= \$1,338 \\ S_{20} &= 1000 \times (1.06)^{20} &&= \$3,207 \\ S_{100} &= 1000 \times (1.06)^{100} &&= \$339,302 \end{aligned}$$



# Future Value of a Single Cash Flow



# Future Value of a Single Cash Flow

The future value of a cash flow depends on the following factors:

- The time period,  $n$ 
  - Future value increases as  $n$  increases
- The interest rate,  $i$ 
  - Future value increases as  $i$  increases
- The method of calculating interest
  - Future value increases as the compounding interval increases (more on this later).

## Present Value of a Single Cash Flow

The present value ( $P_0$ ) at  $i\%$  p.a. of  $\$S_n$  at the end of time  $n$  is the amount which invested today would grow to  $\$S_n$  in time  $n$

$$P_0 = S_n / (1 + i)^n = S_n \times (1 + i)^{-n}$$



*Cash flows occur at the end of the period*

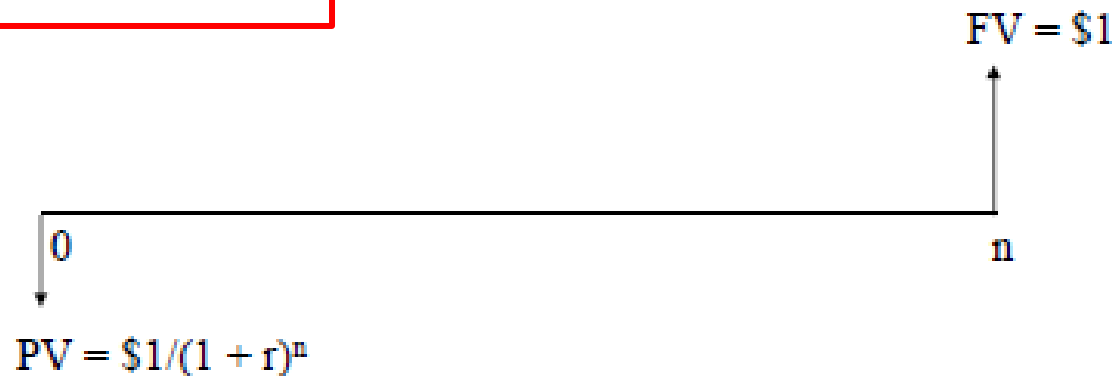
## Present Value of a Single Cash Flow

The present value (PV) at  $r\%$  p.a. of \$1 at the end of time  $n$  is the amount which invested now would grow to \$1 in time  $n$

$$\text{PVIF}_{r,n} = \$1/(1+r)^n = \$1(1+r)^{-n}$$

PVIF is the short for Present Value Interest Factor

Note:  $\text{PVIF}_{r,n} = 1/\text{FVIF}_{r,n}$



*Cash flows occur at the end of the period*

# Present Value of a Single Cash Flow

**Example:** If you needed \$10,000 in (a) five years, (b) ten and (c) twenty years how much would you need to save and invest today if the interest rates were (a) 4% and (b) 6%?

**The present value of \$10,000 in five years**

At 4% p.a.,  $P_0 = 10000/(1.04)^5 = \$8,219.27$

At 6% p.a.,  $P_0 = 10000/(1.06)^5 = \$7,472.58$

**The present value of \$10,000 in ten years**

At 4% p.a.,  $P_0 = 10000/(1.04)^{10} = \$6,755.64$

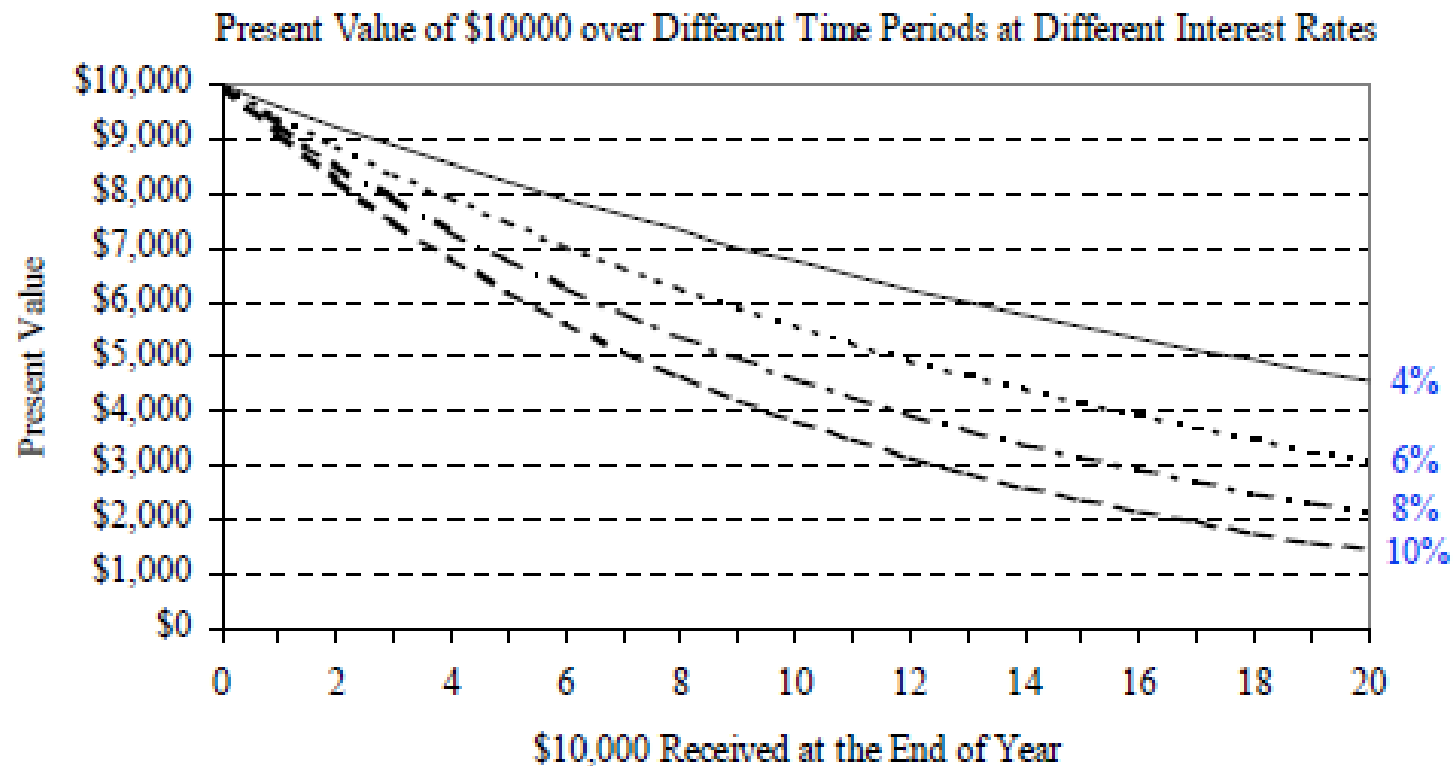
At 6% p.a.,  $P_0 = 10000/(1.06)^{10} = \$5,583.95$

**The present value of \$10,000 in twenty years**

At 4% p.a.,  $P_0 = 10000/(1.04)^{20} = \$4,563.87$

At 6% p.a.,  $P_0 = 10000/(1.06)^{20} = \$3,118.05$

# Present Value of a Single Cash Flow



# Factors Influencing Present and Future Values

The present and future values of a cash flow depend on the following factors

## The time period, $n$

- Future value increases as  $n$  increases
- Present value decreases as  $n$  increases

## The interest rate, $i$

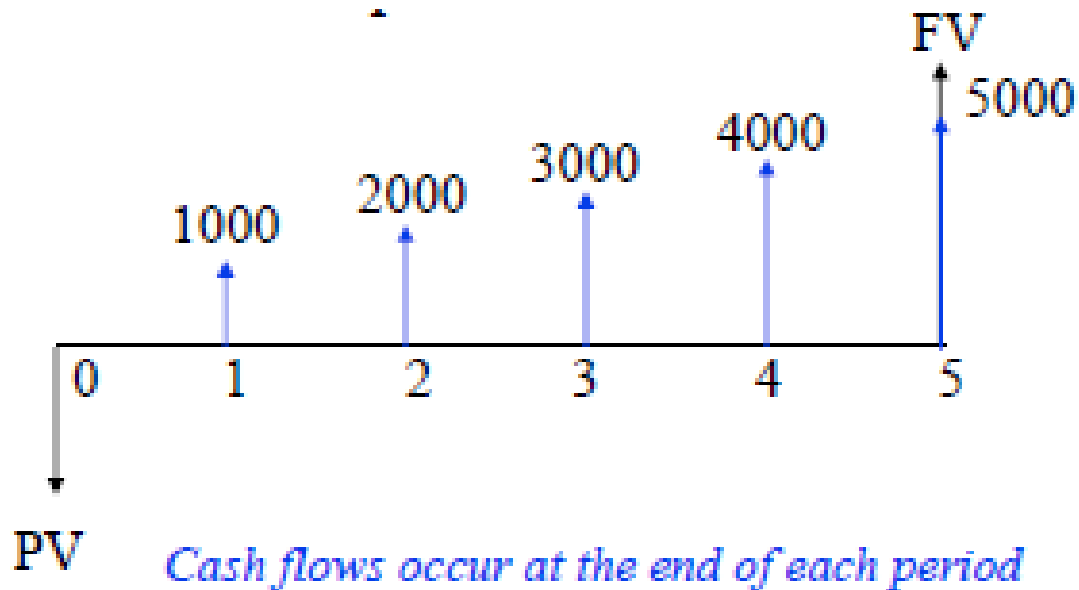
- Future value increases as  $i$  increases
- Present value decreases as  $i$  increases

## The method of calculating interest

- Future value increases as the compounding interval increases
- Present value decreases as the compounding interval increases

# Valuing Unequal Cash Flows

**Class Exercise 1:** You decide to invest \$1,000 at the end of year 1 and then an additional \$1,000 at the end of every year for five years. What is the future value of these cash flows at the end of five years? What equivalent lump-sum amount could you invest today to get this future amount? Assume an interest rate of 10% p.a.





## Answer to Class Exercise 1

To get the total future (present) value of different cash flows occurring at different time periods compute their individual future (present) values and then add across

- Future value of cash flows at the end of five years

- $FV_5 = 1000 \times (1.10)^4 + 2000 \times (1.10)^3 + 3000 \times (1.10)^2 + 4000 \times (1.10) + 5000$

- $FV_5 = \$17,156.10$

- Equivalent single amount that could be invested today to get this future amount

- $PV_0 = 1000/(1.10) + 2000/(1.10)^2 + 3000/(1.10)^3 + 4000/(1.10)^4 + 5000/(1.10)^5 = \$10,652.59$

--or equivalently--

- $PV_0 = FV_5 / (1.10)^5 = 17156.10 / (1.10)^5 = \$10,652.59$

# Contents

1. A closer look at the Fixed income asset classes
2. Bond Market Overview
3. Features in Debt Securities
4. Risks Associated with Investing in Bonds
5. Yield measures, Spot rates and Forward Rates
6. Introduction to the Valuation of Debt Securities
7. Theories of Term Structure of Interest Rates
11. The measurement of Interest Rate Risk

# Asset classes and subcategories

| Equities  | Fixed Income   | Cash  | Alternative Assets  |
|---|--|---|---|
| <b>UK Equities</b> <ul style="list-style-type: none"> <li>- Large capitalisation</li> <li>- Mid capitalisation</li> <li>- Small capitalisation</li> <li>- Micro capitalisation</li> <li>- Growth</li> <li>- Value</li> <li>- Blend (Value and Growth)</li> <li>- Preference shares</li> <li>- Options and futures</li> </ul> <b>Other Developed Markets</b> <ul style="list-style-type: none"> <li>- North America</li> <li>- Europe</li> <li>- Japan</li> <li>- Options and futures</li> </ul> <b>Emerging Markets</b> <ul style="list-style-type: none"> <li>- Africa</li> <li>- Asia ex Japan</li> <li>- Emerging Europe</li> <li>- Latin America</li> <li>- Middle East</li> <li>- Options and futures</li> </ul> | <b>UK Fixed Income</b> <ul style="list-style-type: none"> <li>- UK Treasury bonds</li> <li>- Municipal</li> <li>- Corporate</li> <li>- Mortgage-backed</li> <li>- Asset-backed</li> <li>- Options and futures</li> </ul> <b>High Yield</b><br><b>Convertible Securities</b><br><b>Other Developed Markets</b> <ul style="list-style-type: none"> <li>- North America</li> <li>- Europe</li> <li>- Japan</li> <li>- Options and futures</li> <li>- Interest rate swaps</li> </ul> <b>Emerging Markets</b> <ul style="list-style-type: none"> <li>- Africa</li> <li>- Asia ex Japan</li> <li>- Emerging Europe</li> <li>- Latin America</li> <li>- Middle East</li> <li>- Options and futures</li> </ul> | <b>Cash</b> <ul style="list-style-type: none"> <li>- Physical holdings</li> <li>- Bank balance</li> <li>- UK Treasury bills</li> <li>- Municipal notes</li> <li>- Commercial papers</li> <li>- Certificates of deposit</li> <li>- Repurchase agreement</li> <li>- Banker acceptances</li> <li>- Non UK instruments</li> </ul> | <b>Commodities</b> <ul style="list-style-type: none"> <li>- Commodity trading advisors (CTAs)</li> <li>- Physicals: Agricultural, metal and oil</li> <li>- Options and futures</li> </ul><br><b>Hedge Funds</b> <ul style="list-style-type: none"> <li>- Event driven</li> <li>- Relative value</li> <li>- Market neutral</li> <li>- Long - short</li> <li>- Global macro</li> </ul><br><b>Private Equity</b> <ul style="list-style-type: none"> <li>- Leveraged Buyouts</li> <li>- Venture Capital</li> <li>- Non UK</li> </ul><br><b>Real Estate</b> <ul style="list-style-type: none"> <li>- Residential</li> <li>- Commercial</li> <li>- REITs (Real Estate Investment Trusts)</li> </ul><br><b>Art</b> |

## Fixed Income

| Rationale for Investment                | Risks and Concerns         |
|---|----------------------------|
| Senior claim                            | Lower returns than equity  |
| Low risk                                | Interest rate risk         |
| Higher return than cash                 | Inflation risk             |
| Portfolio diversifier (Low correlation) | Credit risk                |
|   | Reinvestment risk          |
|   | Prepayment risk (Callable) |

## High yield fixed income

| Rationale for Investment  | Risks and Concerns  |
|---|---|
| High return   | Issued to finance leveraged buyouts or ex-investment grade bond consequently downgraded |
| Lower risk than equity  | Credit risk   |
| Irrational (Inefficient) pricing:<br>Possibility to beat the market | Liquidity risk  |
| Claim senior to equity  |   |

## Features of Debt Securities

**Fixed income security:** financial obligation of an entity that promises to pay a specified sum of money at specified future dates.

**Issuer of the security:** Entity that promises to make the payment (e.g. US government, French government, the city of Rio de Janeiro in Brazil, Corporation such Coca-Cola, Sport Institutions such Porto Football Club or supranational governments such as the World Bank.

**Fixed Income securities (two general categories):** debt obligations and preferred stock

**Debt Obligations:** bonds, mortgage-backed securities, asset backed securities and bank loans.

**Bond indenture** (also trust indenture or deed of trust): legal document issued to lenders and describes key terms such as the interest rate, Maturity date, convertibility, pledge, promises, representations, covenants, and other terms of the bond offering.

**Bond Covenant:** designed to protect the interests of both parties. Negative or restrictive covenants forbid the issuer from undertaking certain activities; positive or affirmative covenants require the issuer to meet specific requirements

**Maturity:**

**Term to maturity:** number of years the debt is outstanding or the number of years remaining prior to final principal payment

**Maturity date:** date that the debt will cease to exist

| Type              | Maturity           |
|-------------------|--------------------|
| Short-term        | 1 to 5 years       |
| Intermediate-term | 5 – 12 years       |
| Long-term         | More than 12 years |

**Par Value:** Amount that the issuer agrees to repay the bondholder at or by the maturity date (principal value, face value, redemption value or maturity value).

Because bonds have different par values, the practice is to quote bonds as a percentage of its par value.

| Quoted Price | Price per \$1 of par value<br>(rounded) | Par value | Dollar Price |
|--------------|---|-----------|--------------|
| 90 1/2       | 0.9050                                  | \$1,000   | 905.00       |
| 102 3/4      | 1.0275                                  | \$5,000   | 5,137.50     |
| 70 5/8       | 0.7063                                  | \$10,000  | 7,062.50     |
| 113 11/32    | 1.1334                                  | \$100,000 | 113,343.75   |



**Coupon Rate** (nominal rate): is the interest rate that the issuer agrees to pay each year.

**Coupon:** Annual amount of the interest payments made to bondholders during the term of the bond. Calculated as:

$$\text{coupon} = \text{coupon rate} \times \text{par value}$$

**Example:**

6% coupon rate and a par value of \$1,000

Coupon (interest payment) = \$60

United States (semi-annual instalments), Mortgage and Asset Backed Securities typically pay interest monthly.

**Zero-coupon Bonds:** the holder realizes interest by buying the bond substantially below its par value

## Provisions for Paying off Bonds

**Bullet maturity:** No principal repayments prior to maturity date.

**Amortizing Securities:** Schedule of partial payments until maturity (e.g. fixed income securities backed by pool of loans, mortgage backed securities and asset-backed securities).

**Sinking Fund:** Repayment of the bond may be arranged to repay only a part of the total by the maturity date.

**Call provision:** guarantee the issuer an option to retire all or part of the issue to the stated maturity date (callable bond).

**Convertible bond:** grants the bondholder the right to convert the bond for a specified number of shares of common stock.

**Put Provision:** grants the bondholder the right to sell issue back to the issuer at a specified price on designed dates.

**Currency denomination:** in the USA, dollar-dominated, nondollar denominated issues and dual-currency issues.

# Risks Associated with Investing in Bonds

## Interest-rate risk or market risk

As interest rates **rise**, the price of a bond **fall** (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price).

### Example:

Consider a 6% 20-year bond with a face value of \$100. if the yield investors require to buy this bond is 6%, the price of this bond would be \$100 (selling at par).

If required yield increase to 6.5%, the price of this bond **would decline to \$94.4479**. Thus, for a 50 basis point increase in yield, the bond's price declines by 5.5%. If, instead, the yield declines from 6% to 5.5%, the bond's price **will rise by 6.02% to \$106.0195**.

Coupon rate = yield required by market → price = par value

Coupon rate < yield required by market → price < par value (discount)

Coupon rate > yield required by market → price > par value (premium)

If interest rates increase → price of a bond decreases

If interest rates decrease → price of a bond increases

## Bond Features that affect Interest Rate Risk

**Maturity:** all other factors constant, the longer the bond's maturity, the greater the bond's price sensitivity to changes in interest rates

**Coupon Rate:** all other factors constant, the lower the coupon rate, the greater the bond's price sensitivity to changes in interest rates

### Embedded Options:

**Call option:** As interest rates decline, the price of a callable bond may not increase as much as an otherwise option-free bond

Price of callable bond = price of option-free bond – price of embedded call option

**Yield level:** Bond's that trade at a lower yield are more volatile in both percentage price change and absolute price change (as long as the other bond characteristics are the same).

**Yield curve risk:** bond portfolios have different exposures to how the yield curve shifts.

## Call Risk or Prepayment Risk

Issuer can retire or “call” all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate).

Disadvantages from the investor's perspective:

- 1) The cash flow pattern of a callable bond is **not known with certainty** because it is not known when the bond is called.
- 2) Because the issuer is likely to call the bonds when interest rates have declined below the bond's coupon rate, **the investor is exposed to reinvestment risk** (will have to reinvest the proceeds at a lower interest rate than the bond's coupon rate)
- 3) The price appreciation potential of the bond **will be reduced** relative to an otherwise comparable option-free bond (price compression)

## Reinvestment Risk

Risk that the proceeds received from the payment of interest and principal that are available for reinvestment **must be reinvested at a lower interest rate** than the security that generated the proceeds.

## Credit Risk

Three types of credit risk: default risk, credit spread risk and downgrade risk.

**Default Risk:** Risk that issuer will fail to satisfy the term of the obligations with respect to the timely payment of interest and principal (default rate, recovery rate and expected loss).

**Credit Spread Risk:** The part of the risk premium or yield spread attributable to default risk. The price performance and the return over some time period will depend on how the credit spread changes.

**Downgrade Risk:** Risk that the bond issue or issuer **credit rating** will change.

## Three rating agencies in the United States: Moody's Investors Service Inc, Standard & Poor's Corporation and Fitch Ratings

| Moody's                                   | S&P  | Fitch | Summary Description              |
|---|------|-------|----------------------------------|
| Investment Grade – High Credit Worthiness |      |       |                                  |
| Aaa                                       | AAA  | AAA   | Gilt edge, prime, maximum safety |
| Aa1                                       | AA+  | AA+   | High-grade, high credit quality  |
| Aa2                                       | AA   | AA    |                                  |
| Aa3                                       | AA-  | AA-   |                                  |
| A1  | A+   | A+    | Upper-medium grade               |
| A2  | A    | A     |                                  |
| A3  | A-   | A-    |                                  |
| Baa1                                      | BBB+ | BBB+  | Lower-medium Grade               |
| Baa2                                      | BBB  | BBB   |                                  |
| Baa3                                      | BBB- | BBB-  |                                  |



| Moody's  | S&P         | Fitch          | Summary Description                   |
|--|-------------|----------------|---------------------------------------|
| Speculative – Lower Credit Worthiness                      |             |                |                                       |
| Ba1  | BB+         |                | Low grade, speculative                |
| Ba2  | BB          |                |                                       |
| Ba3  | BB-         |                |                                       |
| B1   | B           | B+             | Highly speculative                    |
| B2   |             | B              |                                       |
| B3   |             | B-             |                                       |
| Predominantly Speculative, Substantial Risk, or in Default |             |                |                                       |
| Caa  | CCC+<br>CCC | CCC+<br>CCC    | Substantial Risk, in poor standing    |
| Ca   | CC          | CC             | May be in default, very speculative   |
| C  | C           | C              | Extremely speculative                 |
|  | Cl          |                | Income bonds – no interest being paid |
|  | D           | DDD<br>DD<br>D | Default                               |

## Liquidity Risk

The risk that the investor will have to **sell a bond below its indicated value**, where the indication is revealed by a recent transaction.

The primary measure of liquidity is the size of the spread between the bid price (the price at which the dealer is willing to buy a security) and the ask price (the price at which a dealer is willing to sell a security).

The wider the bid-ask spread, the greater the liquidity risk.

## Exchange Rate or Currency Risk

Risk of **receiving less of the domestic currency** when investing in a bond issue that makes payments in a currency other than the manager's domestic currency.

## Inflation Risk

Risk of decline in the value of a security's cash flows due to inflation.

**Volatility Risk:** Risk that the “expected yield volatility” will change.

The greater the expected yield volatility, the greater the value (price) of an option.

Price of callable bond = price of option-free bond – price of embedded call option

Price of Putable bond = price of option-free bond + price of embedded put option

| Type of embedded option | Volatility risk due to                   |
|-------------------------|--|
| Callable Bonds          | An increase in expected yield volatility |
| Putable Bonds           | An decrease in expected yield volatility |

## Event Risk

- 1) Natural disaster (earthquake or hurricane) or an industrial accident.
- 2) Takeover or corporate restructuring
- 3) Regulatory change

**Sovereign Risk:** 1) Unwillingness of a foreign government to pay, or 2) inability to pay due to unfavourable economic conditions in the country

# Yield Measures, Spot Rates and Forward Rates

## Sources of Return

- 1) The coupon interest payments made by the issuer
- 2) Any capital gain (or capital loss – negative return) when the security matures, is called or is sold.
- 3) Income from reinvestment of interim cash flows (interest and/or principal payments prior to stated maturity).

## Current yield

Annual dollar coupon interest to a bond's market price

$$\text{Current Yield} = \frac{\text{Annual dollar coupon interest}}{\text{price}}$$

## Yield to Maturity

Interest rate that will make the present value of the bond's cash flows equal to its market price plus accrued interest (is the interest that has accumulated since the previous interest payment)

## Yield to Call

The yield to call assumes the issuer will call a bond on some assumed call date and that the call price is the price specified in the call schedule.

## Yield to Put

Interest rate that will make the present value of the cash flows to the first put date equal to the price plus accrued interest.

## Yield to Worst

Is the lowest of possible yields (YTM, Yield to call and yield to put).

## Spot Rates

A default-free theoretical spot rate curve can be constructed from the observed Treasury yield curve.

The approach for creating a theoretical spot rate curve is called **bootstrapping**.

### Example:

2-year = 1.71%, 5-year = 3.25%, 10-year = 4.35% and 30-year = 5.21%

$$\frac{\text{yield at higher maturity} - \text{yield at lower maturity}}{\text{number of years between two observed maturity points}} = \frac{4.35\% - 3.25\%}{5} = 0.22\%$$

Then,

**Interpolated 6-year yield** = 3.25% + 0.22% = **3.47%**

7, 8 and 9-years yield, 3.69%, 3.91% and 4.13%, respectively

## Forward Rates

Examples of forward rates:

6-month forward rate six months from now

6-month forward rate three years from now

1-year forward rate one year from now

3-year forward rate two years from now

5-year forward rates three years from now, etc, etc....

## Deriving 6-month forward rates

**Arbitrage principle** (if two investments have the same cash flows and have the same risk, they should have the same value).

**Investor with two alternatives:**

- Buy a 1-year Treasury bill or,
- Buy a 6-month Treasury bill and when it matures in six months, buy another 6-month Treasury bill.

Spot rate on the 6-month Treasury bill = 3.0% ( $Z_1$ )

Spot rate on the 1-year Treasury bill = 3.3% ( $Z_2$ )

6-month forward rate on in six months from now = ?

$$(1 + Z_1) \times (1 + f_{1,1}) = (1 + Z_2)^2$$

$$f_{1,1} = \frac{(1 + Z_2)^2}{(1 + Z_1)} - 1 = \frac{(1 + 0.0165)^2}{(1 + 0.015)} - 1 = 1.80\%$$



# The Valuation Principle

The price of a security **today** is the present value of all future expected cash flows discounted at the “appropriate” required rate of return (or discount rate)

## The valuation variables are

1. Current price
2. Future expected cash flows - Face value and/or coupons
3. Yield or required rate of return

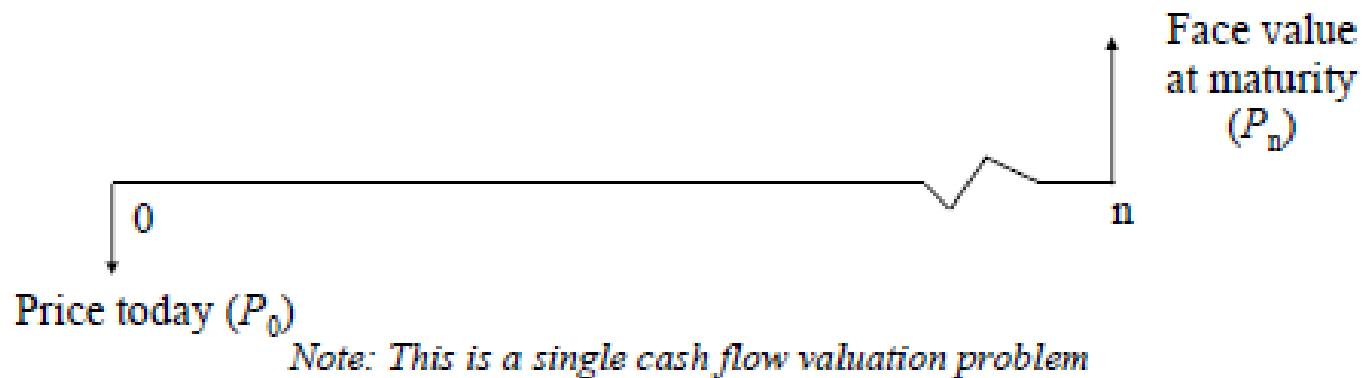
## The valuation problem is to

1. Estimate the price; given the future cash flows and required rate of return, or
2. Estimate the required rate of return; given the future cash flows and price

# Zero Coupon Securities

Zero coupon bonds are long-term securities paying the face value at maturity

- No coupon or interest payment made
- Issued at deep discount to face value
- Return earned is based on the appreciation in bond's value (price) over time



# Pricing Zero Coupon Securities

## Example:

Consider a zero coupon bond which matures in 5 years with a face value of \$1,000

a) If the bond has a yield to maturity of 8% what price should it be selling for today?

b) Suppose interest rates change suddenly and the price of these bonds rises to \$700. What has happened to the yield to maturity of the bonds and why?

Given:  $P_n = \$1,000$ ,  $n = 5$  years, and  $k_d = \text{YTM} = 8\%$

a)  $P_0 = \frac{1,000}{(1.08)^5} = \$680.58$

b) The price has risen so you'd expect the YTM to be lower

$$\text{New price, } P_0 = 700 = \frac{1,000}{(1 + k_{d*})^5} \quad k_{d*} = \left( \frac{1,000}{700} \right)^{1/5} = 7.39\%$$

# Coupon Paying Securities

Fixed coupon payment, typically every six months

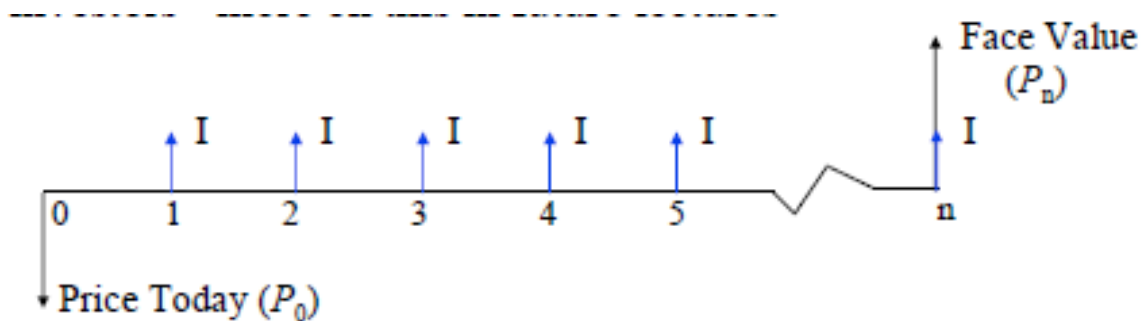
- Non coupon paying bonds called zero coupon bonds

Repayment of face value at maturity

Typically issued at face value

- **Examples:** Treasury bonds, corporate bonds

Market price depends on the rate of return required by investors



*Note: This is a single cash flow plus annuity valuation problem*

## Pricing a Bond

Equal to the present value of the expected cash flows from the financial instrument. Determining the price requires:

An estimate of the **expected cash flows**

An estimate of the **appropriate required yield**

The price of the bond is the present value of the cash flows, it is determined by adding these two present values:

- i) The present value of the semi-annual coupon payments
- ii) The present value of the par or maturity value at the maturity date

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}$$

$$P = \sum_{t=1}^n \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

P = Price

n = number of periods (nr of years times 2, if semi-annual)

C = semi-annual coupon payment

r = periodic interest rate (required annual yield divided by 2, if semi-annual)

t = time period when payment is to be received

Because the semi-annual coupon payments are equivalent to an **ordinary annuity**, applying the equation for the present value of an ordinary annuity gives the present value of the coupon payments:

$$C \times \left[ \frac{1 - \frac{1}{(1+r)^n}}{r} \right]$$

Consider a 20 year 10% coupon bond with a par value of \$1,000. The required yield on this bond is 11%.

$$50 \times \left[ \frac{1 - \frac{1}{(1 + 0.055)^{40}}}{0.055} \right] = \text{\$802.31}$$

The PV of the par or maturity value of \$1,000 received 40 six-month periods from now, discounted at 5.5%, is \$117.46, as follows:

$$\frac{\$1,000}{(1.055)^{40}} = \frac{\$1,000}{8.51332} = \text{\$117.46}$$

$$\begin{aligned} \text{Price} &= \text{PV coupon payments} + \text{PV of par (maturity value)} \\ &\text{\$802.31} + \text{\$117.46} = \text{\$919.77} \end{aligned}$$

## Holding Period Yield

### Example

Consider a 30-year zero coupon bond with a face value of \$100. If the bond is priced at a yield-to-maturity of 10%, it will cost \$5.73 today (the present value of this cash flow). Over the coming 30 years, the price will advance to \$100, and the annualized return will be 10%.

Suppose that over the first 10 years of the holding period, interest rates decline, and the yield-to-maturity on the bond falls to 7%.

With 20 years remaining to maturity, the price of the bond will be \$25.84.

Even though the yield-to-maturity for the remaining life of the bond is just 7%, and the yield-to-maturity bargained for when the bond was purchased was only 10%, the return earned over the first 10 years is 16.26%. This can be found by evaluating:

$$(1 + r) = (25.84 / 5.73)^{1/10} = 1.1626$$



Over the remaining 20 years of the bond, the annual rate earned is not 16.26%, but 7%

This can be found by evaluating:

$$25.84 = 100 / (1 + r)^{20}$$

$$(1 + i) = (100 / 25.84)^{0.05} = 1.07$$

Over the entire 30 year holding period, the original \$5.73 invested matured to \$100, so 10% annually was made, irrespective of interest rate changes in between

# Theories of Term Structure of Interest Rates

What is the information in the yield curve?

How can it be explained and interpreted changes in the yield curve?

Three main theories:

- 1) The pure expectation theory (unbiased expectations theory)
- 2) The liquidity preferences theory (or liquidity premium theory)
- 3) The market segmentation theory

## Pure Expectations Theory

Makes simple link between the yield curve and investors' expectations about future interest rates.

Also, because long-term interest rates are possible linked to investors expectations about future inflation, it also address economic interpretations.

Explains the term structure in terms of expected future short-term interest rates.

The market will sets the yield on a two year-bond so that the return on a two-year bond is approximately equal to the return on a one-year bond plus the expected return on a one-year bond purchased one year from today.

Therefore:

Raising term structure indicates that the market expects short-term rates to raise in the future

| Shape of term structure     | Implications according to pure expectations theory |
|-----------------------------|--|
| Upward sloping (normal)     | Rates expected to rise                             |
| Downward sloping (inverted) | Rates expected to decline                          |
| Flat                        | Rates not expected to change                       |

Under the hypothesis that interest rates reflect the sum of a relatively **stable real rate of interest plus a premium for expected inflation**:

If short-term rates are **expected to rise**, investors expect **inflation to rise** as well.

**Shortcomings:** assumes that investors are **indifferent** to interest rate risk and any other factors associated with investing in bonds with different maturities.

## Liquidity Preference Theory

Market participants want to be **compensated** for the interest rate risk associated with holding longer-term bonds.

Therefore, the term structure of interest rates is determined by:

- 1) **Expectations** about future interest rates
- 2) **Yield premium** for interest rate risk (more interest rate risk, the less the liquidity)

Since **interest rate risk** increases with maturity, **yield premium** increases with maturity

| Shape of term structure             | Implications according to Liquidity Preference Theory   |
|-------------------------------------|---|
| Upward sloping (normal)             | Rates expected to rise, or will be unchanged or even fall (but with yield premium increasing with maturity fast enough to produce an upward sloping of yield curve) |
| Downward sloping (inverted) or flat | Rates expected to fall, given the theory's prediction that the yield premium for interest rate risk increases with maturity   |

## Market Segmentation Theory

Each maturity “sector” is an independent or segmented market for purposes of determining the interest rate in the maturity “sector”.

Two major groups of investors:

- 1) Those who manage funds versus a broad-based bond market index, and
- 2) those that manage funds against liabilities.

The 2<sup>nd</sup> group will restrict their activities to the maturity sector that provides the best match with the maturity of their liabilities (basic principle of asset-liability management).

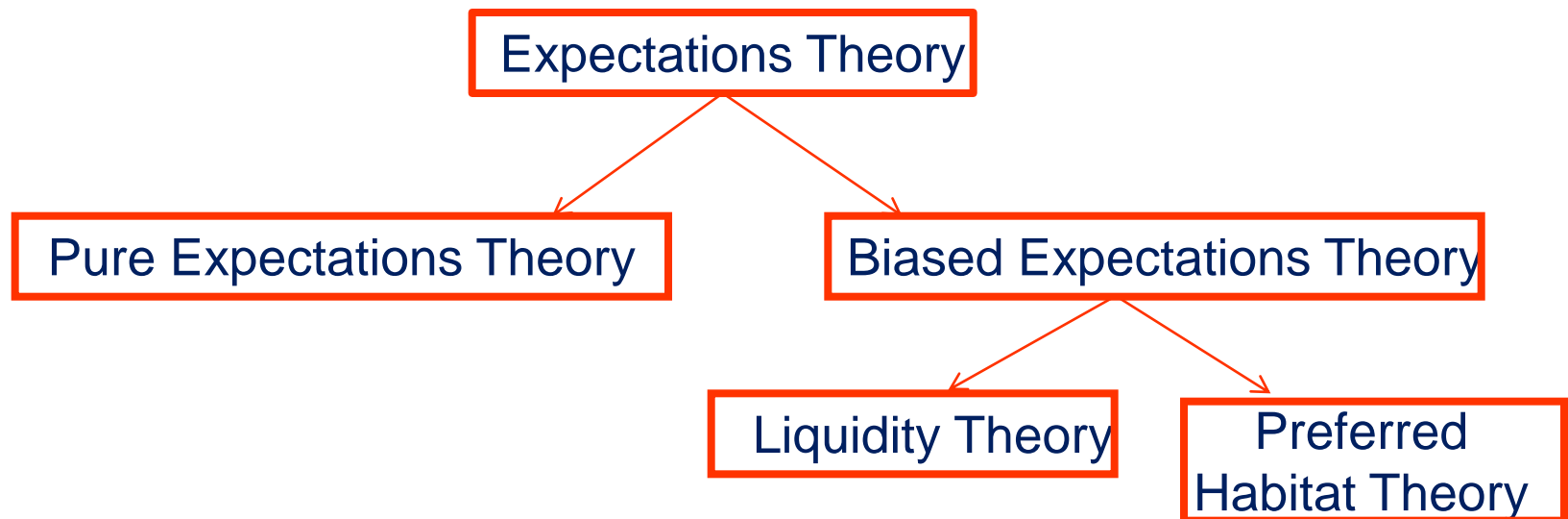
Defined benefit pension fund: investment in long-term maturity sector of the bond market

Commercial banks: will focus in short-term fixed income investments (since their liabilities are mainly short-term).

**Preferred habitat theory:** variant of the market segmentation theory

Investors might be willing to shift out for their preferred maturity sector if a **incentive yield premium** exist.

**Implication:** Under the preferred habitat theory **any shape** of the yield curve is possible.



# The measurement of Interest Rate Risk

Most obvious way: Re-value the bond when interest rates change

Example:

\$10 million par value position in a 9% coupon 20-year bond (option free).

Current price: 134.6722 for a YTM of 6%. Market value of the position is \$13,467,220

Three scenarios:

- 1) 50 basis point increase
- 2) 100 basis point increase
- 3) 200 basis point increase

| Scenario | Yield change (bp) | New yield | New Price | New Market Value (\$) | Percentage Change in Market Value (%) |
|----------|-------------------|-----------|-----------|-----------------------|---------------------------------------|
| 1        | 50                | 6.5%      | 127.7606  | 12,776,050            | -5.13%                                |
| 2        | 100               | 7.0%      | 121.3551  | 12,135,510            | -9.89%                                |
| 3        | 200               | 8.0%      | 109.8964  | 10,989,640            | -18.40%                               |



## Price Volatility Characteristics of Bonds

Price of bond changes in the opposite direction to a change in the bond's yield.

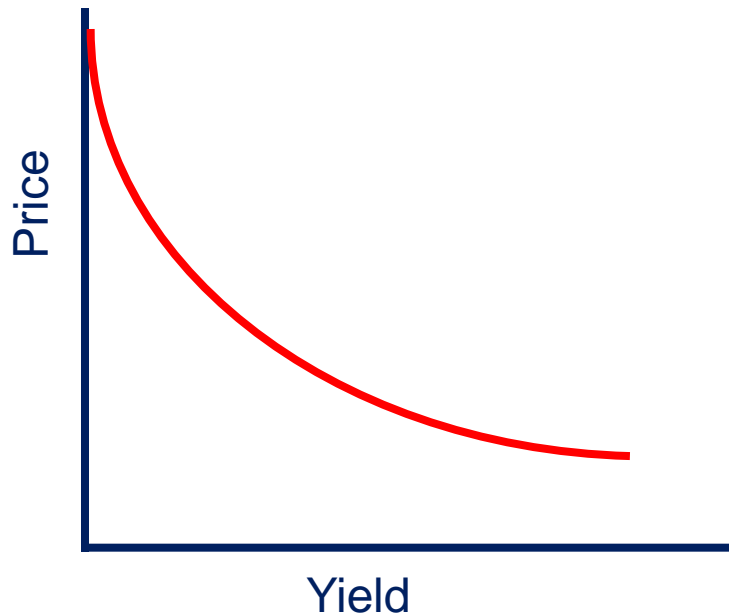
The percentage price change is not the same to all bonds

|           | Price (\$) |            |           |            |
|-----------|------------|------------|-----------|------------|
| Yield (%) | 6%/ 5 year | 6%/20 year | 9%/5 year | 9%/20 year |
| 4.00      | 108.9826   | 127.3555   | 122.4565  | 168.3887   |
| 5.00      | 104.3760   | 112.5514   | 117.5041  | 150.2056   |
| 5.50      | 102.1600   | 106.0195   | 115.1201  | 142.1367   |
| 5.90      | 100.4276   | 101.1157   | 113.2556  | 136.1193   |
| 5.99      | 100.0427   | 100.1157   | 112.8412  | 134.8159   |
| 6.00      | 100.0000   | 100.0000   | 112.7953  | 134.6722   |
| 6.01      | 99.9574    | 99.8845    | 112.7494  | 134.5287   |
| 6.10      | 99.5746    | 98.8535    | 112.3373  | 133.2472   |
| 6.50      | 97.8944    | 94.4479    | 110.5280  | 127.7605   |
| 7.00      | 95.8417    | 89.3225    | 108.3166  | 121.3551   |
| 8.00      | 91.8891    | 80.2072    | 104.0554  | 109.8964   |

**Instantaneous Percentage Price Change for Four Hypothetical Bonds**  
(Initial yield for all four bonds is 6.%)

|           | <b>Percentage Price Change</b> |            |           |            |
|-----------|--------------------------------|------------|-----------|------------|
| Yield (%) | 6%/ 5 year                     | 6%/20 year | 9%/5 year | 9%/20 year |
| 4.00      | 8.98                           | 27.36      | 8.57      | 25.04      |
| 5.00      | 4.38                           | 12.55      | 4.17      | 11.53      |
| 5.50      | 2.16                           | 6.02       | 2.06      | 5.54       |
| 5.90      | 0.43                           | 1.17       | 0.41      | 1.07       |
| 5.99      | 0.04                           | 0.12       | 0.04      | 0.11       |
| 6.01      | -0.04                          | -0.12      | -0.04     | -0.11      |
| 6.10      | -0.43                          | -1.15      | -0.41     | -1.06      |
| 6.50      | -2.11                          | -5.55      | -2.01     | -5.13      |
| 7.00      | -4.16                          | -10.68     | -3.97     | -9.89      |
| 8.00      | -8.11                          | -19.79     | -7.75     | -18.40     |

## Price/yield relationship for a hypothetical option-free bond



- 1) Although the price moves in the opposite direction from the change in the yield, percentage change is **not the same** for all bonds
- 2) For **small changes** in the yield, the percentage price changes for a given bond is **roughly the same**, whether the yield increases or decreases.
- 3) For **large changes** in yield, the percentage price change is **not the same** for an increase in yield as it is for a decrease in yield
- 4) For a **given large change** in yield, the percentage price increase is **greater** than the percentage price decrease.

## Duration/Convexity approach

**Duration** is a measure of the approximate price sensitivity of a bond to interest rate changes. It is the approximate percentage change in price for a 100 basis points change in rate.

**Duration:** average lifetime of a debt security's stream of payments

Two bonds with the same term to maturity **does not mean** that they have the same interest-rate risk

### Macaulay Duration

Is the **weighted average time-to-maturity** of the cash flows of a bond. In the Macaulay, and all other duration measures, the weighting of the cash flows is based on their discounted present value, rather than their nominal value.

$$\text{Macaulay Duration} = \frac{\sum(n)(PV \text{ of Cash Flows})}{k \times \text{Bond Price}}$$

**Where:**

**n** is the number of periods until each cash flow is paid

**k** is the number of times coupon interest is paid per year

## Calculating Macaulay Duration on a \$1,000 ten-year 10% Coupon Bond when its Interest Rate is 10%

| (1)          | (2)                                  | (3)                               | (4)                     | (5)                            |
|--------------|--------------------------------------|-----------------------------------|-------------------------|--------------------------------|
| Year         | Cash Payments<br>(Zero-Coupon Bonds) | Present Value of Cash<br>Payments | Weights<br>(% of total) | Weighted Maturity<br>(1×4)/100 |
| 1            | 100                                  | 90.91                             | 9.091                   | 0.09091                        |
| 2            | 100                                  | 82.64                             | 8.264                   | 0.16528                        |
| 3            | 100                                  | 75.13                             | 7.513                   | 0.22539                        |
| 4            | 100                                  | 68.30                             | 6.830                   | 0.27320                        |
| 5            | 100                                  | 62.09                             | 6.209                   | 0.31045                        |
| 6            | 100                                  | 56.44                             | 5.644                   | 0.33864                        |
| 7            | 100                                  | 51.32                             | 5.132                   | 0.35924                        |
| 8            | 100                                  | 46.65                             | 4.665                   | 0.37320                        |
| 9            | 100                                  | 42.41                             | 4.241                   | 0.38169                        |
| 10           | 100                                  | 38.55                             | 3.855                   | 0.38550                        |
| 10           | 1000                                 | 385.54                            | 38.554                  | 3.85500                        |
| <b>Total</b> |                                      | <b>1,000.00</b>                   | <b>100.00</b>           | <b>6.75850 (years)</b>         |

## Effective Duration

If the yield increased by a small amount  $\Delta r$ , from  $r_0$  to  $r_+$ , the price of the bond will decrease from  $P_0$  to  $P_-$ .

A bond's effective duration measures how sensitive the return on the bond (measured as the percentage change in its price) will be to the change in interest rates.

$$DE = \frac{\text{Percentage Change in Price of Bond}}{\text{Change in Interest Rates}}$$

$$DE = \frac{\Delta P/P}{\Delta r} = \frac{(P_- - P_+)/P_0}{2\Delta r}$$

$$DE = \frac{P_- - P_+}{2P_0\Delta r}$$

## Example:

A 7% coupon, 5-year bond, yielding 6% is priced at 104.265. If its yield declines by 25 basis points to 5.75%, the bond's price will increase to 105.366. On the other hand, if its yield increases by 25 basis points to 6.25%, the price of the bond will decline to 103.179. Compute the effective duration of the bond under these conditions.

$$DE = \frac{P_- - P_+}{2P_0\Delta r} = \frac{105.366 - 103.179}{2(104.265)(.0025)} = 4.2$$

If this bond's yield increases by 1% (apparently because interest rates have increased by 100 basis points), the price of the bond will fall by approximately 4.2%.

## Dollar Duration

Measures the dollar market value change resulting from a 100 basis point change in yield:

$$\text{Dollar duration} = -D_E \times (\$ \text{ Market Value}) \times \Delta r$$

## Example:

A manager has a holding of XYZ bond with a current market value of \$25 million and a duration of 5.4. If the bond's yield dropped by 100 basis points, what would be the change in the market value.

$$\begin{aligned}\text{Dollar duration} &= -D_E \times (\$ \text{ Market Value}) \times \Delta r \\ &= -5.4 \times (\$25,000,000) \times -0.0100 \\ &= +\$1,350,000\end{aligned}$$

## Application of Effective Duration

**From the basic formula:** percentage change in the price of bond will approximately equal its effective duration times any change that occurs in its yield, but in opposite direction

$$\% \text{ Change in Price of Bond} = \frac{\Delta P_B}{P_B} = -D_E \times \Delta r$$



## Example:

A 7% coupon, 5-year bond, priced at 104.265 with duration of 4.2 has a YTM of 6%. Estimate the percentage change in the price and the new price of the bond if its YTM declines by 50 basis points from 6% to 5.5%.

$$\% \text{ Change in Price of Bond} = \frac{\Delta P_B}{P_B} = -D_E \times \Delta r = -4.2(-.005) = \mathbf{2.1\%}$$

$$P_{\text{NEW}} = P_0 \left( 1 + \frac{\Delta P_B}{P_B} \right) = 104.265 \times (1.021) = 106.455$$

## Modified Duration

Is an adjusted measure of the Macaulay duration that produces a more accurate estimate of how much the percentage change in the price of a bond will be per 100 basis points change in the interest rate.

$$\text{Modified Duration (D}^*) = \frac{\text{Macaulay Duration}}{(1 + \text{yield} / k)}$$

### Where

Yield is the yield-to-maturity of the bond

K is the number of periodic payment (compounding) periods per year

# The arbitrage-free Approach to Bond Valuation

The traditional valuation approach is deficient because it uses a single discount rate (the appropriate YTM) to find the present value of the future cash flows with no regard given to the timing of those cash flows.

Cash flows received in year 1 on a 20 year bond are discounted at the same rate as the cash flows received in 20 years!

## Arbitrage Free Valuation Model

$$P = \frac{CF_1}{(1 + r_1)} + \frac{CF_2}{(1 + r_2)^2} + \frac{CF_3}{(1 + r_3)^3} + \dots + \frac{CF_n}{(1 + r_n)^n}$$

This model treats each separate cash flow paid by a fixed-income security as if it were a stand-alone zero-coupon bond. These discount rates are called **spot rates**.

## Example

Give the following Treasury spot rates, calculate the arbitrage-free value of a 5% coupon, 2 year treasury note.

| Maturity  | Spot Rate |
|-----------|-----------|
| 0.5 years | 4.0%      |
| 1.0       | 4.4%      |
| 1.5       | 5.0%      |
| 2.0       | 5.2%      |

The arbitrage-free price of the note is:

$$P = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_2)^2} + \frac{CF_3}{(1+r_3)^3} + \dots + \frac{CF_n}{(1+r_n)^n}$$

$$P = \frac{\$2.50}{(1.020)} + \frac{\$2.5}{(1.022)^2} + \frac{\$2.5}{(1.025)^3} + \frac{\$102.5}{(1.026)^4} = \$99.66 \text{ per } \$100 \text{ of par value}$$

# Contents

Overview equity returns

Valuation of Equity Securities

Estimating the Intrinsic Value

# Asset classes and subcategories

| Equities  | Fixed Income   | Cash  | Alternative Assets  |
|---|--|---|---|
| <b>UK Equities</b> <ul style="list-style-type: none"> <li>- Large capitalisation</li> <li>- Mid capitalisation</li> <li>- Small capitalisation</li> <li>- Micro capitalisation</li> <li>- Growth</li> <li>- Value</li> <li>- Blend (Value and Growth)</li> <li>- Preference shares</li> <li>- Options and futures</li> </ul> <b>Other Developed Markets</b> <ul style="list-style-type: none"> <li>- North America</li> <li>- Europe</li> <li>- Japan</li> <li>- Options and futures</li> </ul> <b>Emerging Markets</b> <ul style="list-style-type: none"> <li>- Africa</li> <li>- Asia ex Japan</li> <li>- Emerging Europe</li> <li>- Latin America</li> <li>- Middle East</li> <li>- Options and futures</li> </ul> | <b>UK Fixed Income</b> <ul style="list-style-type: none"> <li>- UK Treasury bonds</li> <li>- Municipal</li> <li>- Corporate</li> <li>- Mortgage-backed</li> <li>- Asset-backed</li> <li>- Options and futures</li> </ul> <b>High Yield</b><br><b>Convertible Securities</b><br><b>Other Developed Markets</b> <ul style="list-style-type: none"> <li>- North America</li> <li>- Europe</li> <li>- Japan</li> <li>- Options and futures</li> <li>- Interest rate swaps</li> </ul> <b>Emerging Markets</b> <ul style="list-style-type: none"> <li>- Africa</li> <li>- Asia ex Japan</li> <li>- Emerging Europe</li> <li>- Latin America</li> <li>- Middle East</li> <li>- Options and futures</li> </ul> | <b>Cash</b> <ul style="list-style-type: none"> <li>- Physical holdings</li> <li>- Bank balance</li> <li>- UK Treasury bills</li> <li>- Municipal notes</li> <li>- Commercial papers</li> <li>- Certificates of deposit</li> <li>- Repurchase agreement</li> <li>- Banker acceptances</li> <li>- Non UK instruments</li> </ul> | <b>Commodities</b> <ul style="list-style-type: none"> <li>- Commodity trading advisors (CTAs)</li> <li>- Physicals: Agricultural, metal and oil</li> <li>- Options and futures</li> </ul><br><b>Hedge Funds</b> <ul style="list-style-type: none"> <li>- Event driven</li> <li>- Relative value</li> <li>- Market neutral</li> <li>- Long - short</li> <li>- Global macro</li> </ul><br><b>Private Equity</b> <ul style="list-style-type: none"> <li>- Leveraged Buyouts</li> <li>- Venture Capital</li> <li>- Non UK</li> </ul><br><b>Real Estate</b> <ul style="list-style-type: none"> <li>- Residential</li> <li>- Commercial</li> <li>- REITs (Real Estate Investment Trusts)</li> </ul><br><b>Art</b> |

# Valuation

At different levels business decisions involves valuation

**Capital Budgeting:** involves consideration of how a particular project will affect firm value.

**Strategic planning:** focuses on how value is influenced by larger sets of actions.

**Security analysts:** conduct valuation to support their buy/sell decisions, and potential acquirers.

## Basics in Valuation Approaches

- perception that markets are **inefficient** and make mistakes in assessing value
- an assumption about **how and when** these inefficiencies will get corrected

In an efficient market, the **market price is the best estimate** of value.

The purpose of any valuation model is then the **justification** of this value.



---

*Valuation objective is to search for “true” value*

Valuations are biased. The question is how much and in which direction.

The direction and magnitude of the bias is *directly proportional* to who pays you and how much you are paid.

# Valuation Approaches

**Discounted Cash Flow:** value of any asset is estimated by computing the PV of the expected cash flows on that asset, discounted back at a rate that reflects the riskiness of the cash flows (measure of the intrinsic value of an asset).

**Relative Valuation:** The value of any asset can be estimated by looking how similar assets are priced in the market place.

## Academic Studies

Mainly focus on the comparison of **two model approaches**:

- Discounted Dividends
- Discounted Cash Flows

Ratios or multiple based models are discussed in isolation or in addition of the three previous models.

## Main valuation models:

**Discounted Dividends:** This approach expresses the value of firm's equity as the present value of forecasted future dividends.

**Discounted Cash Flow (DCF):** involves detailed production of multiple year forecasts of cash flows. Cash Flows are then discounted at the firm's estimated cost of capital to arrive at an estimated present value.

**\*Discounted Abnormal Earnings:** Value of firm's equity is expressed as the sum of its book value and the present value of the forecasted abnormal earnings.

**\*Discounted abnormal earnings growth:** Value of the firm's equity as the sum of its capitalized next-period earnings forecast and the present value of forecasted abnormal earnings growth beyond the next period.

**\*Real Options:** Contingent Claim (Option) Valuation

---

**Valuation based on price multiples:** Current measure of performance or single forecast of performance is converted into a value by applying an appropriate **price multiple derived from the value of comparable firms.**

**Example:** firm value can be estimated by applying a **price-to-earnings ratio** to a forecast of the firm's earnings for the coming year. Other commonly used multiples include **price-to-book ratios and price-to-sales ratios.**

## Discounted Cashflow Valuation

$$\text{Value} = \sum_{t=1}^n \frac{CF_t}{(1+r)^t}$$

Where

- $CF_t$  is the cash flow in period  $t$ ,
- $r$  is the discount rate appropriate given the riskiness of the cash flow, and
- $t$  is the life of the asset.

For an asset to have value, the expected cash flows have to be **positive** some time over the **life of the asset**.

Assets that generate cash flows early in their life will **be worth more than assets that generate cash flows later**; the later may however have greater growth and higher cash flows to compensate.

# Characteristics of Ordinary Shares

Ordinary shares typically provide investors with an infinite stream of uncertain cash flows or dividends -  $D_1, D_2, \dots, D_n, \dots$ . The price of ordinary shares today is the present value of all future expected dividends discounted at the “appropriate” required rate of return (or discount rate)

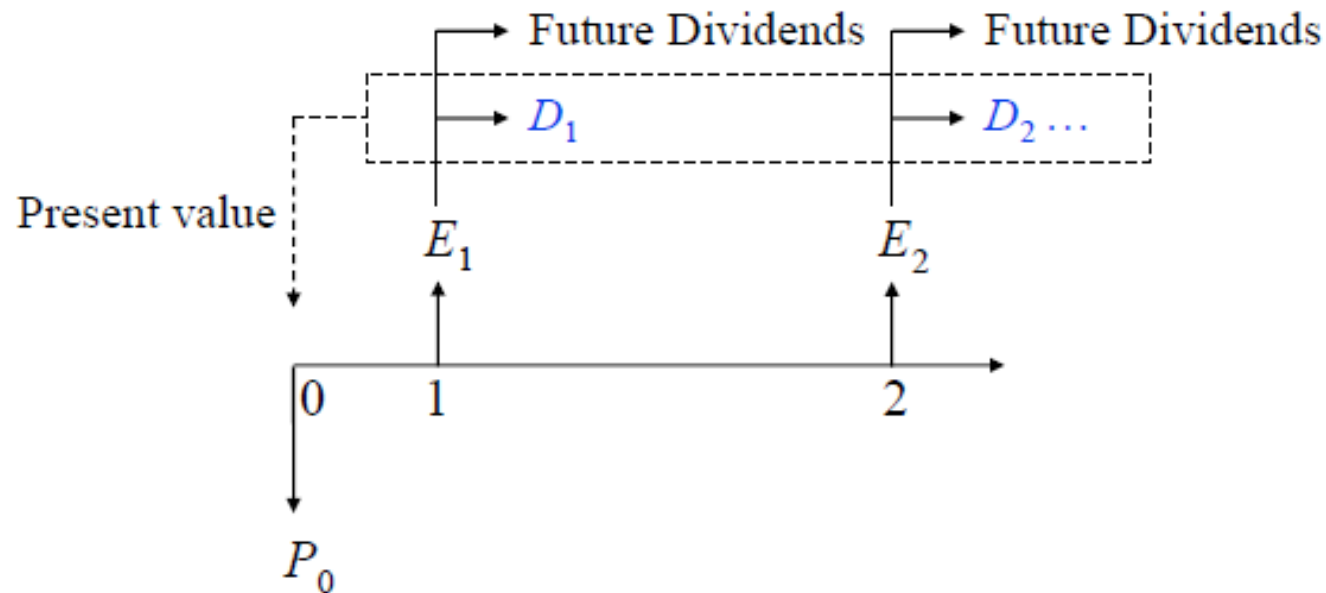
$$P_0 = \sum_{t=1}^N \frac{CF_t}{(1 + k_e)^t}$$

where  $k_e$  is the rate of return required by investors for the time value and risk associated with the security's cash flows ( $CF_t$ )

What cash flows are relevant?

# Characteristics of Ordinary Shares

Need to consider dividends, which are paid from earnings





## Pricing Ordinary Shares

In a one period framework, the stock price is equal to the sum of the next period's dividend and the expected price discounted at the required return

$$P_0 = \frac{D_1 + P_1}{1 + k_e} \quad \text{and} \quad P_1 = \frac{D_2 + P_2}{1 + k_e} \quad \text{and} \quad P_t = \frac{D_{t+1} + P_{t+1}}{1 + k_e}$$

Over any period, the expected rate of return ( $k_e$ ) is

$$k_e = \frac{D_{t+1} + P_{t+1}}{P_t} - 1 = \frac{D_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$k_e = \text{Dividend yield} + \text{Percent price change}$

## Pricing Ordinary Shares

**Example:** The price and dividend per share for OzCo Ltd next period are expected to be \$5.00 and \$0.50, respectively. If the expected return on these shares is 10% p.a. what is OzCo's current stock price? If the current price changes to \$4.80 what has happened to the expected return on these shares? Why?

Given:  $P_{t+1} = \$5.00$ ,  $D_{t+1} = \$0.50$  and  $k_e = 10\%$

$$P_t = \frac{0.50 + 5.00}{1 + 0.10} = \$5.00$$

If the current price changes to \$4.80, the expected return rises to

$$k_e = \frac{0.50 + 5.00}{4.80} - 1 = 14.6\%$$

**Note that prices and expected returns are inversely related**

## Pricing Ordinary Shares

The stock price over periods 0, 1 and 2 can be written as

$$P_0 = \frac{D_1 + P_1}{1 + k_e} \quad \text{and} \quad P_1 = \frac{D_2 + P_2}{1 + k_e} \quad \text{and} \quad P_2 = \frac{D_3 + P_3}{1 + k_e}$$

Substituting  $P_2$  and  $P_1$  recursively, we get  $P_0$  as

$$P_0 = \frac{D_1}{1 + k_e} + \frac{D_2}{(1 + k_e)^2} + \frac{D_3 + P_3}{(1 + k_e)^3}$$

- The **current dividend** ( $D_0$ ) is not relevant to our estimate of the **current price** - all prices estimated are ex-dividend prices
- Ex-dividend prices are prices after the current period's dividend has been paid

## Pricing Ordinary Shares

Extending the above process to H periods, we get

$$P_0 = \frac{D_1}{1 + k_e} + \frac{D_2}{(1 + k_e)^2} + \frac{D_3}{(1 + k_e)^3} + \dots + \frac{D_H + P_H}{(1 + k_e)^H}$$

$$P_0 = \sum_{t=1}^H \frac{D_t}{(1 + k_e)^t} + \frac{P_H}{(1 + k_e)^H}$$

$$\text{As } H \rightarrow \infty, PV(P_H) \rightarrow 0$$

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_e)^t}$$

Market analysts often make simplifying assumptions about future expected dividends

## Constant Dividend Growth Model

A constant growth rate in dividends implies

$$D_2 = D_1(1 + g), D_3 = D_1(1 + g)^2, \dots, D_t = D_1(1 + g)^{t-1}$$

Substituting the above dividends in the expression for  $P_0$  we get

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_e)^t} = \sum_{t=1}^{\infty} \frac{D_1(1 + g)^{t-1}}{(1 + k_e)^t}$$

$$\text{As } t \rightarrow \infty, \sum_{t=1}^{\infty} \frac{(1 + g)^{t-1}}{(1 + k_e)^t} \rightarrow \frac{1}{k_e - g}$$

The above expression simplifies to

$$P_0 = \frac{D_1}{k_e - g} \quad \text{where } k_e > g \quad \text{or} \quad P_t = \frac{D_{t+1}}{k_e - g}$$

## Constant Dividend Growth Model

**Application 1:** Assume that year 0 is the end of 2004. Telstra Ltd is expected to pay annual dividends of \$0.26 in 2005 (year 1). Assume that this dividend grows at an annual rate of 5% in the foreseeable future and investors require a return of 10% p.a.

- a) Estimate Telstra's stock price today
- b) What is Telstra's price expected to be at the end of 2005?
- c) Based on Telstra's current price of \$4.75, what is the constant dividend growth rate implied?
- d) How sensitive is the price estimate to different assumptions regarding the growth in dividends over time?
- e) How sensitive is the price estimate to different assumptions regarding the required rate of return?

# Constant Dividend Growth Model

Given:  $D_1 = 0.26$ ,  $g = 0.05$  and  $k_e = 0.10$

a)  $P_0 = 0.26 / (0.10 - 0.05) = \$5.20$

b)  $P_1 = D_2 / (k_e - g) = 0.26(1.05) / (0.10 - 0.05) = \$5.46$  (a 5% rise)

c)  $k_e = D_1 / P_0 + g$  or  $g = k_e - D_1 / P_0$   
 $g = 0.10 - 0.26 / 4.75 = 0.0453$  or 4.5%

d) Sensitivity of Telstra's price to changes in expectations of  $g$

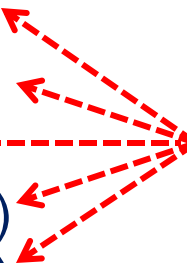
$g = 3\%: P_0 = 0.26 / (0.10 - 0.03) = \$3.71$  (-28.7%)

$g = 4\%: P_0 = 0.26 / (0.10 - 0.04) = \$4.33$  (-16.7%)

$g = 5\%: P_0 = 0.26 / (0.10 - 0.05) = \$5.20$

$g = 6\%: P_0 = 0.26 / (0.10 - 0.06) = \$6.50$  (+25.0%)

$g = 7\%: P_0 = 0.26 / (0.10 - 0.07) = \$8.67$  (+66.7%)



# Constant Dividend Growth Model

Sensitivity of Telstra's price to changes in  $k_e$

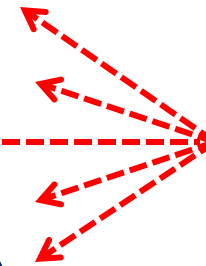
$$k_e = 8\%: P_0 = 0.26 / (0.08 - 0.05) = \$8.67 (+66.7\%)$$

$$k_e = 9\%: P_0 = 0.26 / (0.09 - 0.05) = \$6.50 (+25.0\%)$$

$$k_e = 10\%: P_0 = 0.26 / (0.10 - 0.05) = \$5.20$$

$$k_e = 11\%: P_0 = 0.26 / (0.11 - 0.05) = \$4.33 (-16.7\%)$$

$$k_e = 12\%: P_0 = 0.26 / (0.12 - 0.05) = \$3.71 (-28.7\%)$$



- Price estimates are very sensitive to assumptions regarding future dividends, growth in dividends and required rate of return
- It is often more realistic to assume a variable growth rate in dividends with higher initial growth in dividends followed by subsequent lower (or zero) growth in dividends



## Variable Dividend Growth Model

**Application 2:** In the previous application, assume that Telstra's current dividend of \$0.25 grows at 10% for 3 years and then stabilizes at 5% thereafter. What price should Telstra shares sell for today if the required rate of return remains at 10%?

**Three step procedure to estimate  $P_0$**

**Step 1:** Compute the dividends up to the point where  $g$  becomes constant (over years 1 to 4 in this case)

**Step 2:** Compute the price at the end of the year after which dividends grow at a constant rate (year 3 in this case)

**Step 3:** Add the present value of dividends from Step 1 to the present value of the price from Step 2 to get  $P_0$

# Variable Dividend Growth Model

Given:  $D_0 = \$0.25$ ,  $g_1 = 10\%$  over years 1 - 3,  
 $g_2 = 5\%$  from year 4 onwards,  $k_e = 10\%$

Step 1: Obtain dividends up to where  $g$  becomes constant

$$D_1 = 0.2500(1.10) = \$0.2750$$

$$D_2 = 0.2750(1.10) = \$0.3025$$

$$D_3 = 0.3025(1.10) = \$0.3328$$

$$D_4 = 0.3328(1.05) = \$0.3494$$

Step 2: Obtain  $P_n$  (after which dividend growth is constant)

$$P_3 = D_4 / (k_e - g_2) = 0.3494 / (0.10 - 0.05) = \$6.988$$

Step 3: Add the present values of dividends and  $P_n$  to get  $P_0$

$$P_0 = D_1 / (1 + k_e) + D_2 / (1 + k_e)^2 + (D_3 + P_3) / (1 + k_e)^3$$

$$P_0 = 0.2750 / 1.1 + 0.3025 / 1.1^2 + (0.3328 + 6.988) / 1.1^3 = \$6.00$$

## Equity versus Firm Valuation

- Value just the **equity stake** in the business.
- Value the entire business, which includes, besides equity, the **other claimholders in the firm**

# Equity Valuation

The value of equity is obtained by:

Discounting expected *cashflows to equity* (the residual cashflows after meeting all expenses, tax obligations and interest and principal payments) *at the cost of equity* (required return to shareholders).

$$\text{Value of Equity} = \sum_{t=1}^n \frac{\text{CF to Equity}_t}{(1+k_e)^t}$$

CF to Equity<sub>t</sub> = Expected Cashflow to Equity in period t

k<sub>e</sub> = Cost of Equity

**Note:** The dividend discount model is a specialized case of equity valuation

## Firm Valuation

The value of the firm is obtained by:

Discounting expected *cashflows to the firm* (the residual cashflows after meeting all operating expenses and taxes, but prior to debt payments) *at the WACC* (cost of the different components of financing used by the firm, weighted by their market value proportions)

$$\text{Value of Firm} = \sum_{t=1}^n \frac{\text{CF to Firm}_t}{(1+\text{wacc})^t}$$

### Adjusted Present Value approach:

Firm Value = Unlevered Firm Value + PV of tax benefits of debt - Expected Bankruptcy Cost

# Generic DCF Valuation Model

Firm is a stable growth  
Grows at constant rate  
forever

## Cash Flows

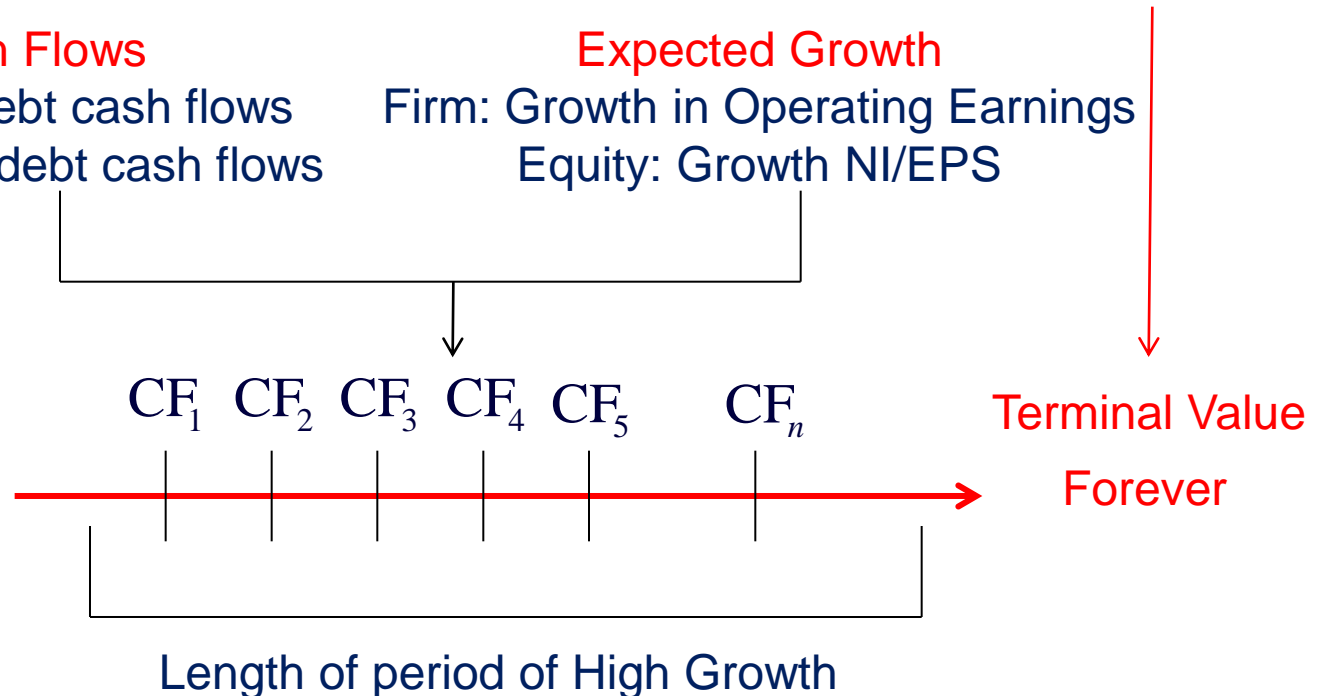
Firm: Pre-debt cash flows  
Equity: After-debt cash flows

## Expected Growth

Firm: Growth in Operating Earnings  
Equity: Growth NI/EPS

## Value

Firm: Value of Firm  
Equity: Value of Equity



## Discount Rate

Firm: Cost of Capital  
Equity: Cost of Equity

# Estimating the Intrinsic Value

Most investment valuation involves:

Estimating the **amount** and **timing** of the cash flows

Interest, dividends, and capital gains

Estimating the **growth rate** of returns

common stock / Real estate  
(Can grow over time)

Preferred Stock / Bonds  
(fixed)

Applying an appropriate discount rate to the cash flows to estimate the investment's intrinsic value

The required return for the **risk** assumed **Amount & timing of cash flow**

Comparing the intrinsic value to the **market price**

If estimated intrinsic value > market price, then **BUY!**

## Preferred Stock

# Common Stock

## Constant (Gordon) Growth DDM • $g \leq g_{\text{economy}}$

Cesario MATEUS 2015