Fixed Income Investment

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(Morning)

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Lecture 1

1. A closer look at the different asset classes
2. Bond Market Overview
3. Features in Debt Securities
4. Risks Associated with Investing in Bonds
5. Yield measures, Spot rates and Forward Rates
6. Introduction to the Valuation of Debt Securities
7. The arbitrage-free Approach to Bond Valuation
8. Profiting from Arbitrage Opportunities: Stripping and Reconstituting Bonds
9. Holding Period Yield

A closer look at the different asset classes

What is an Investment?

- Commitment of funds to assets that will be held over the future time period.

- Real assets vs. financial assets
  - Real assets are physical, tangible assets such as gold or real estate
  - Financial assets are paper (electronic) claims on some issuer (e.g. corporation or government)
  - Type of financial assets investors are mainly interested in are marketable securities

- Investment refers in general to financial assets and in particular to marketable securities.
  - Marketable securities are financial assets that are easily traded on the organized exchanges
  - Impersonal trading

Why Do We Invest?

The purpose is to increase one's wealth

- Wealth = current income or funds + present value of all income in the future
- We are concerned with monetary wealth

Get a return on the money, do not hold cash

- Opportunity cost
- Inflation
- Purchasing power diminishes

Protect yourself from inflation, taxes, etc. and **MAKE MONEY**!!!

One Classification of Financial Assets

Assets with random cashflows:

- Equity (share, common stock)
- Type of financial asset that enables the holder to receive dividend payments, after creditors and preference shareholders are paid, and any capital gain (loss) that may arise at the disposal of the asset.
- Equity holders are residual claimants of a company
- Shares are irredeemable, thus having an indefinite life
- A share represents the unit of ownership in the company
- Not known if investor will receive dividend, as dividends are paid out of earnings

Assets with known cashflows

- Fixed income securities (Money Market Instruments, Bonds)
- Characteristics: coupon rate, principal amount, time to maturity
- Bond is a promise made by a bond issuer to make regular coupon payments and repay a principal amount at the maturity date to the bondholder.
- A failure to fulfill that promise results in a default of a bond

Assets with contingent cashflows

- Derivative securities: forwards, futures, options and swaps
- Cashflows are dependent on the price movements of the underlying assets

Asset classes and subcategories

Equities	Fixed Income	Cash	Alternative Assets
UK Equities - Large capitalisation - Mid capitalisation - Small capitalisation - Micro capitalisation - Growth - Value - Blend (Value and Growth) - Preference shares - Options and futures **Other Developed Markets** - North America - Europe - Japan - Options and futures **Emerging Markets** - Africa - Asia ex Japan - Emerging Europe - Latin America - Middle East - Options and futures	**UK Fixed Income** - UK Treasury bonds - Municipal - Corporate - Mortgage-backed - Asset-backed - Options and futures **High Yield** **Convertible Securities** **Other Developed Markets** - North America - Europe - Japan - Options and futures - Interest rate swaps **Emerging Markets** - Africa - Asia ex Japan - Emerging Europe - Latin America - Middle East - Options and futures	**Cash** - Physical holdings - Bank balance - UK Treasury bills - Municipal notes - Commercial papers - Certificates of deposit - Repurchase agreement - Banker acceptances - Non UK instruments	**Commodities** - Commodity trading advisors (CTAs) - Physicals: Agricultural, metal and oil - Options and futures **Hedge Funds** - Event driven - Relative value - Market neutral - Long - short - Global macro **Private Equity** - Leveraged Buyouts - Venture Capital - Non UK **Real Estate** - Residential - Commercial - REITs (Real Estate Investment Trusts) **Art**

Fixed Income

Rationale for Investment	Risks and Concerns
Senior claim	Lower returns than equity
Low risk	Interest rate risk
Higher return than cash	Inflation risk
Portfolio diversifier (Low correlation)	Credit risk
	Reinvestment risk
	Prepayment risk (Callable)

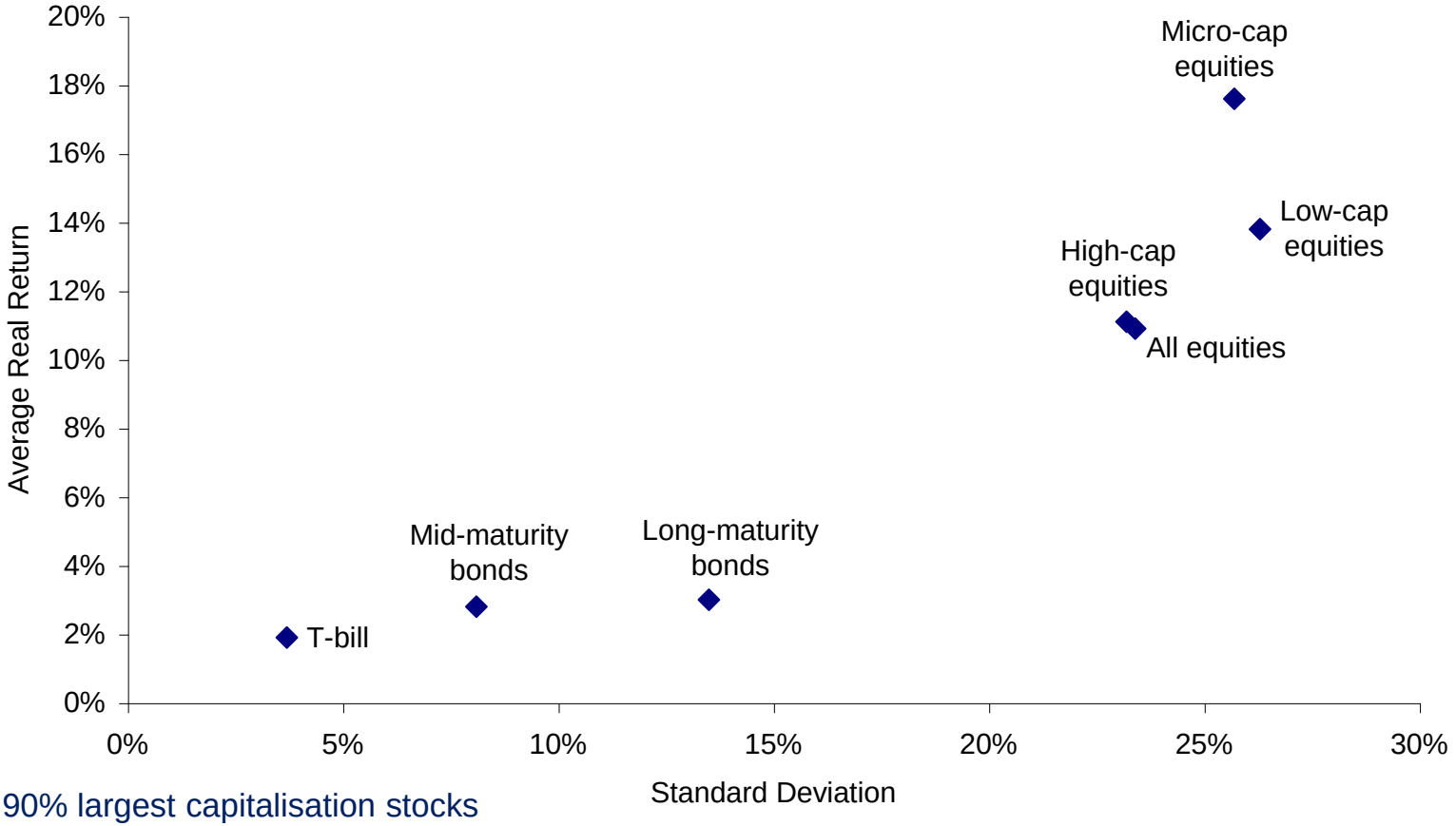
High yield fixed income

Rationale for Investment	Risks and Concerns
High return	Issued to finance leveraged buyouts or ex-investment grade bond consequently downgraded
Lower risk than equity	Credit risk
Irrational (Inefficient) pricing: Possibility to beat the market	Liquidity risk
Claim senior to equity	

Convertible preference shares and convertible bonds

Rationale for Investment	Risks and Concerns
Equity-debt hybrid	Prepayment risk (Callable)
Claim senior to equity	Claim junior to bond
Portfolio diversifier (Low correlation with bonds)	Complicated valuation

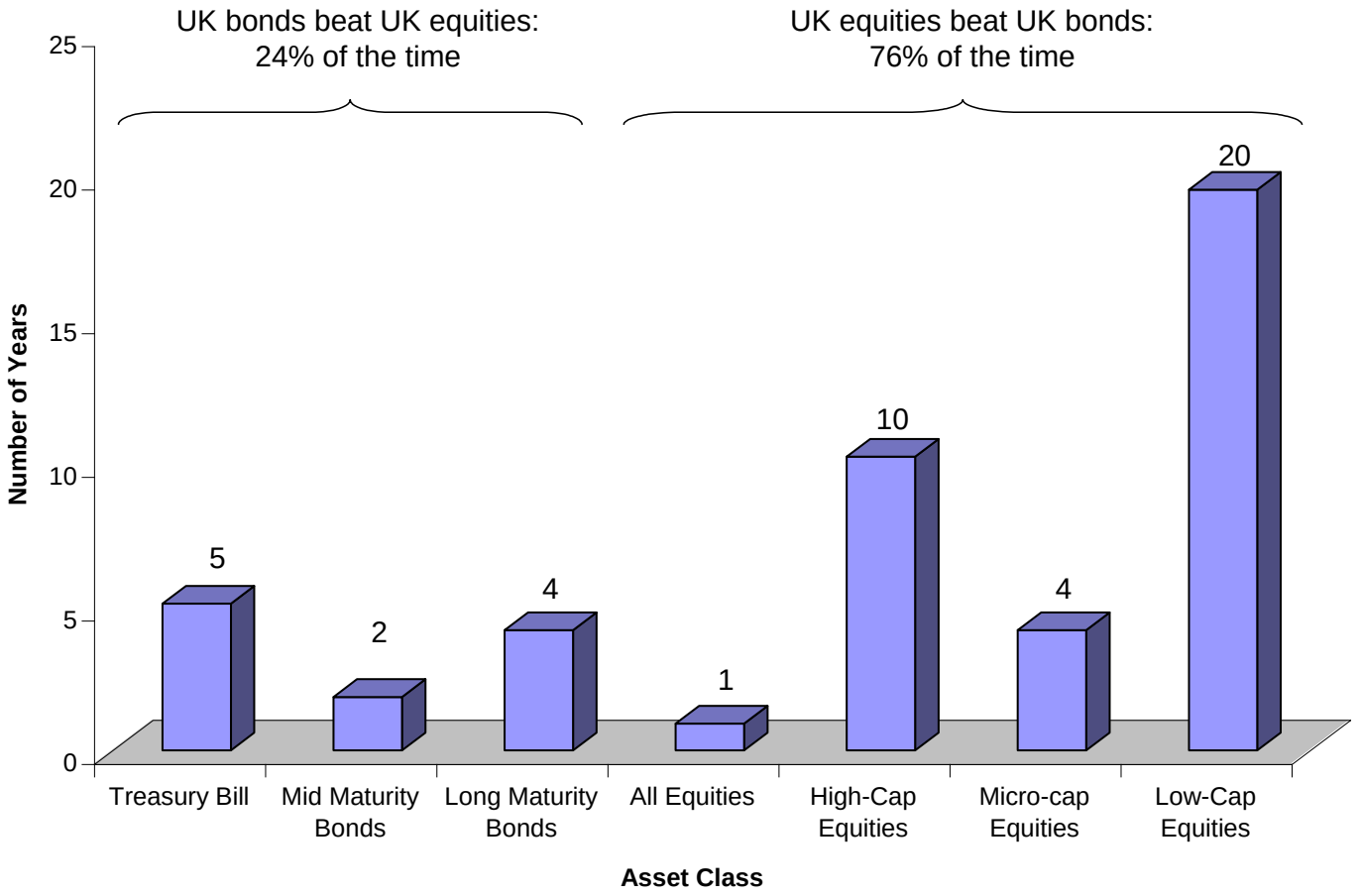
UK financial market real returns and risks: 1955 – 2000

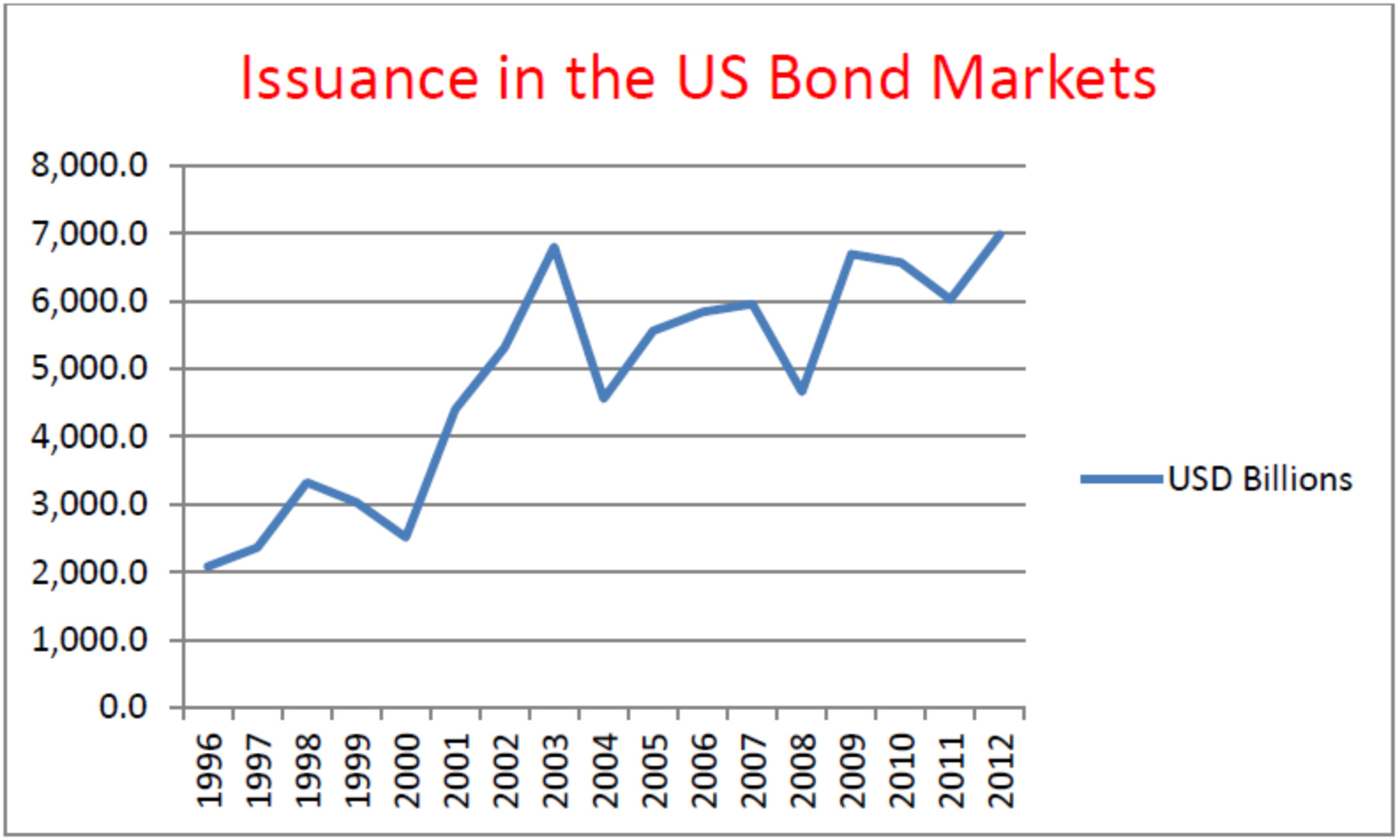


High cap: 90% largest capitalisation stocks
Low cap: Next 9% largest stocks
Micro cap: 1% smallest stocks
Market capitalisation = Number of stocks * Share price

Source: Dimson and Marsh, 2001

Number of years each UK asset class performed the best
(1955 – 2000)




Issuance in the US Bond Markets
8,000.0
7,000.0
6,000.0
5,000.0
4,000.0
3,000.0
2,000.0
1,000.0
0.0
1996
1997
1998
1999
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2006
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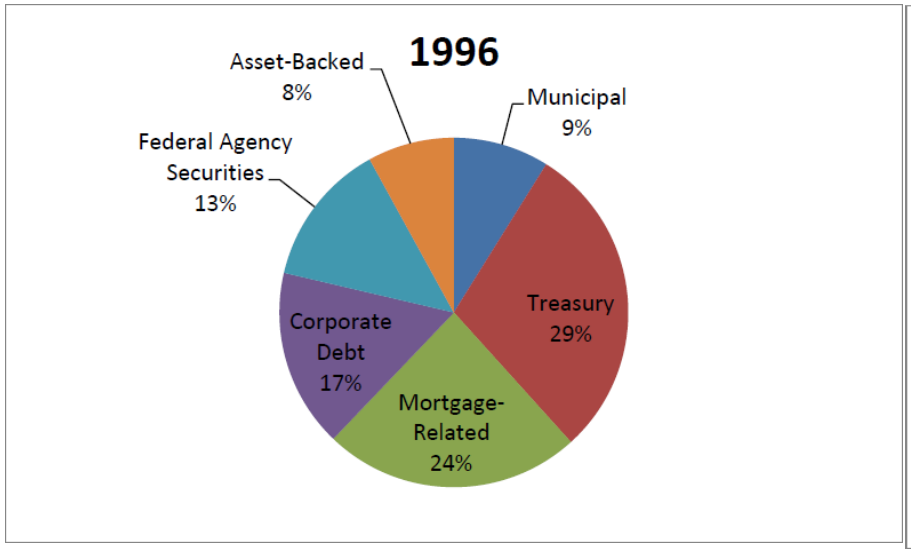
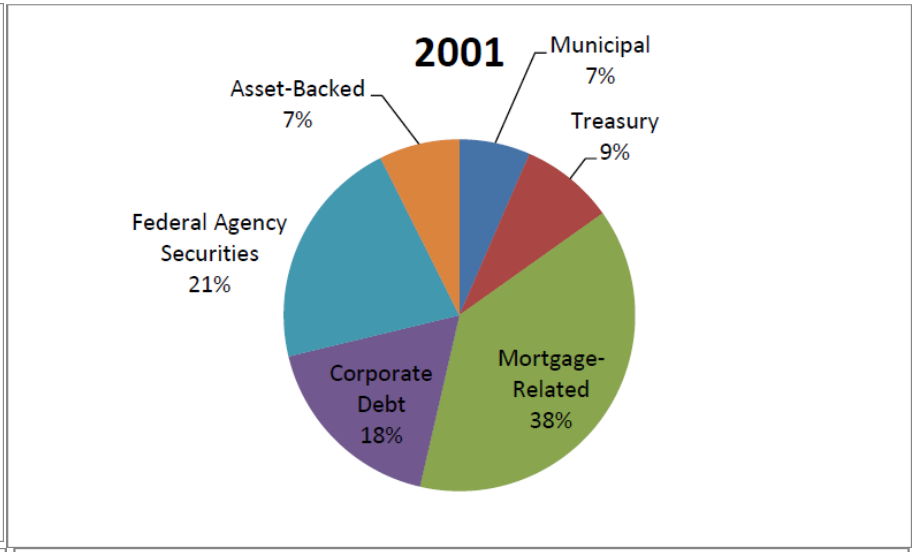
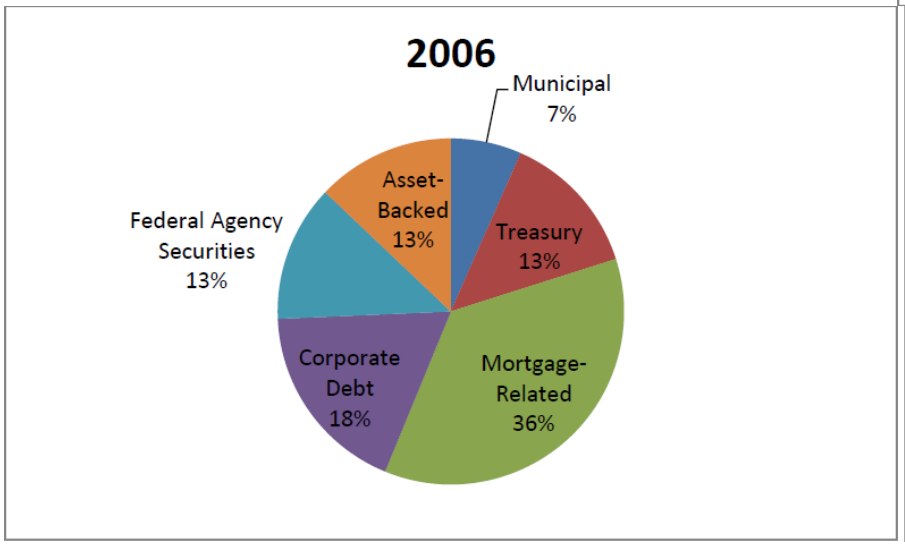
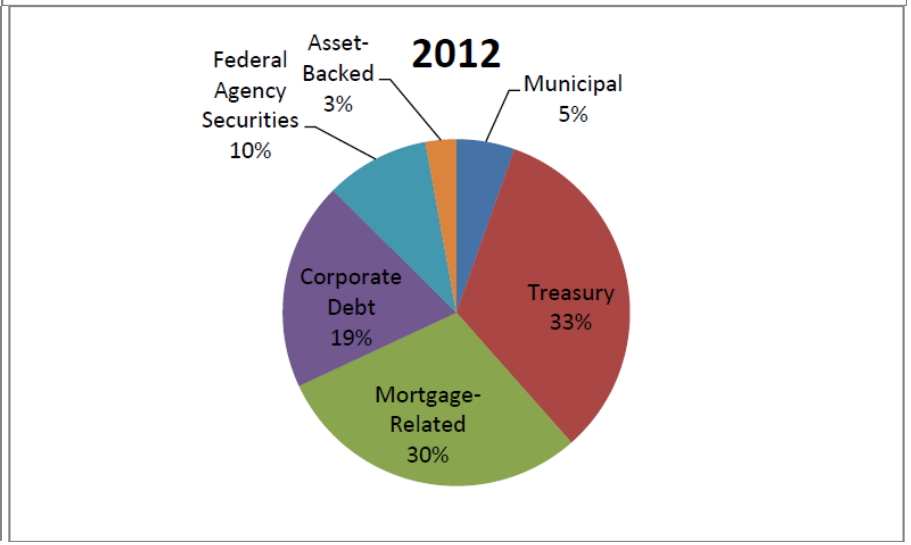
2008
2009
2010
2011
2012
USD Billions

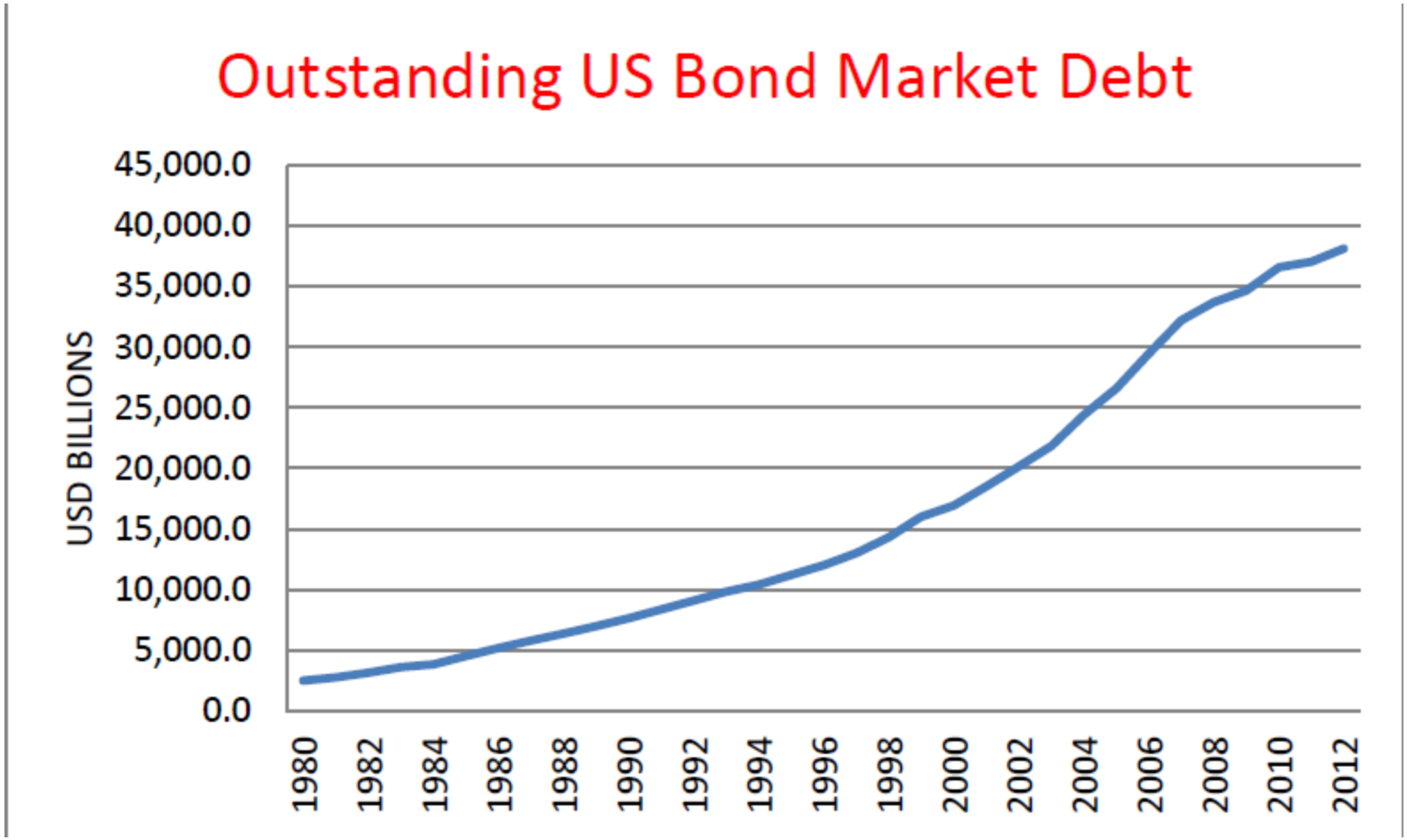


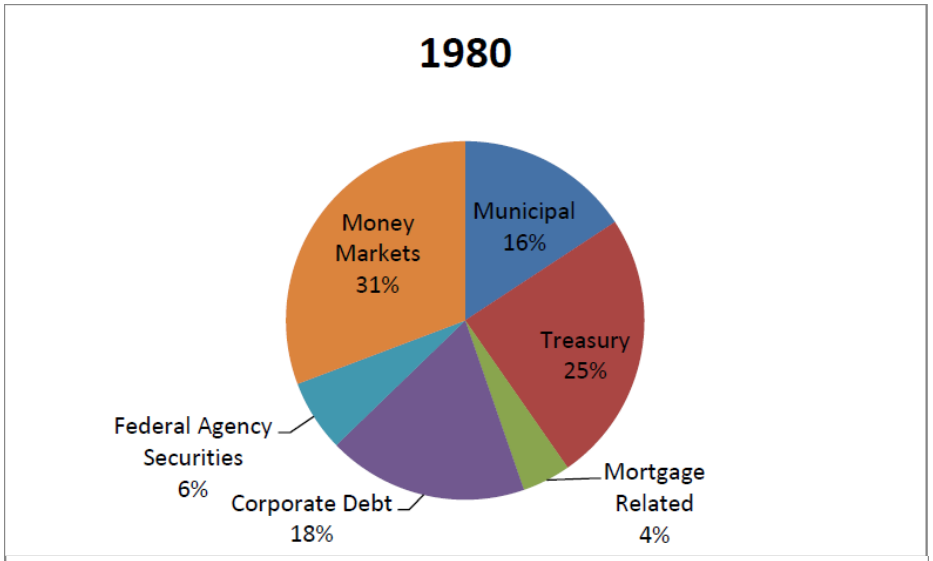


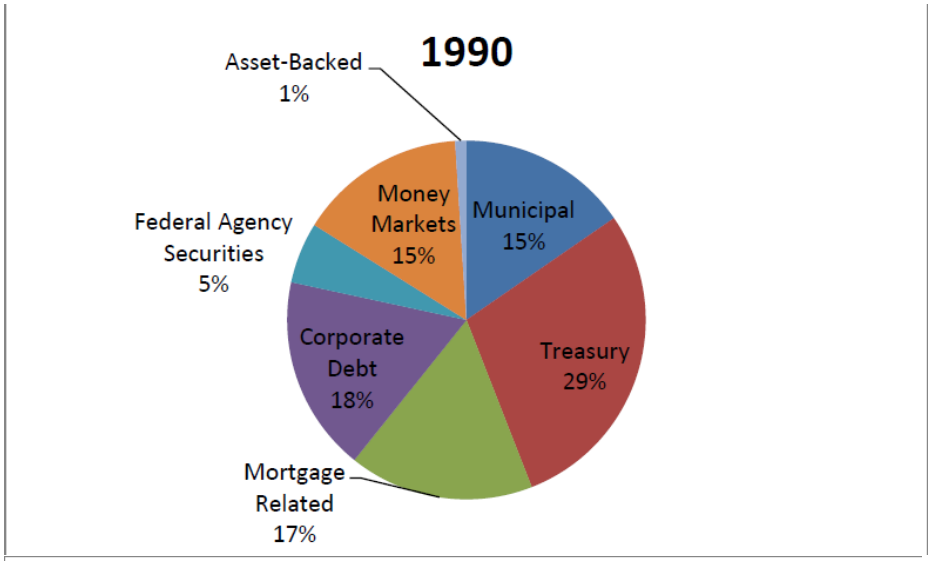


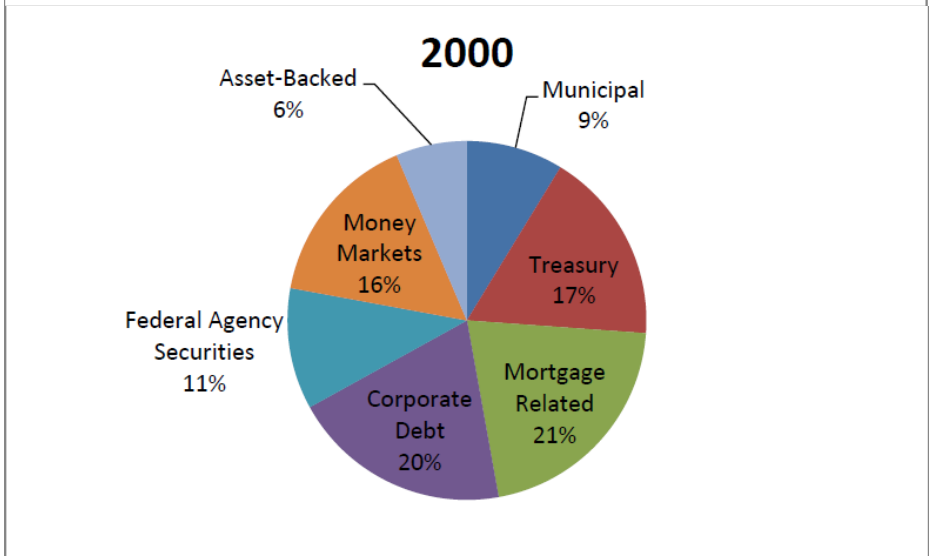


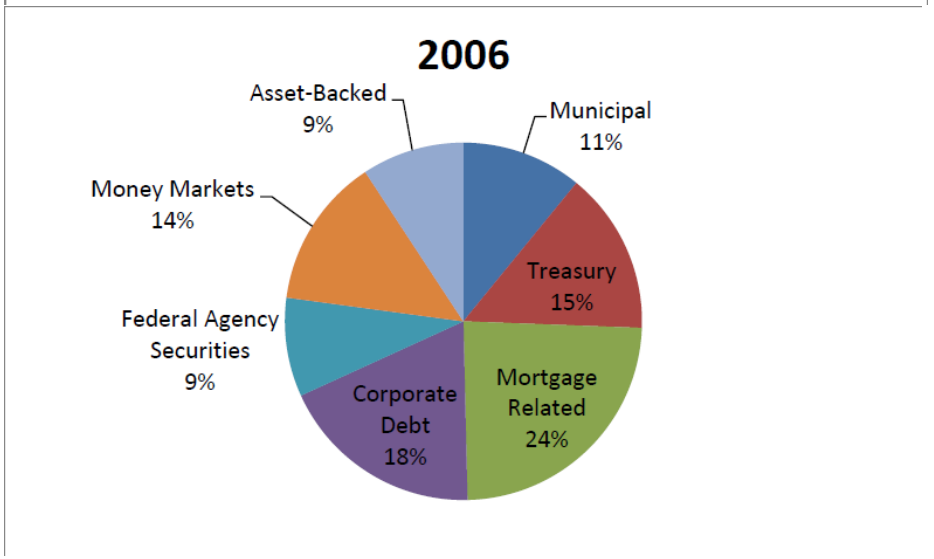
Source: SIFMA (Bloomberg, Dealogic, Thomson Reuters, U.S. Treasury, Fannie Mae, Freddie Mac, Ginnie Mae)

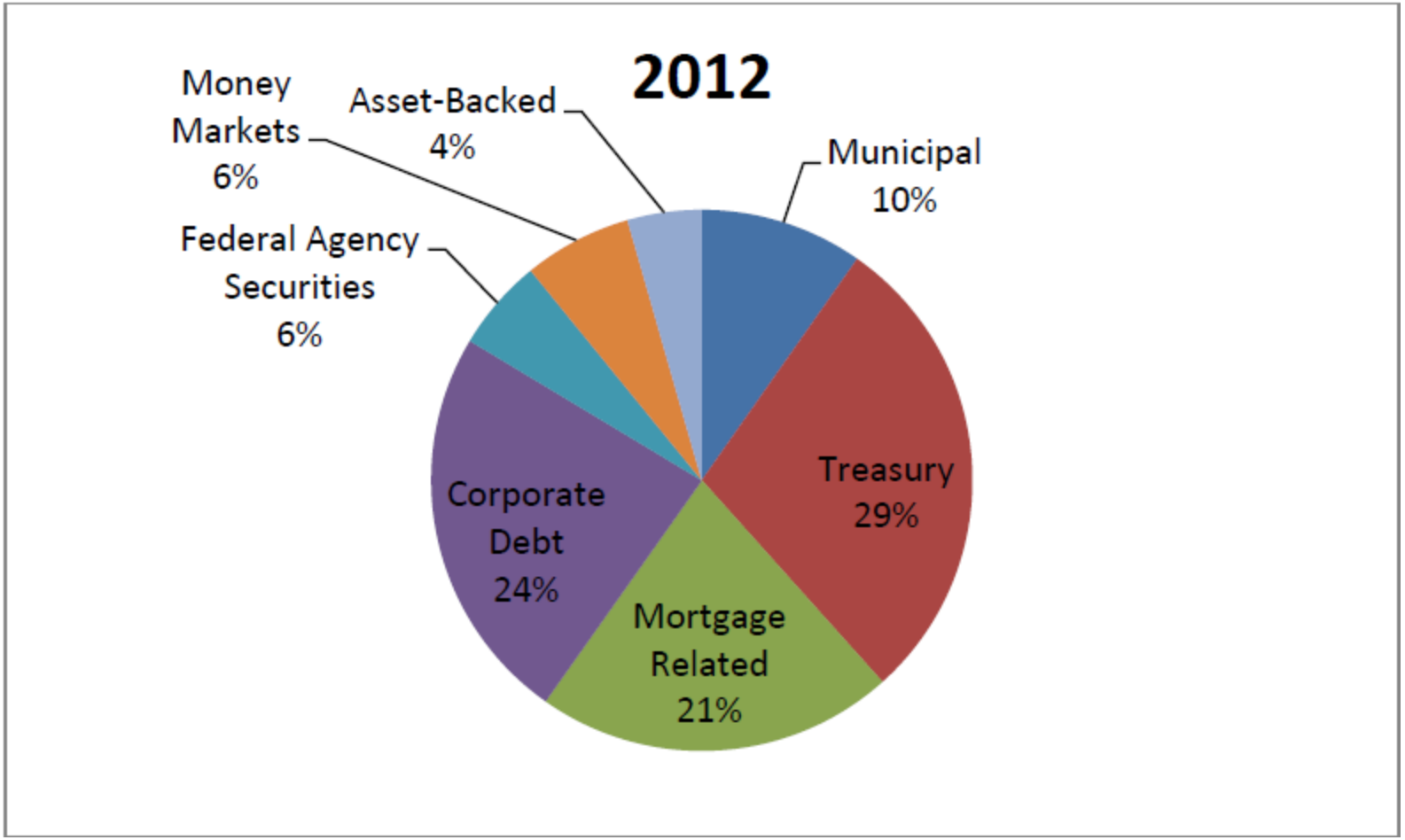

Outstanding US Bond Market Debt
USD BILLIONS
45,000.0
40,000.0
35,000.0
30,000.0
25,000.0
20,000.0
15,000.0
10,000.0
5,000.0
0.0
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012


1980
Money Markets 31%
Municipal 16%
Treasury 25%
Federal Agency Securities 6%
Corporate Debt 18%
Mortgage Related 4%

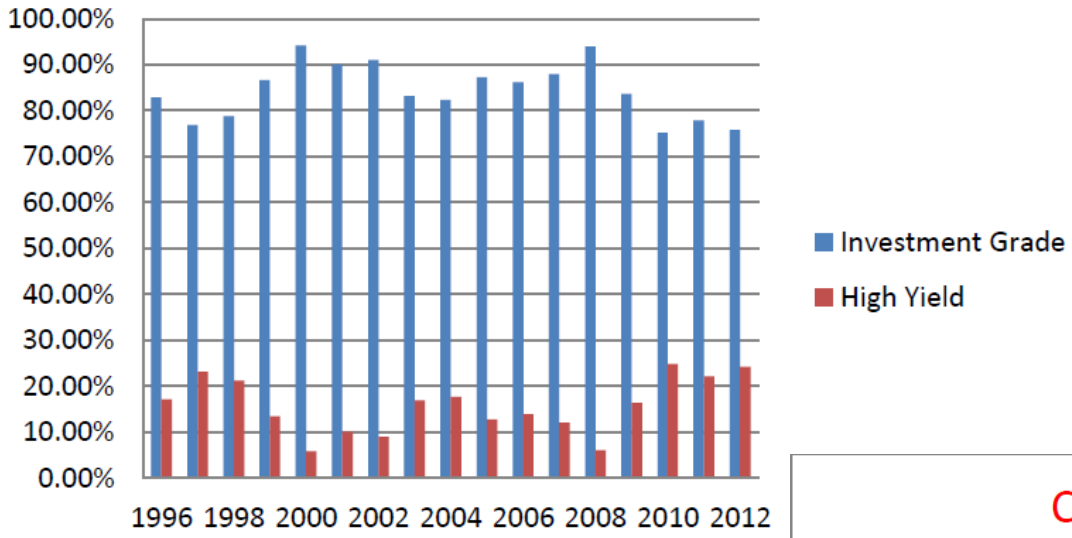

1990
Asset-Backed 1%
Money Markets 15%
Municipal 15%
Federal Agency Securities 5%
Corporate Debt 18%
Treasury 29%
Mortgage Related 17%


2000
Asset-Backed 6%
Municipal 9%
Money Markets 16%
Treasury 17%
Federal Agency Securities 11%
Corporate Debt 20%
Mortgage Related 21%

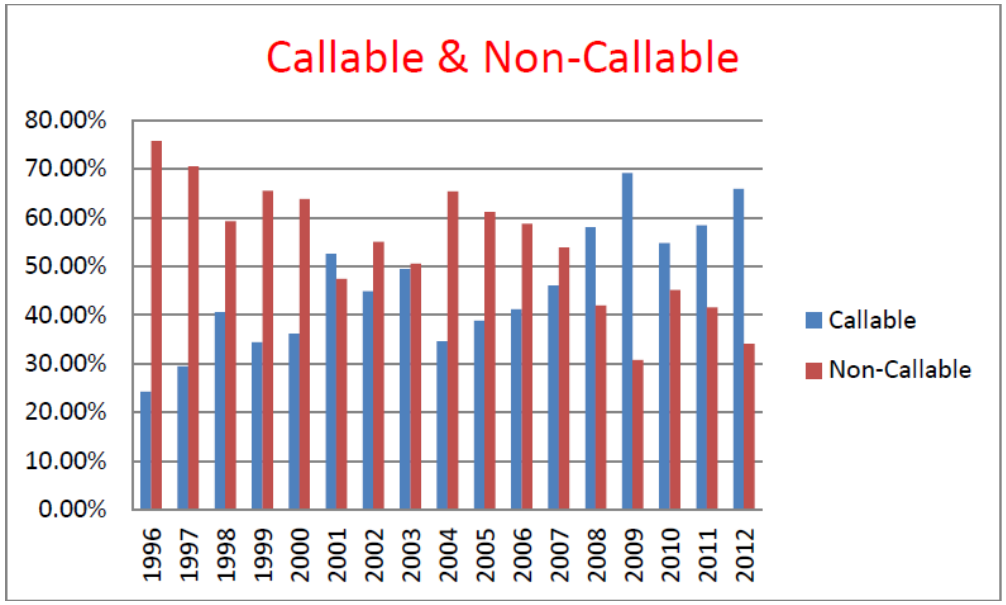

2006
Asset-Backed 9%
Municipal 11%
Money Markets 14%
Treasury 15%
Federal Agency Securities 9%
Corporate Debt 18%
Mortgage Related 24%


2012
Money Markets 6%
Asset-Backed 4%
Municipal 10%
Federal Agency Securities 6%
Corporate Debt 24%
Treasury 29%
Mortgage Related 21%

Investment Grade vs. High Yield



Callable & Non-Callable



Features of Debt Securities

Fixed income security: financial obligation of an entity that promises to pay a specified sum of money at specified future dates.

Issuer of the security: Entity that promises to make the payment (e.g. US government, French government, the city of Rio de Janeiro in Brazil, Corporation such Coca-Cola, Sport Institutions such Porto Football Club or supranational governments such as the World Bank.

Fixed Income securities (two general categories): debt obligations and preferred stock

Debt Obligations: bonds, mortgage-backed securities, asset backed securities and bank loans.

Bond indenture (also trust indenture or deed of trust): legal document issued to lenders and describes key terms such as the interest rate, Maturity date, convertibility, pledge, promises, representations, covenants, and other terms of the bond offering.

Bond Covenant: designed to protect the interests of both parties. Negative or restrictive covenants forbid the issuer from undertaking certain activities; positive or affirmative covenants require the issuer to meet specific requirements

Maturity:

Term to maturity: number of years the debt is outstanding or the number of years remaining prior to final principal payment

Maturity date: date that the debt will cease to exist

Type	Maturity
Short-term	1 to 5 years
Intermediate-term	5 – 12 years
Long-term	More than 12 years

Par Value: Amount that the issuer agrees to repay the bondholder at or by the maturity date (principal value, face value, redemption value or maturity value).

Because bonds have different par values, the practice is to quote bonds as a percentage of its par value.

Quoted Price	Price per $1of par value (rounded)	Par value	Dollar Price
90 1/2	0.9050	$1,000	905.00
102 3/4	1.0275	$5,000	5,137.50
70 5/8	0.7063	$10,000	7,062.50
113 11/32	1.1334	$100,000	113,343.75

Coupon Rate (nominal rate): is the interest rate that the issuer agrees to pay each year.

Coupon: Annual amount of the interest payments made to bondholders during the term of the bond. Calculated as:

$$coupon = coupon\ rate \times par\ value$$

Example:
6% coupon rate and a par value of $1,000
Coupon (interest payment) = $60

United States (semi-annual instalments), Mortgage and Asset Backed Securities typically pay interest monthly.

Zero-coupon Bonds: the holder realizes interest by buying the bond substantially below its par value

Provisions for Paying off Bonds

Bullet maturity: No principal repayments prior to maturity date.

Amortizing Securities: Schedule of partial payments until maturity (e.g. fixed income securities backed by pool of loans, mortgage backed securities and asset-backed securities).

Sinking Fund: Repayment of the bond may be arranged to repay only a part of the total by the maturity date.

Call provision: guarantee the issuer an option to retire all or part of the issue to the stated maturity date (callable bond).

Convertible bond: grants the bondholder the right to convert the bond for a specified number of shares of common stock.

Put Provision: grants the bondholder the right to sell issue back to the issuer at a specified price on designed dates.

Currency denomination: in the USA, dollar-dominated, nondollar denominated issues and dual-currency issues.

Risks Associated with Investing in Bonds

Interest-rate risk or market risk

As interest rates rise, the price of a bond fall (vice-versa)

If an investor has to sell a bond prior to the maturity date, an increase in interest rates will mean the realization of a loss (i.e. selling the bond below the purchase price).

Example:

Consider a 6% 20-year bond with a face value of $100. if the yield investors require to buy this bond is 6%, the price of this bond would be $100 (selling at par).

If required yield increase to 6.5%, the price of this bond would decline to $94.4479. Thus, for a 50 basis point increase in yield, the bond's price declines by 5.5%. If, instead, the yield declines from 6% to 5.5%, the bond's price will rise by 6.02% to $106.0195.

Coupon rate = yield required by market → price = par value

Coupon rate < yield required by market → price < par value (discount)

Coupon rate > yield required by market → price > par value (premium)

If interest rates increase → price of a bond decreases

If interest rates decrease → price of a bond increases

Bond Features that affect Interest Rate Risk

Maturity: all other factors constant, the longer the bond's maturity, the greater the bond's price sensitivity to changes in interest rates

Coupon Rate: all other factors constant, the lower the coupon rate, the greater the bond's price sensitivity to changes in interest rates

Embedded Options:

Call option: As interest rates decline, the price of a callable bond may not increase as much as an otherwise option-free bond

Price of callable bond = price of option-free bond – price of embedded call option

Yield level: Bond's that trade at a lower yield are more volatile in both percentage price change and absolute price change (as long as the other bond characteristics are the same).

Yield curve risk: bond portfolios have different exposures to how the yield curve shifts.

Call Risk or Prepayment Risk

Issuer can retire or “call” all or part of the issue before the maturity date (Issuer usually retains this right in order to have flexibility to refinance the bond in the future if the market interest rate drops below the coupon rate).

Disadvantages from the investor’s perspective:

1) The cash flow pattern of a callable bond is not known with certainty because it is not known when the bond is called.
2) Because the issuer is likely to call the bonds when interest rates have declined below the bond’s coupon rate, the investor is exposed to reinvestment risk (will have to reinvest the proceeds at a lower interest rate than the bond’s coupon rate)
3) The price appreciation potential of the bond will be reduced relative to an otherwise comparable option-free bond (price compression)

Reinvestment Risk

Risk that the proceeds received from the payment of interest and principal that are available for reinvestment must be reinvested at a lower interest rate than the security that generated the proceeds.

Credit Risk

Three types of credit risk: default risk, credit spread risk and downgrade risk.

Default Risk: Risk that issuer will fail to satisfy the term of the obligations with respect to the timely payment of interest and principal (default rate, recovery rate and expected loss).

Credit Spread Risk: The part of the risk premium or yield spread attributable to default risk. The price performance and the return over some time period will depend on how the credit spread changes.

Downgrade Risk: Risk that the bond issue or issuer credit rating will change.

Three rating agencies in the United States: Moody's Investors Service Inc, Standard &Poor's Corporation and Fitch Ratings

Moody's	S&P	Fitch	Summary Description
Investment Grade – High Credit Worthiness			
Aaa	AAA	AAA	Gilt edge, prime, maximum safety
Aa1	AA+	AA+	High-grade, high credit quality
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Uper-medium grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower-medium Grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	

Moody's	S&P	Fitch	Summary Description
Speculative – Lower Credit Worthiness			
Ba1	BB+		Low grade, speculative
Ba2	BB		
Ba3	BB-		
B1	B	B+	Highly speculative
B2		B	
B3		B-	
Predominantly Speculative, Substantial Risk, or in Default			
Caa	CCC+ CCC	CCC+ CCC	Substantial Risk, in poor standing
Ca	CC	CC	May be in default, very speculative
C	C	C	Extremely speculative
	CI		Income bonds – no interest being paid
	D	DDD DD D	Default

Liquidity Risk

The risk that the investor will have to sell a bond below its indicated value, where the indication is revealed by a recent transaction.

The primary measure of liquidity is the size of the spread between the bid price (the price at which the dealer is willing to buy a security) and the ask price (the price at which a dealer is willing to sell a security).

The wider the bid-ask spread, the greater the liquidity risk.

Exchange Rate or Currency Risk

Risk of receiving less of the domestic currency when investing in a bond issue that makes payments in a currency other than the manager's domestic currency.

Inflation Risk

Risk of decline in the value of a security's cash flows due to inflation.

Volatility Risk: Risk that the "expected yield volatility" will change.
The greater the expected yield volatility, the greater the value (price) of an option.

Price of callable bond = price of option-free bond – price of embedded call option

Price of Putable bond = price of option-free bond + price of embedded put option

Type of embedded option	Volatility risk due to
Callable Bonds	An increase in expected yield volatility
Putable Bonds	An decrease in expected yield volatility

Event Risk

1) Natural disaster (earthquake or hurricane) or an industrial accident.
2) Takeover or corporate restructuring
3) Regulatory change

Sovereign Risk: 1) Unwillingness of a foreign government to pay, or 2) inability to pay due to unfavourable economic conditions in the country

Yield Measures, Spot Rates and Forward Rates

Sources of Return

1) The coupon interest payments made by the issuer
2) Any capital gain (or capital loss – negative return) when the security matures, is called or is sold.
3) Income from reinvestment of interim cash flows (interest and/or principal payments prior to stated maturity).

Current yield

Annual dollar coupon interest to a bond's market price

$$Current\ Yield = \frac{Annual\ dollar\ coupon\ interest}{price}$$

Yield to Maturity

Interest rate that will make the present value of the bond's cash flows equal to its market price plus accrued interest (is the interest that has accumulated since the previous interest payment

Yield to Call

The yield to call assumes the issuer will call a bond on some assumed call date and that the call price is the price specified in the call schedule.

Yield to Put

Interest rate that will make the present value of the cash flows to the first put date equal to the price plus accrued interest.

Yield to Worst

Is the lowest of possible yields (YTM, Yield to call and yield to put).

Spot Rates

A default-free theoretical spot rate curve can be constructed from the observed Treasury yield curve.

The approach for creating a theoretical spot rate curve is called bootstrapping.

Example:

2-year = 1.71%, 5-year = 3.25%, 10-year = 4.35% and 30-year = 5.21%

$$\frac{yield\ at\ higher\ maturity - yield\ at\ lower\ maturity}{number\ of\ years\ between\ two\ observed\ maturity\ points} = \frac{4.35\% - 3.25\%}{5} = 0.22\%$$

Then,

Interpolated 6-year yield = 3.25% + 0.22% = 3.47%

7, 8 and 9-years yield, 3.69%, 3.91% and 4.13%, respectively

Forward Rates

Examples of forward rates:

6-month forward rate six months from now
6-month forward rate three years from now
1-year forward rate one year from now
3-year forward rate two years from now
5-year forward rates three years from now, etc, etc….

Deriving 6-month forward rates

Arbitrage principle (if two investments have the same cash flows and have the same risk, they should have the same value).

Investor with two alternatives:

- Buy a 1-year Treasury bill or,
- Buy a 6-month Treasury bill and when it matures in six months, buy another 6-month Treasury bill.

Spot rate on the 6-month Treasury bill = 3.0% (Z_1)
Spot rate on the 1-year Treasury bill = 3.3% (Z_2)

6-month forward rate on in six months from now = ?

$$(1 + Z_1) \times (1 + f_{1,1}) = (1 + Z_2)^2$$

$$f_{1,1} = \frac{(1 + Z_2)^2}{(1 + Z_1)} - 1 = \frac{(1 + 0.0165)^2}{(1 + 0.015)} - 1 = 1.80\%$$

The Valuation Principle

The price of a security today is the present value of all future expected cash flows discounted at the “appropriate” required rate of return (or discount rate)

The valuation variables are

1. Current price
2. Future expected cash flows - Face value and/or coupons
3. Yield or required rate of return

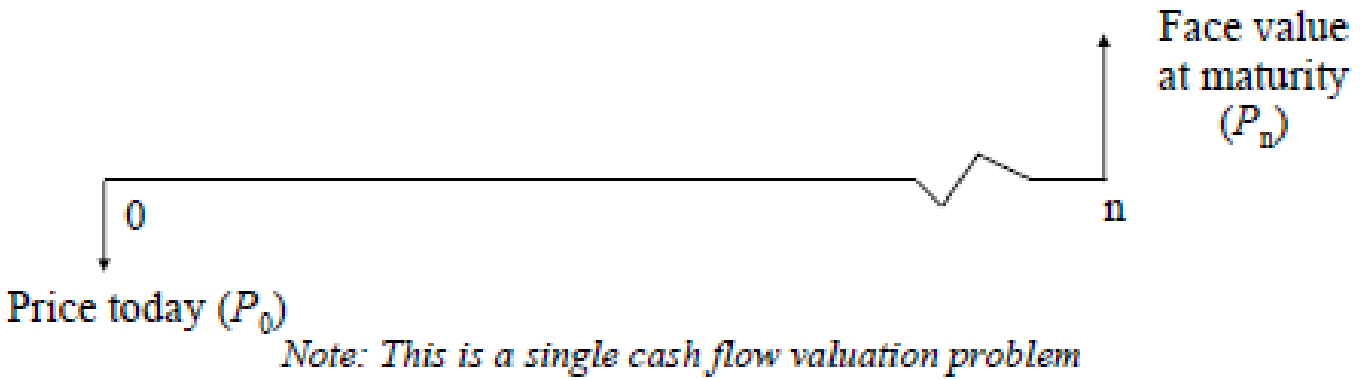
The valuation problem is to

1. Estimate the price; given the future cash flows and required rate of return, or
2. Estimate the required rate of return; given the future cash flows and price

Zero Coupon Securities

Zero coupon bonds are long-term securities paying the face value at maturity

- No coupon or interest payment made
- Issued at deep discount to face value
- Return earned is based on the appreciation in bond's value (price) over time



Note: This is a single cash flow valuation problem

Pricing Zero Coupon Securities

Example:

Consider a zero coupon bond which matures in 5 years with a face value of $1,000

a) If the bond has a yield to maturity of 8% what price should it be selling for today?

b) Suppose interest rates change suddenly and the price of these bonds rises to $700. What has happened to the yield to maturity of the bonds and why?

Given: P_n = $1,000, n = 5 years, and k_d = YTM = 8%

a) $P_0 = \frac{1{,}000}{(1.08)^5} = \$680.58$

b) The price has risen so you'd expect the YTM to be lower

$$New\ price, P_0 = 700 = \frac{1{,}000}{(1+k_{d*})^5} \qquad k_{d*} = \left(\frac{1{,}000}{700}\right)^{1/5} = 7.39\%$$

Coupon Paying Securities

Fixed coupon payment, typically every six months

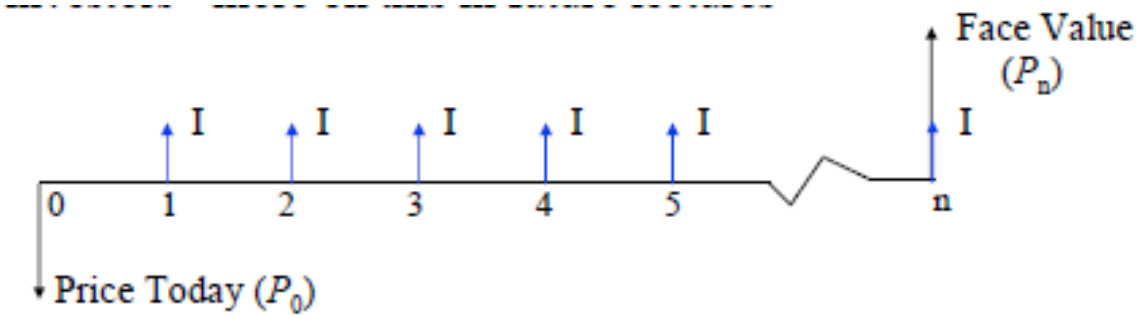
- Non coupon paying bonds called zero coupon bonds

Repayment of face value at maturity

Typically issued at face value

- Examples: Treasury bonds, corporate bonds

Market price depends on the rate of return required by investors



Note: This is a single cash flow plus annuity valuation problem

Pricing a Bond

Equal to the present value of the expected cash flows from the financial instrument. Determining the price requires:

An estimate of the expected cash flows

An estimate of the appropriate required yield

The price of the bond is the present value of the cash flows, it is determined by adding these two present values:

i) The present value of the semi-annual coupon payments

ii) The present value of the par or maturity value at the maturity date

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}$$

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^n}$$

P = Price

n = number of periods (nr of years times 2, if semi-annual)

C = semi-annual coupon payment

r = periodic interest rate (required annual yield divided by 2, if semi-annual)

t = time period when payment is to be received

Because the semi-annual coupon payments are equivalent to an ordinary annuity, applying the equation for the present value of an ordinary annuity gives the present value of the coupon payments:

$$C \times \left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right]$$

Consider a 20 year 10% coupon bond with a par value of \$1,000. The required yield on this bound is 11%.

$$50 \times \left[\frac{1 - \frac{1}{(1+0.055)^{40}}}{0.055} \right] = \$802.31$$

The PV of the par or maturity value of \$1,000 received 40 six-month periods from now, discounted at 5.5%, is \$117.46, as follows:

$$\frac{\$1{,}000}{(1.055)^{40}} = \frac{\$1{,}000}{8.51332} = \$117.46$$

Price = PV coupon payments + PV of par (maturity value)

\$802.31 + \$117.46 = \$919.77

The arbitrage-free Approach to Bond Valuation

The traditional valuation approach is deficient because it uses a single discount rate (the appropriate YTM) to find the present value of the future cash flows with no regard given to the timing of those cash flows.
Cash flows received in year 1 on a 20 year bond are discounted at the same rate as the cash flows received in 20 years!

Arbitrage Free Valuation Model

$$P = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_2)^2} + \frac{CF_3}{(1+r_3)^3} + \cdots + \frac{CF_n}{(1+r_n)^n}$$

This model treats each separate cash flow paid by a fixed-income security as if it were a stand-alone zero-coupon bond. These discount rates are called spot rates.

Example

Give the following Treasury spot rates, calculate the arbitrage-free value of a 5% coupon, 2 year treasury note.

Maturity	Spot Rate
0.5 years	4.0%
1.0	4.4%
1.5	5.0%
2.0	5.2%

The arbitrage-free price of the note is:

$$P = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_2)^2} + \frac{CF_3}{(1+r_3)^3} + \cdots + \frac{CF_n}{(1+r_n)^n}$$

$$P = \frac{\$2.50}{(1.020)} + \frac{\$2.5}{(1.022)^2} + \frac{\$2.5}{(1.025)^3} + \frac{\$102.5}{(1.026)^4} = \$99.66 \text{ per } \$100 \text{ of par value}$$

Profiting from Arbitrage Opportunities: Stripping and Reconstituting Bonds

Stripping

Suppose the same 2-year, 5% coupon Treasury note is priced at $95.00, which is below its arbitrage-free value of $99.66. What action should arbitrageurs take and what will be the affect of their actions?

Because the note is priced below its arbitrage-free value, its zero-coupon cash flow “pieces ”are worth more than the note it self. Therefore, an arbitrage profit could be earned by:

- Buying the undervalued note at $95.00
- Stripping the note of its individual cash flows
- Selling the individual cash flows “pieces” as zero-coupon bonds for $99.66 and earning arbitrage profit of $4.66 per $100 of investment.

As this arbitrage is performed:

The increased demand for the notes will cause their prices to increase and their yields maturity to fall

The increased supply of zero-coupon bond “pieces” will cause the prices of zero-coupon bonds to fall and their yields (spot rates) to rise.

These forces will quickly eliminate the arbitrage opportunity

Reconstituting

The 2-year, 5% coupon Treasury note is priced at $100. Its arbitrage-free value is $99.66. What action should arbitrageurs take and what will be the effect of their actions?

Because the note is priced above its arbitrage-free value, it is overpriced relative to the value of its zero-coupon cash flow “pieces”. Therefore, an arbitrage profit can be earned by:

Buying the zero coupon “pieces” in the zero-coupon treasury market for $99.66

Reconstituting the note from these zero-coupon Treasuries.

Selling the reconstituted note for $100 to earn an arbitrage profit of $0.34 for every $99.66 of original investment.

As dealers perform this arbitrage:

The increased demand for Treasury zero-coupon bonds will drive their prices up and their yields (spot rates) down.

The increased supply of reconstituted 2-year, 55 coupon treasuries will drive their prices down and their yields-to-maturity up

These forces will quickly eliminate the arbitrage opportunity

Arbitrage Example

We observe two types of bonds: T-bills and coupon bonds. A one-year T-bills pays 1000 in one year, a two year T-bill pay 1000 in two years and a three year T-bill pays 1000 in three years.

There are no coupon interests on T-bills. The coupon bond is a 5% three-year bond with a face value of 1000. Thus the cash flow from the coupon bond are: In the first year, you receive 50, in the second year 50, and in the last year 1050.

We observe the following prices:

Type of Bond	Price	Yield
One year T-bill	943.4	$\frac{1000}{1+r} = 943{,}3 \rightarrow r = 6\%$
Two year T-bill	873.44	$\frac{1000}{(1+r)^2} = 873{,}4 \rightarrow r = 7\%$
Three year T-bill	793.83	$\frac{1000}{(1+r)^3} = 793{,}83 \rightarrow r = 8\%$
Coupon bond	1.000	

Pricing using discounted cash flow

$$P = \frac{50}{(1+0.06)} + \frac{50}{(1+0.07)^2} + \frac{1{,}050}{(1+0.08)^3} = 924.37$$

Assumption: We can borrow funds at the above rates and short sell the securities without any costs

No arbitrage profit: Using no wealth, No risk and Positive return

The first condition requires a long and short position

To satisfy the second condition (no risk) we need to match the cash flows from the long and short positions

Short sell (borrow) 20 coupon bonds and buy 1 one year T-bill, 1 two year T-bill and 21 three year T-bills → we do not use any of our own wealth.

Pricing by arbitrage

Cash flows from bond transactions

			Cash flows at time:			
	Number of Bonds	Price	0	1	2	3
Coupon Bonds	20	1,000	20,000	-1,000	-1,000	-21,000

Short position - Loan **Long Position**

One year T-bill	1	943,4	-943,4	1,000	0	0
Two year T-bill	1	873,44	-873,44	0	1,000	0
Three year T-bill	21	793,83	-16,670.43	0	0	21,000
TOTAL	3		1,512.73	0	0	0

We have an arbitrage profit of 1,512.73, with no risk and using none of our own funds

What will happen:

- Investors will sell the coupon bonds → prices start to drop
- Investors will buy T-bills → price start to increase
- The price of the coupon bond that is consistent with the no *arbitrage condition is*

924.3635

Holding Period Yield

Example

Consider a 30-year zero coupon bond with a face value of $100. If the bond is priced at a yield-to-maturity of 10%, it will cost $5.73 today (the present value of this cash flow). Over the coming 30 years, the price will advance to $100, and the annualized return will be 10%.

Suppose that over the first 10 years of the holding period, interest rates decline, and the yield-to-maturity on the bond falls to 7%.

With 20 years remaining to maturity, the price of the bond will be $25.84.

Even though the yield-to-maturity for the remaining life of the bond is just 7%, and the yield-to-maturity bargained for when the bond was purchased was only 10%, the return earned over the first 10 years is 16.26%. This can be found by evaluating:

$$(1 + r) = \left({}^{25.84}/_{5.73}\right)^{1/10} = 1.1626$$

Over the remaining 20 years of the bond, the annual rate earned is not 16.26%, but 7%

This can be found by evaluating:

$$25.84 = {}^{100}/_{(1+\mathrm{r})^{20}}$$

$$(1+i)=({}^{100}/_{25.84})^{0.05}=1.07$$

Over the entire 30 year holding period, the original $5.73 invested matured to $100, so 10% annually was made, irrespective of interest rate changes in between