

Corporate Finance

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Session 2 – 04.12.2014

- Risk and Return: Technicalities to start from
- The Capital Asset Pricing Model (CAPM)
- The Hypothesized Relationship between Risk and Return
- Examine the concept of capital market efficiency
- Examine the types of information related to market efficiency
- Examine the role of market analysis in an efficient market
- Outline some tests of market efficiency and the evidence
- Examine the implications of market efficiency

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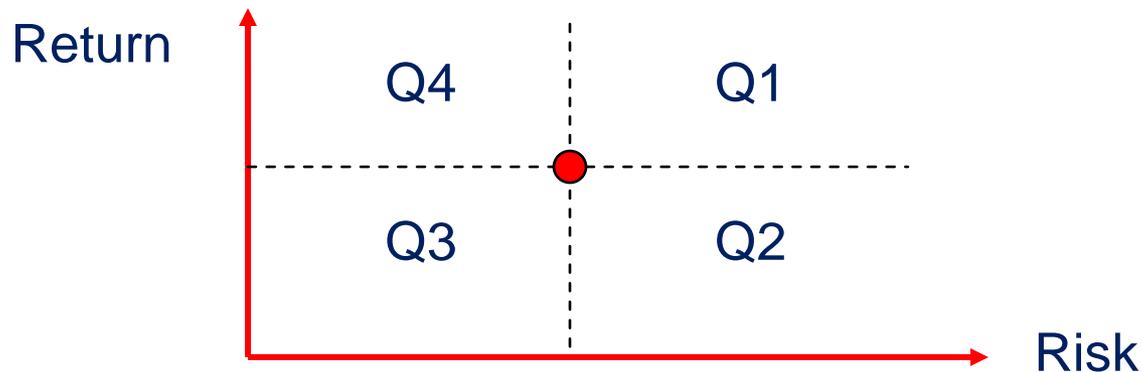
- Capital Budgeting/Project Evaluation
- Outline the capital budgeting process
- Define, use and interpret the net present value
- Define, use and interpret the internal rate of return
- Comparing the IRR and NPV Methods
- Payback Period
- Analyze mutually exclusive projects with different lives

Risk and Return

Technicalities to start from

Facts about Risk and Return

- Concepts defined from the **perspective of investor not issuer.**
- Assessment of risk and return represent the central issue for investment management
- Investors **are risk averse** (like returns and dislike risk)



Risk and Risk Premium

Every investment involves **some degree of uncertainty**

- future selling price is unknown, future dividends are unknown, unknown future cash flows, ...
- might have to sell assets due to emergency
- reinvestment rate might change (fall)
- increase in inflation changes the purchasing power of money / investment receipts

Expected holding period return ($t = 0$) is not the same as actual holding period return ($t = 1$)

$$E_t H_{t+1} = H_{t+1} + e_{t+1}$$

Scenario Analysis and Probability Distribution

To quantify risk, address two questions:

- What Holding period return is possible ?
- How likely is it ?

Scenario Analysis

- Assess different economic scenarios (outcomes)

Probability Distribution

- Assign probabilities to possible outcomes

Example : Scenario Analysis

Measurement of Equity Returns			
State of the Economy	Probability (prob)	Expected Return (ER)	Prob.×ER
Bad times	$\frac{1}{4}$	-10%	-2.5%
Normal	$\frac{1}{2}$	10%	5%
Good times	$\frac{1}{4}$	20%	5%
Expected Return = 7.5%			

Dispersion / Variability

Analyzing the data from the previous table, what surprises await us?

Definition of “surprise”:

$$\text{Surprise} = \text{Return} - \text{Expected Return}$$

Measures of dispersion of actual return from expected return: **variance** (defined as average square surprises) or **standard deviation** (square root of variance).

The reason why we use expected (mean) return and standard deviation as **return and risk measures** in investment decision making process is because we assume that **past asset returns follow normal distribution**.

Converting Prices into Rate of Returns

Financial data is usually reported as prices (Bloomberg, Datastream, etc)

For our statistical analysis and to be able to compare different investments we have to convert prices into returns

Example :

Suppose two shares with the following share price

- ABC share price ($P = \text{£}1.10$)
- XYZ share price ($P = \text{£}4.35$)
- Difficult to compare as they are measured in different 'units' (different base).

Converting Prices → Returns

– Arithmetic Rate of Return

$$R_t = (P_t + D_{t-1} - P_{t-1}) / P_{t-1}$$

– Geometric Rate of Return (continuously compounded return).

$$R_t = \ln [(P_t + D_{t-1}) / P_{t-1}]$$

Converting Prices into Returns - Example

Suppose that stock ABC had the following end-of-month prices:

245p on 31st August 2011

256p on 30th September 2011

Anticipated dividend for 2009 is 10p, therefore on a monthly basis it will be $10p/12=0.83333p$

The arithmetic monthly rate of return of this stock is:

$$R_t = (256 + 0.833 - 245)/245 = 0.048 = 4.8\%$$

The geometric monthly rate of return of this stock is:

$$R_t = \ln [(256 + 0.833)/ 245] = 0.047 = 4.7\%$$

Advantages of Continuously Compounded Rate of Return

Geometric rate of return is also known as **continuously compounded return**.

Differences of calculating the arithmetic or continuously compounded rate of **return are small**, especially for daily, weekly or even monthly data

If the formula for Geometric return is solved for P , continuously compounded rate of return will never give negative prices.

The Capital Asset Pricing Model

Asset Pricing: how assets are priced?

The equilibrium concept

Portfolio Theory

- ANY individual investor's optimal selection of portfolio (*partial equilibrium*)

CAPM

- Equilibrium of ALL individual investors (*general equilibrium*)

Risky asset i : Its price is such that

$$E(r) = R_F + \text{Risk Premium specific to asset } i$$

$$R_F + (\text{market price of risk}) \times (\text{quantity of risk of asset } i)$$

The Capital Asset Pricing Model

The *amount of risk* is measured by the covariance of the asset with the market portfolio

The *market price of risk* is the return above the risk-free rate that investors earn for holding the (risky) market portfolio

The *risk premium* can be thought of as a “price” times “quantity” relationship

Higher the market price of risk and/or higher the amount of risk, **greater the risk premium**

The Capital Asset Pricing Model: What is it?

Hypothesizes that investors require higher rates of return for greater levels of relevant risk

There are no prices on the model, instead it hypothesizes the relationship between risk and return for individual securities.

It is often used, however, to price securities and investments

Assumptions

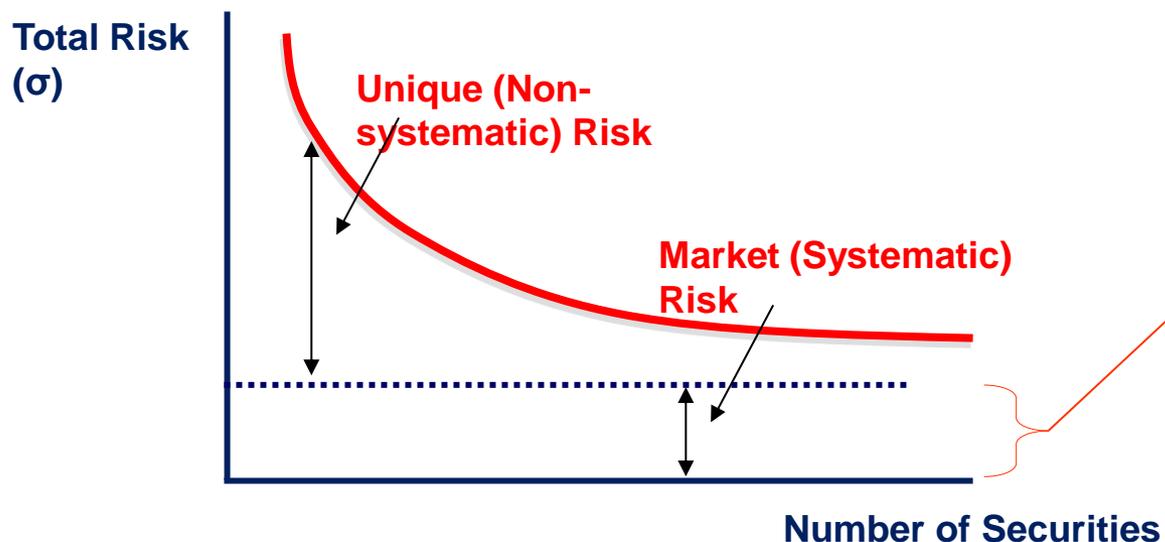
- one period investment horizon
- rational, risk-averse investors
- unlimited borrowing and lending is allowed at a risk free rate that is the same for all investors
- there are no taxes
- there are no transaction costs and inflation
- all assets are infinitely divisible
- free flow and instant availability of information
- there are many investors on the market
- all assets are marketable
- all investors have homogeneous expectations about expected returns, variances and covariances of assets

Diversifiable and Non-Diversifiable Risk

CML applies to efficient portfolios

Volatility (risk) of *individual security returns* are caused by two different factors:

- **Non-diversifiable risk** (system wide changes in the economy and markets that affect all securities in varying degrees)
- **Diversifiable risk** (company-specific factors that affect the returns of only one security)



Market or systematic risk is risk that **cannot** be eliminated from the portfolio by investing the portfolio into more and different securities

Relevant Risk

Previous figure demonstrates that an individual securities' volatility of return comes from two factors:

- Systematic factors
- Company-specific factors

When combined into portfolios, company-specific risk is diversified away.

Since all investors are 'diversified' then in an efficient market, no-one would be willing to pay a 'premium' for company-specific risk.

Relevant risk to diversified investors then is systematic risk.

Systematic risk is measured using the Beta Coefficient

Measuring Systematic Risk

The Beta Coefficient

What is the Beta Coefficient?

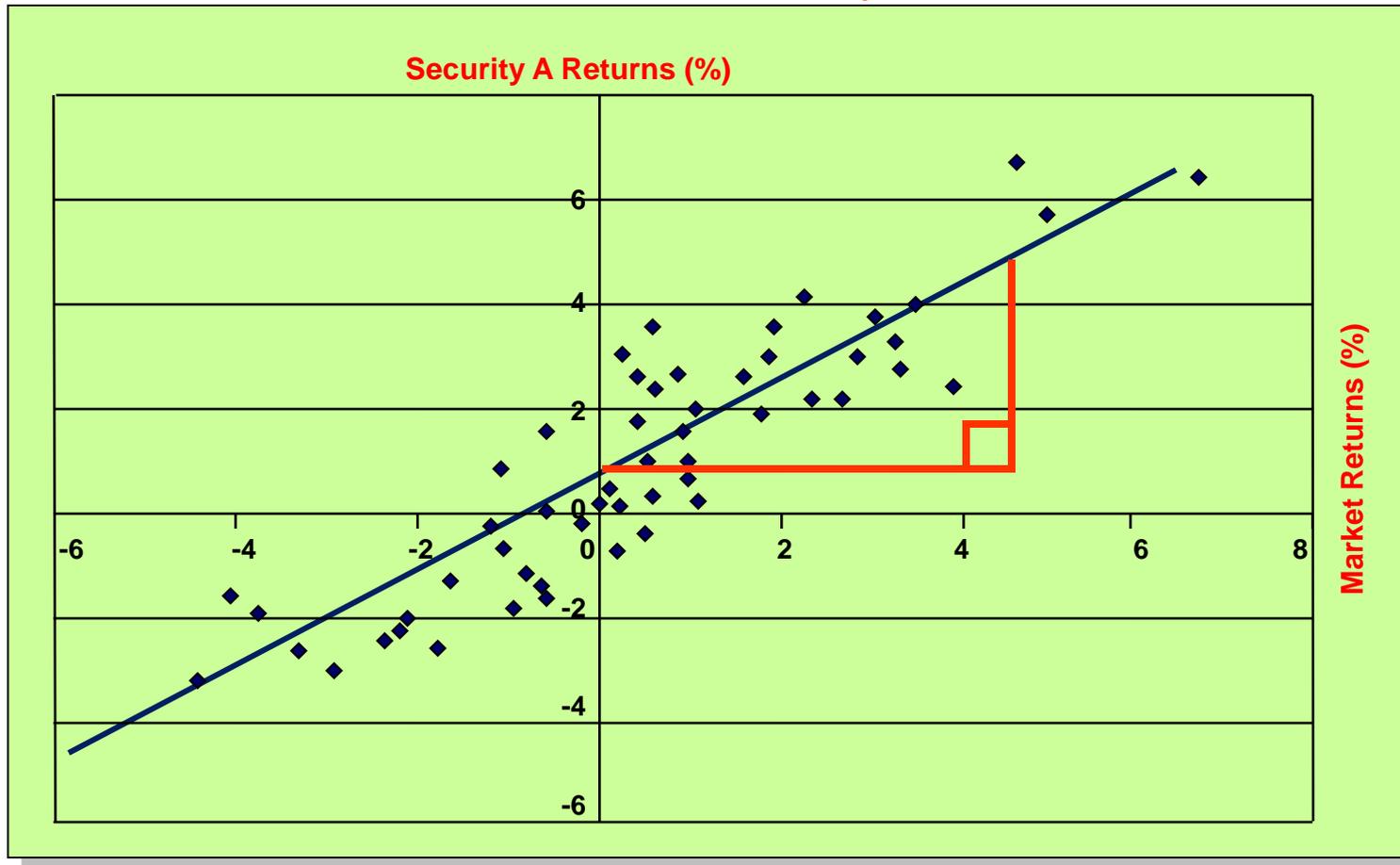
A measure of **systematic** (non-diversifiable) **risk**

As a '**coefficient**' the beta is a pure number and has no units of measure.

How Can We Estimate the Value of the Beta Coefficient?

- Using a formula (and subjective forecasts)
- Use of regression (using past holding period returns)

The Characteristic Line for Security A



The plotted points are the coincident rates of return earned on the investment and the market portfolio over past periods

The Formula for the Beta Coefficient

Beta is equal to the covariance of the returns of the stock with the returns of the market, divided by the variance of the returns of the market

$$\beta_i = \frac{\text{Cov}_{i,M}}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i}{\sigma_M}$$

How is the Beta Coefficient Interpreted?

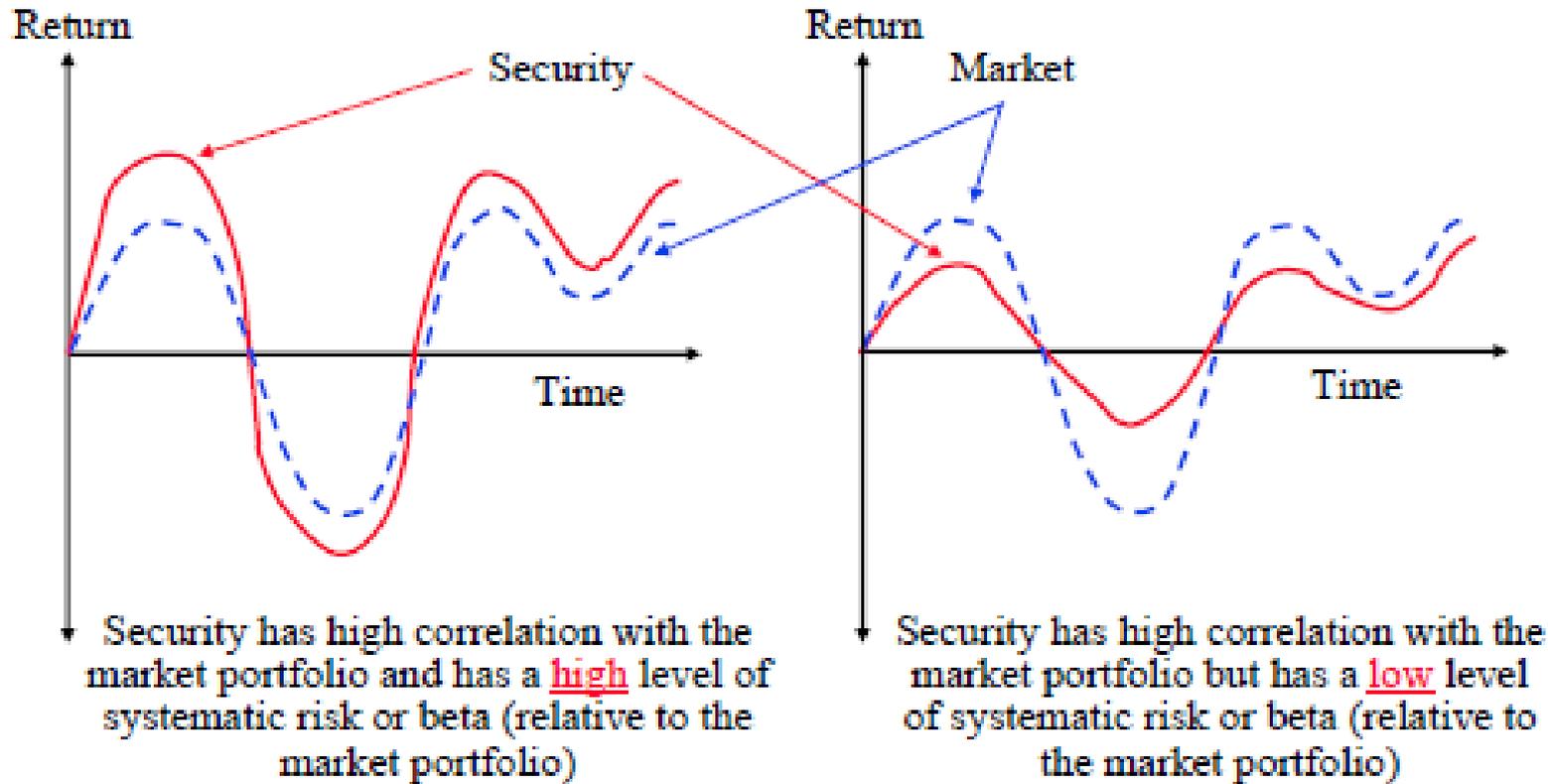
The beta of the market portfolio is **ALWAYS = 1.0**

The **beta of a security** compares the volatility of its returns to the volatility of the market returns:

- $\beta_s = 1.0$ - the security has the same volatility as the market as a whole
- $\beta_s > 1.0$ - aggressive investment with volatility of returns greater than the market
- $\beta_s < 1.0$ - defensive investment with volatility of returns less than the market
- $\beta_s < 0.0$ - an investment with returns that are negatively correlated with the returns of the market

Betas and Correlations

Beta is not the same as the correlation between a security (portfolio) and the market portfolio



The Beta of a Portfolio

The beta of a portfolio is simply the weighted average of the betas of the individual asset betas that make up the portfolio

$$\beta_P = w_A\beta_A + w_B\beta_B + \dots + w_n\beta_n$$

Weights of individual assets are found by dividing the value of the investment by the value of the total portfolio.

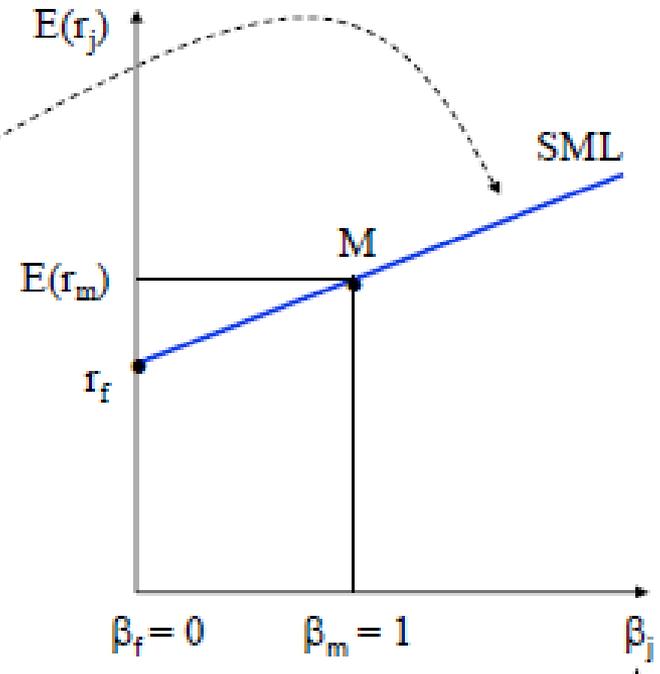
The Security Market Line

In equilibrium, all risky securities are priced so that their expected returns plot on the SML

$$E(R_i) = R_F + \beta_j [E(R_M) - R_F]$$

Assets with β_j less (more) than 1 earn an expected return lower (higher) than the market Portfolio

Note: The x-axis of the CML (used to “price” efficient portfolios) differs from the x-axis of the SML (used to “price” individual assets)



Relationship Between Prices and Returns

Class Exercise 1:

Oz Ltd's dividend is expected to be \$1.00 per share next year and remain unchanged in the future (i.e., $g = 0$). The following information is given:

Oz Ltd's beta = 1.2

Riskfree rate: $R_F = 6\%$

Expected market risk premium: $[E(R_M) - R_F] = 7\%$

[

- a) What price should Oz Ltd be selling for today?
- b) What will happen to Oz price if, after a market crash, analysts change their estimate of Oz beta to 1.5 and no other change occurs? Explain
- c) What general relationship between prices and returns is being illustrated here?

Relationship Between Prices and Returns

a) Based on the CAPM

$$E(r) = 0.06 + 0.07(1.2) = 0.144 \text{ or } 14.4\%$$

$$P_0 = 1.00/0.144 = \$6.94$$

b) Based on the new beta estimate of 1.5, we have

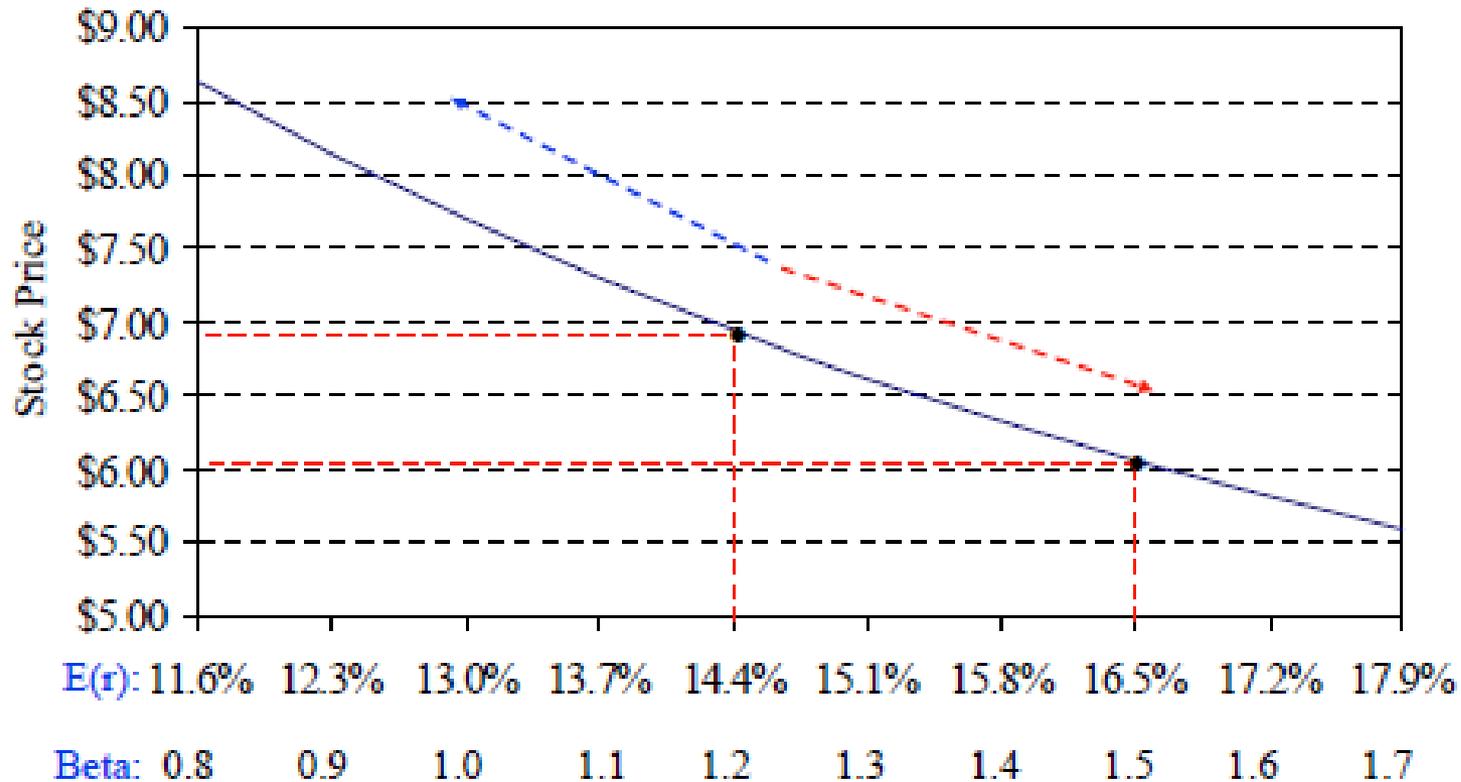
$$\text{Revised } E(r) = 0.06 + 0.07(1.5) = 0.165 \text{ or } 16.5\%$$

$E(r)$ has increased but at \$6.94 investors earn only 14.4% Investors will move funds to other similar risk securities which offer a higher expected return of 16.5%

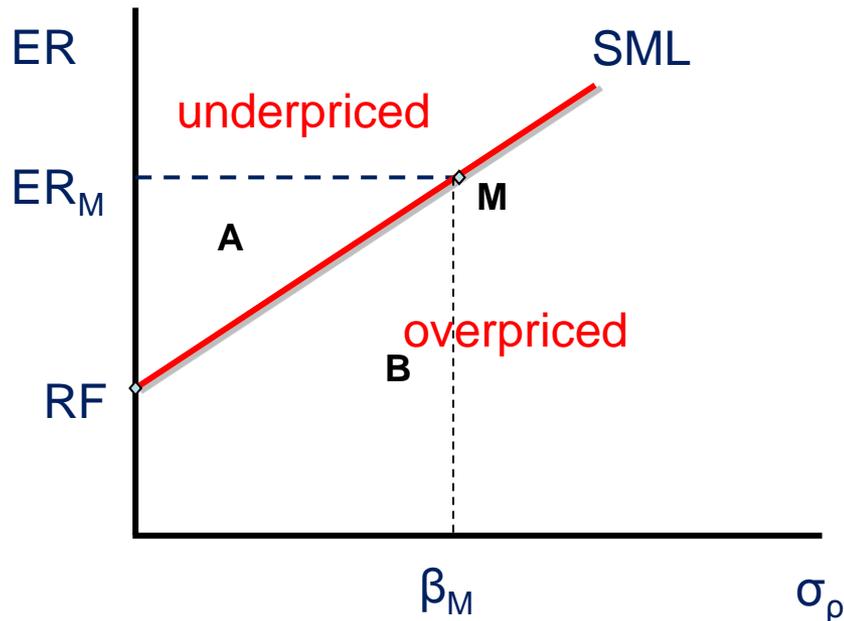
The selling pressure results in a new price

$$\text{New } P_0 = 1.00/0.165 = \$6.06$$

Relationship Between Prices and Returns



SML and Overvalued/Undervalued Securities



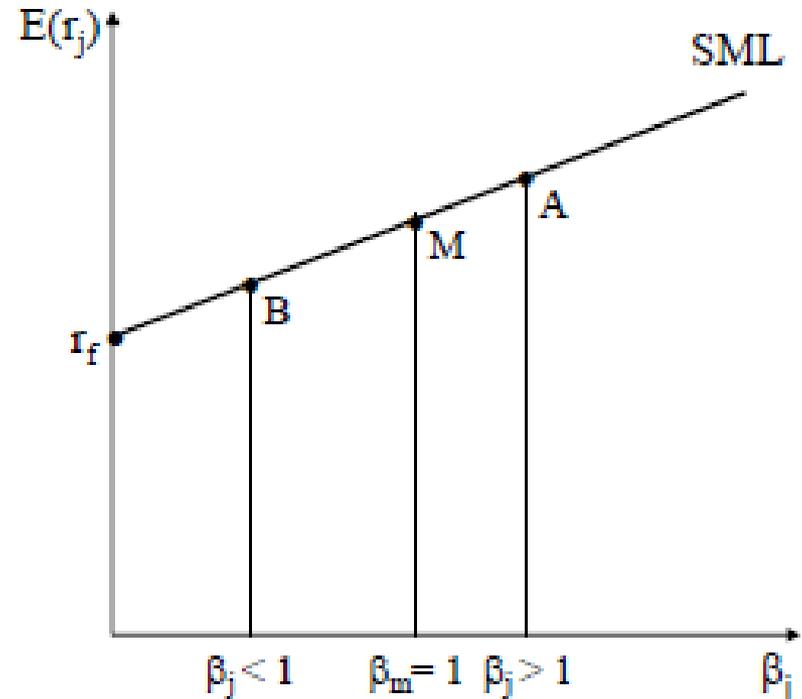
Undervalued Securities: plotted above the SML because they offer greater expected return for a given level of risk, implying that their prices are low. Investors will recognize the arbitrage opportunity and they will start buying those securities. The increase in the demand will drive prices of underpriced securities up, their returns down and the security will eventually be driven to the SML level.

The opposite process will happen to overvalued securities.

Movements in the Security Market Line

Application: What happens to the SML in the following cases

- a) There is an unexpected increase in the market risk premium
- b) There is an unexpected decrease in the risk-free rate



Movements in the Security Market Line

a) An unexpected increase in the market risk premium

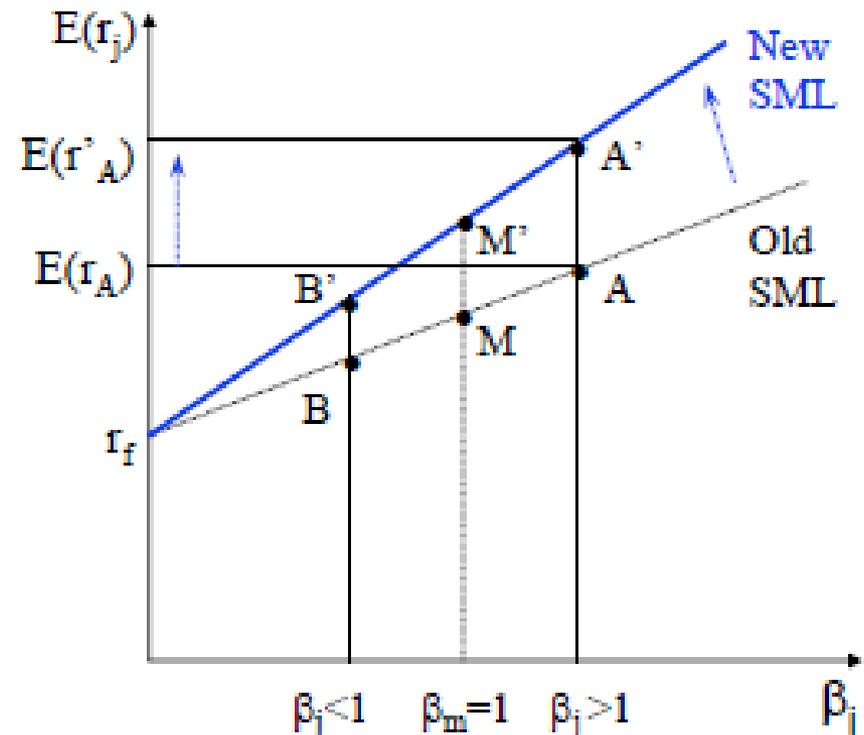
$[E(R_M) - R_F]$ increases

The SML is steeper
(assuming R_F unchanged)

$E(R)$ of asset A increases so A's price will fall

$E(R)$ of the lower risk asset B will rise less than the $E(R)$ of the higher risk asset A

$E(R_M)$ also increases so the market will fall in value



Movements in the Security Market Line

b) An unexpected decrease in the risk-free rate

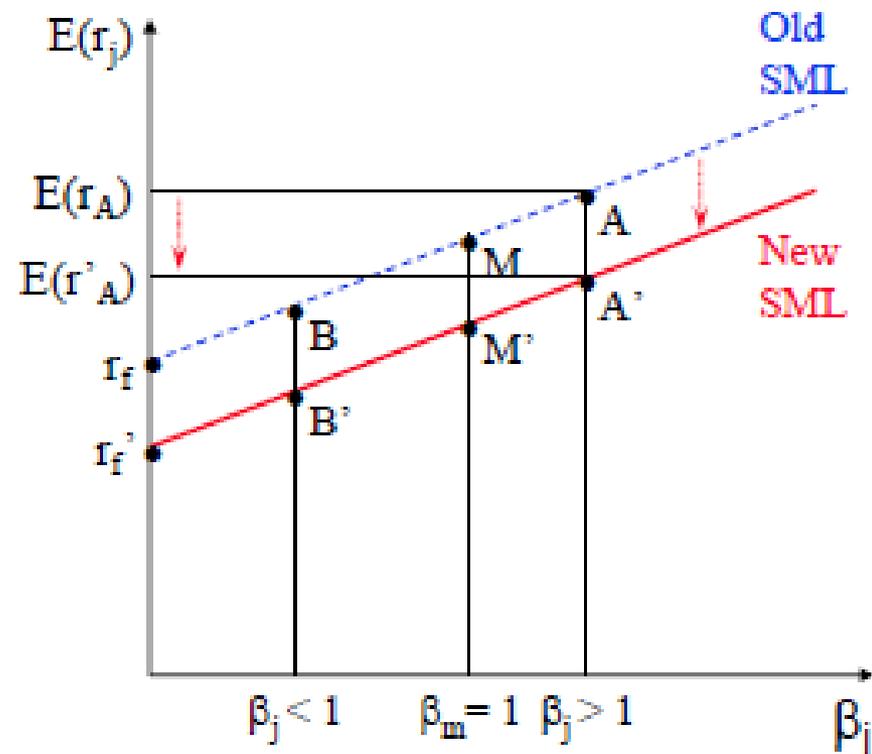
R_F decreases: assume no change in the market risk premium $[E(R_M) - R_F]$

Implies a downward, parallel shift in the SML

E_R of Asset A decreases so the price of A will rise

$E(R_M)$ also falls so the market will rise in value

Expected fall in $E(R_B) =$ Expected fall in $E(R_A)$



CAPM and Market Anomalies

The existence of market anomalies is inconsistent with the CAPM

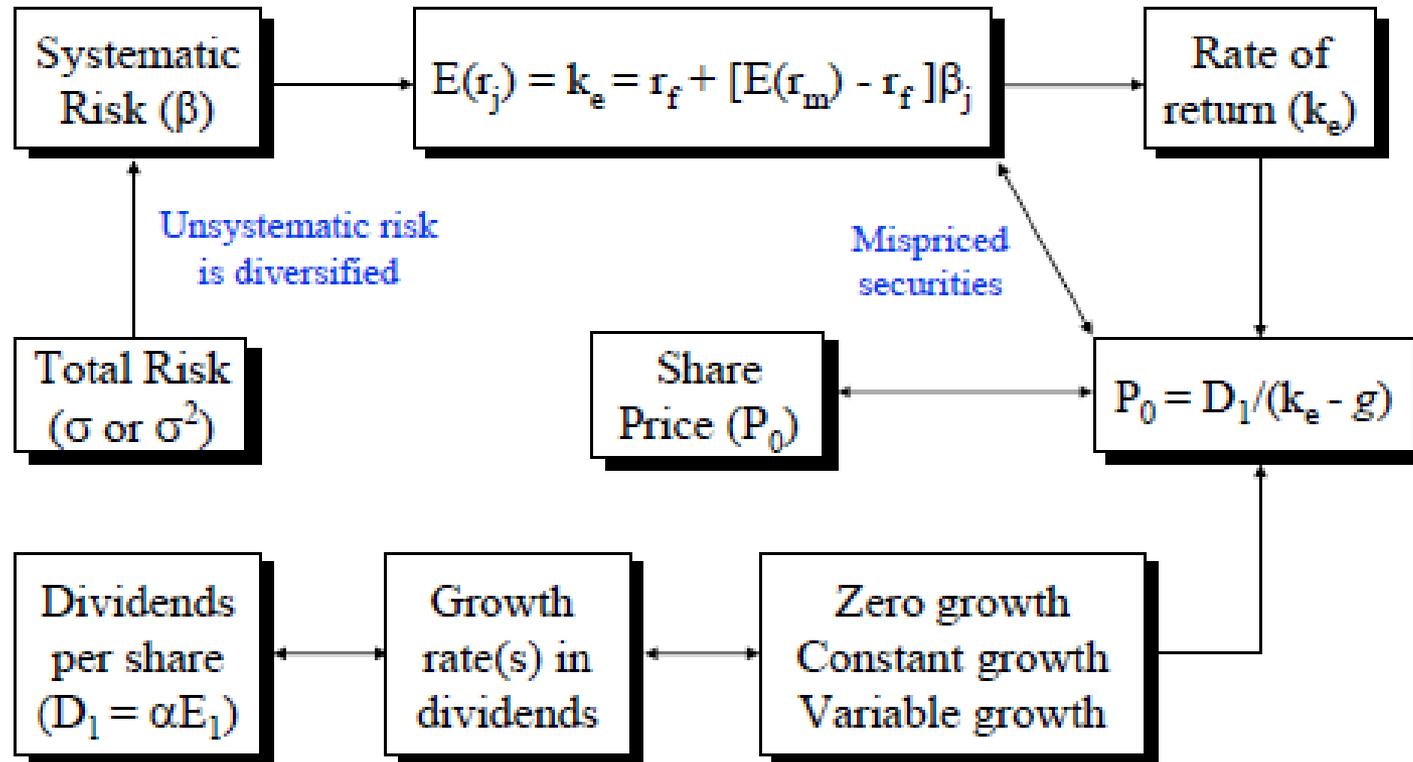
Some findings across time

- Returns lower on Mondays than on other days
- Returns higher in January compared to other months (especially for small firms)
- Returns higher the day before a holiday
- Returns higher at the beginning and end of the trading day

Some findings across securities (holding β constant)

- Returns higher for firms with “low” price-earnings ratios
- Returns higher for smaller firms compared to larger firms
- Returns higher for firms with higher book-to-market value of equity ratios

Putting it all Together



Concept of Capital Market Efficiency

A market is *informationally efficient* if prices instantaneously and *unbiasedly* reflect all available, relevant information

- An **instantaneous price reaction**
 - Any **unexpected “news”** is fully reflected in the price by the time of the next trade
 - **Unexpected news** arrives randomly and can be “good” or “bad”
- An **unbiased price reaction**
 - A biased reaction occurs when the price “**overreacts**” or “**under reacts**” to “**news**”
 - **Example:** Bad news hits the market which implies that the price of a stock should fall **by \$1.50 from \$5.00**. Some market participants “panic” and offer to sell their shares at \$3.00
 - In an **unbiased market** the stock should trade at \$3.50 after the news announcement

Market Efficiency - Main Assumptions

A large number of **profit-maximizing participants** are analyzing and valuing securities independently of each other

New information comes to the market in a **random manner** and the **timing of news announcements is independent** of each other

Market participants **adjust** their estimates of **security prices rapidly** to reflect their **interpretation of the new information received**

Does not mean that market participants **correctly adjust prices**

Some participants may **over-adjust** and others may **under-adjust**, but overall their price adjustments **will be unbiased**

Types of Capital Market Efficiency

Definition of capital market efficiency is quite restrictive

Capital markets may be efficient, but not all the time and in all cases

Efficient Market Hypothesis initially **proposed by Fama (1970)**

Weak form

Semi-strong form

Strong form



For each type of market efficiency we need to

Define the classification

Explain **how that type of efficiency** may be tested

Explain the **implications** of each type for investment purposes

Weak Form of Market Efficiency

Information on past prices is **fully reflected in current prices**

Past prices **cannot** help investors earn returns in excess of what other investors are earning on similar risk securities

- **Implication:** The best predictor of tomorrow's price P_{t+1} is the price today P_t

-

Prices **follow a random walk** - successive price changes are **random (irregular)** over time

$$P_{t+1} = P_t + \varepsilon_t, \text{ where } \varepsilon_t \text{ is a random error term}$$

$$\text{Alternatively, } \varepsilon_t = P_{t+1} - P_t$$

Semi-Strong Form of Market Efficiency

All publicly available information is fully and instantaneously reflected in current market prices

Examples: Announcements of earnings and dividends, share buybacks, stock splits, mergers, takeovers, etc

Implication: Past and currently available information is fully reflected in current market prices

Investors cannot use any publicly available information to *“beat the market”*

Note: A market cannot be semi-strong form efficient if it is weak form inefficient

Strong Form of Market Efficiency

All information, **public and private**, is fully reflected in prices

The market does not neglect any relevant information

Implication: Since all information is impounded in prices fully and instantaneously **it will be useless** in predicting future prices (and returns)

Implications of strong form inefficiency: Company insiders with inside information may exploit their private information to earn “**excess**” or “**abnormal**” returns/profits

Note 1: A market can be **semi-strong form efficient** but not necessarily strong form efficient

Note 2: Stock exchanges typically actively monitor and **prevent insider trading**

Market Analysis and Market Efficiency

The type of **information analysts** use depends on their belief regarding what information is reflected in market prices

- Technical analysts
- Fundamental analysts
- “Middle of the road” analysts

Technical analysts (chartists) believe weak form inefficiency

- It's possible to “**beat the market**” trading on past price movements and trends

Fundamental analysts believe in weak form efficiency

Earning “**abnormal**” **returns/profits** requires **gathering** and **analyzing** information

Forecasting future earnings, dividends and other fundamentals **better than other investors increases** the chance of earning abnormal returns/profits

Market Analysis and Market Efficiency

Most empirical evidence indicates **neither** type of analysis has been effective in earning abnormal returns consistently, **after transactions costs**

- **Continuous market analysis is what makes financial markets efficient!**

Recent evidence on the existence of market anomalies may indicate the prevalence of ***“pockets” of market inefficiency***

- Can these anomalies be exploited **consistently** over time **and** after **all costs** are taken into account?

Testing market efficiency typically involves using some model of asset prices (e.g., the CAPM)

- Such tests are **joint tests** of market efficiency and the model
- One can reject market efficiency if the asset pricing model is **mis-specified even if** the market is efficient

Weak Form Market Efficiency

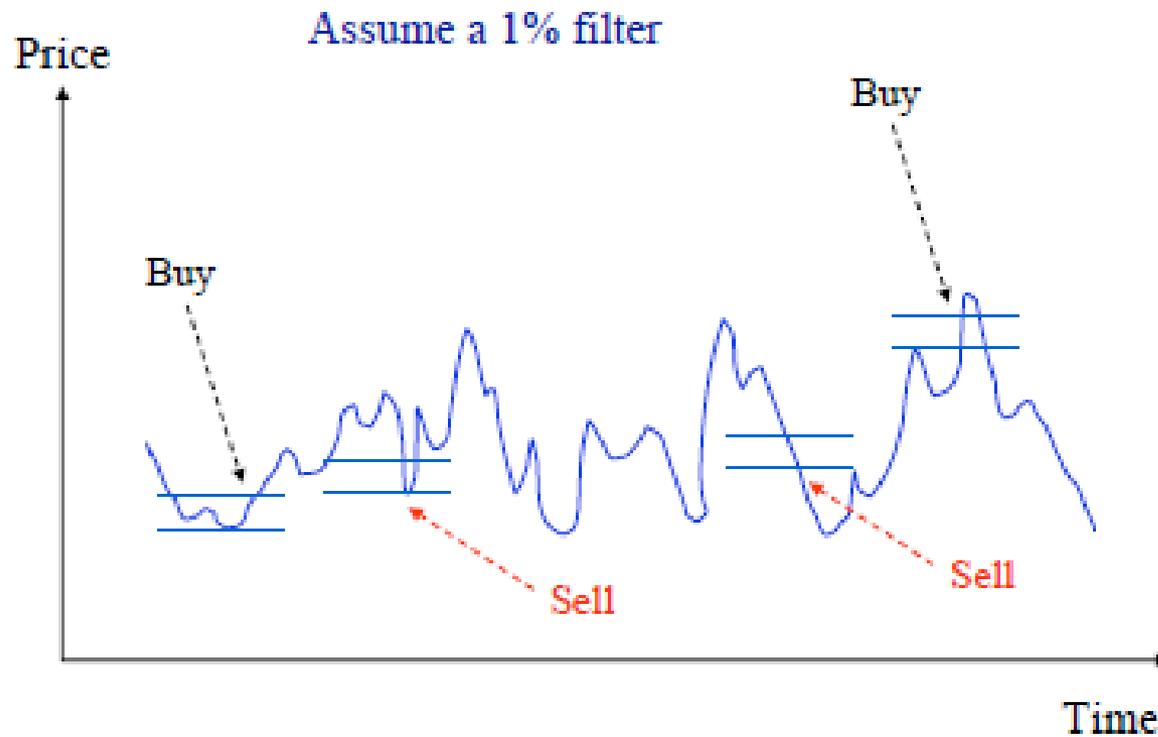
Typical studies analyze the “random walk” nature of prices

- Can past prices be used to predict future prices?
- Is there a predictive relationship between past price changes and subsequent price changes?

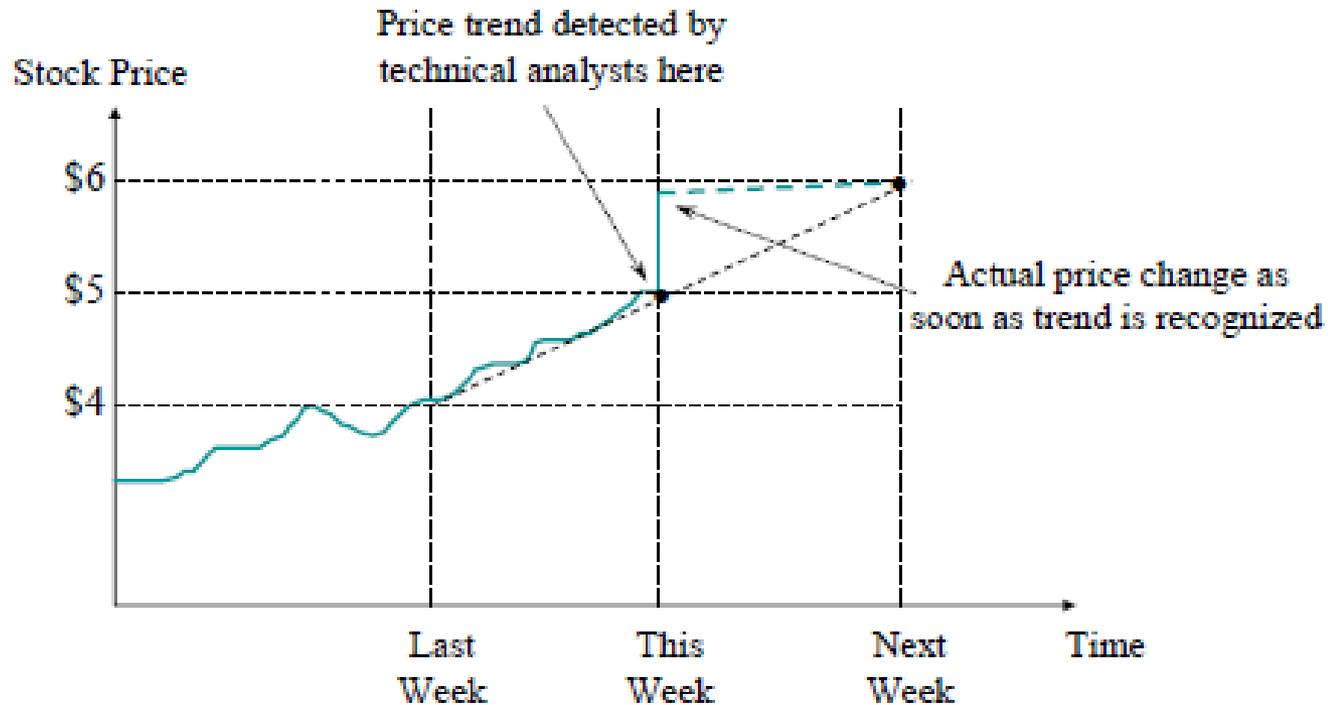
Filter rules used to analyze the profits from specific trading strategies of buying/selling securities depending on how their prices change over time

- Buy (sell) a stock if price rises (falls) by 1%, 5%, etc.
- Evidence indicates that some “small” filters may work, but not after transactions costs

Example of a Filter Rule



Weak Form Market Efficiency



Semi Strong Form Market Efficiency

Event studies used to test semi strong form efficiency

- Studies analyze market responses to **new** information
- Identify an event which involves release of “news”
- e.g., unexpectedly high/low earnings or dividends

In an efficient market, “good” (“bad”) news means an instantaneous upward (downward) price adjustment

Observed as “abnormal” returns at the announcement date

Abnormal returns (AR_t) typically estimated as the difference between observed returns and returns predicted by a **model like the CAPM**

$$AR_t = R_t - [R_F + (R_M - R_F)\beta]$$

$AR_t = 0$ on non-event days

$AR_t > 0$ for “good” news event on event day

$AR_t < 0$ for “bad” news event on event day

The Event Study Approach

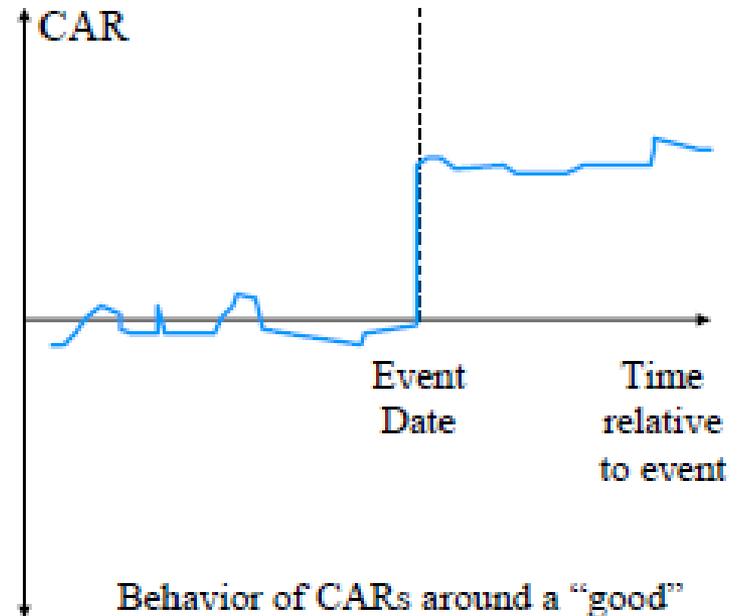
Cumulative abnormal returns (CARs) calculated by adding **abnormal returns** over time

$$CAR_t = CAR_{t-1} + AR_t$$

CARs show the total abnormal return earned over longer “event windows”

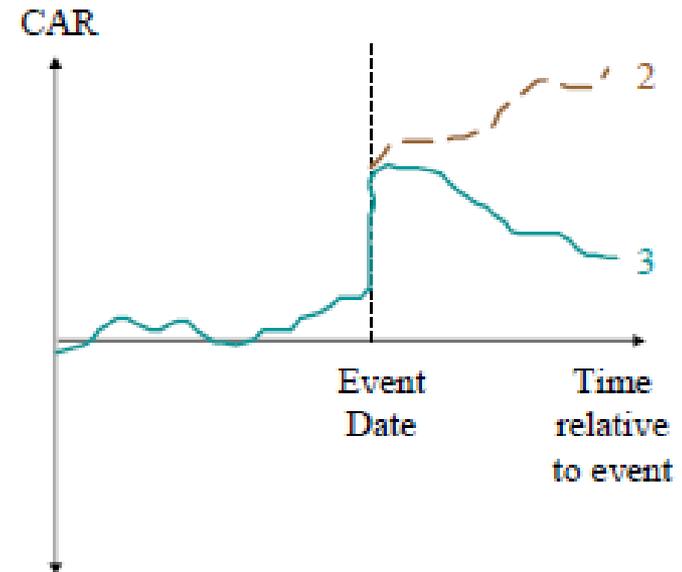
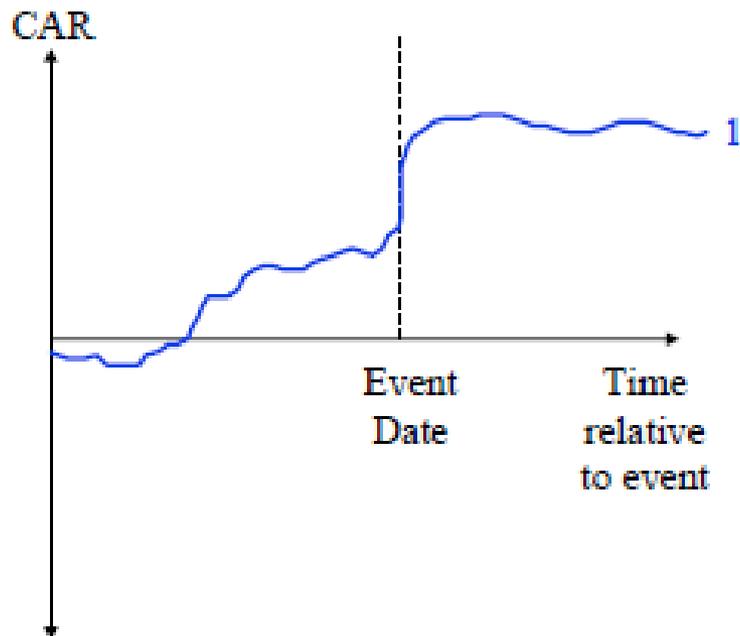
In a **semi-strong form efficient market**, the CARs should

- Have no discernible movement away from **zero prior to the event**
- **Jump** at the event date
- Have no **discernible** trend after the event date



Different Types of Market Reaction

What type of market reactions are these consistent with?



Strong Form Market Efficiency

Trades by corporate insiders (US market)

- Insider purchasers have earned abnormal returns on their trades
- Outsiders following these trades can also earn abnormal returns
- Abnormal returns mainly in the 1960s and 1970s, **not anymore**
- Preliminary Australian evidence (since 1995) **indicates that insider purchasers outperform the market**

Analysts recommendations such as in the “**Heard on the Street**” column of the Wall Street Journal have a **significant effect** on stock prices on the day they appear

- Analysts “**sell**” **recommendations** contain more information

Implications of Market Efficiency

In an efficient market any financial asset's NPV is zero

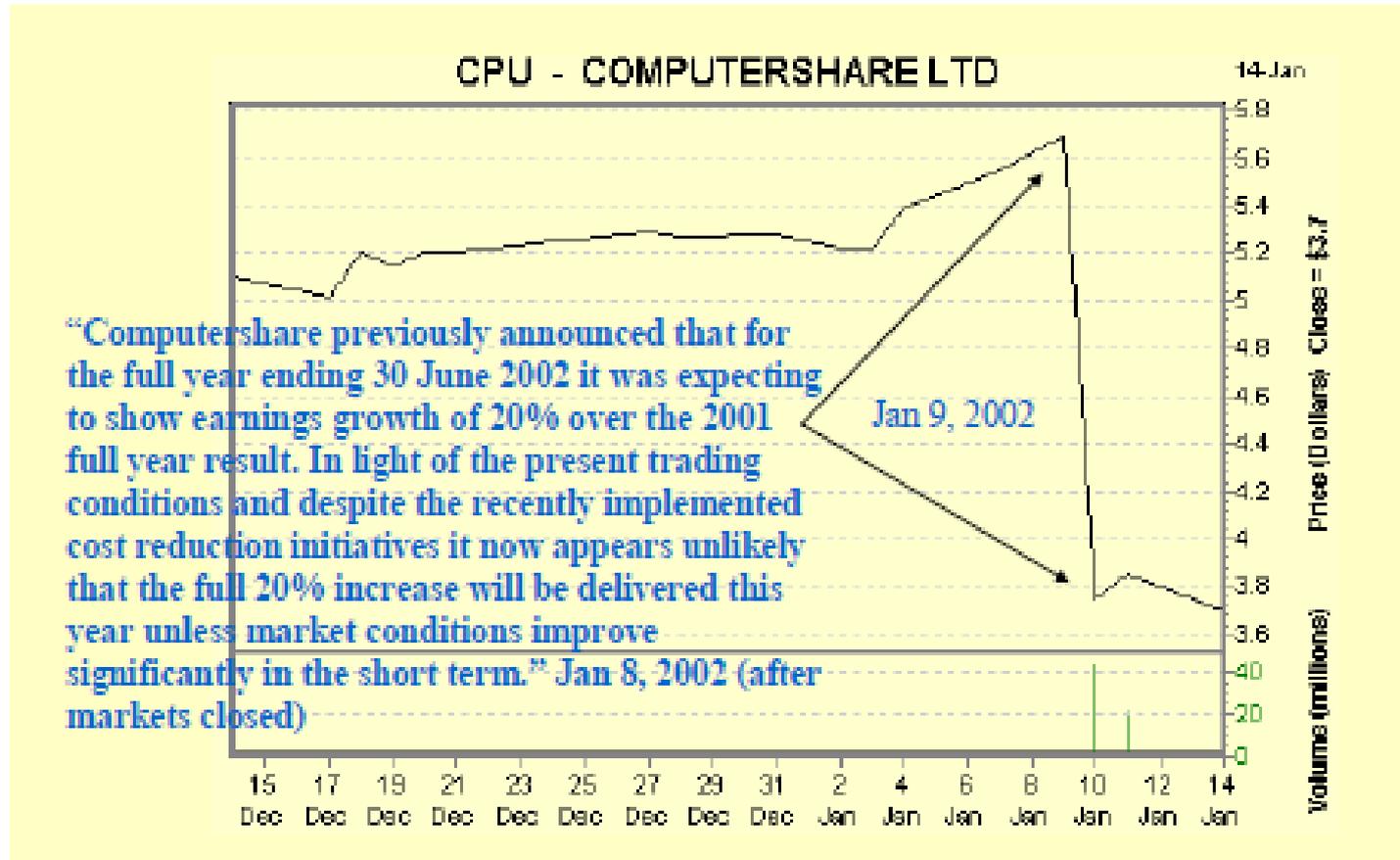
- $NPV = PV(\text{Cash inflows}) - PV(\text{Cash outflows})$
- So, $NPV = PV(\text{Cash inflows}) - \text{Price} = 0$
- So, Price or Value = $PV(\text{Cash inflows})$

Assets are priced appropriately for their market risk

Security prices react virtually instantaneously to any new information and reach a new equilibrium before investors can exploit that information for abnormal profit/return

- This does not imply that individual investors can never make abnormal profits
- Market efficiency implies that the profits cannot be made consistently after we take into account the cost of gathering and analyzing information

Information and Prices – An Example



Key Concepts

Assets are priced appropriately for their market risk

Security prices react virtually instantaneously to any new information and reach a new equilibrium before investors can exploit that information for abnormal profit/return

Markets generally efficient with respect to past and public information. Not always with respect to insider information

This does not imply that individual investors can never make abnormal profits

Market efficiency implies that the profits cannot be made consistently after we take into account the cost of gathering and analyzing information
In an efficient market any financial asset's NPV is zero

Fundamental versus Technical Analysis

Fundamental Analysis: involves analysing its income statements, financial statements, its management and competitive advantages and its competitors and markets

- Research the value of stocks using NPV and other measurements of cash flow
- The analysis is performed on historical and present data, but with the goal to make financial projections
- *Technical analysis:* the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends
 - Forecast stock prices based on the watching of the fluctuations in historical prices
 - Academics such as *Eugene Fama* say the evidence for technical analysis is **sparse** and is **refuted by the efficient market hypothesis**

Behavioral Finance

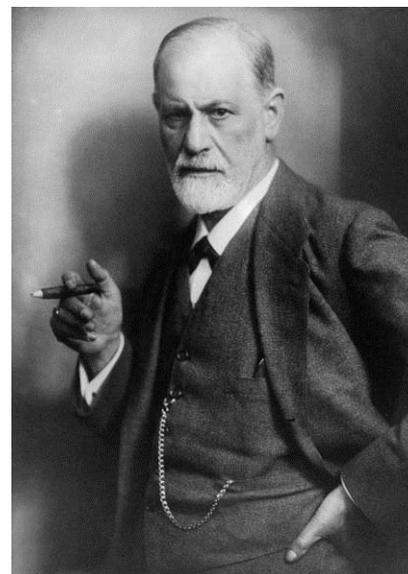
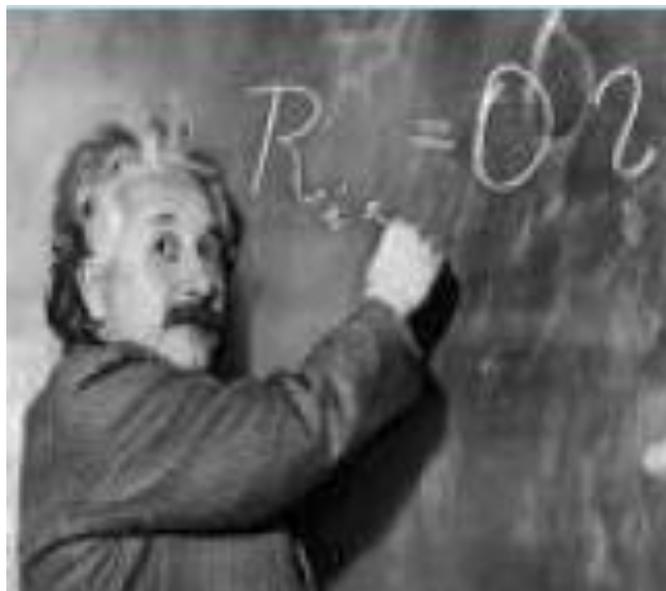
Argues that market participants suffer from **systematic psychological biases** that result in sub-optimal decisions

Investors **underreact to new information that contradicts prior beliefs** (e.g., dramatic change in earnings).

Investors **overreact to a string of similar information** (e.g., investors expect recent trends to continue).

Investors are **overly confident in their ability** to identify mis-valued stocks

“Is Behavioural Finance Inconsistent with Efficient Market Hypothesis”



Homo-Economicus

Investor can consistently rank all investments based on expected return and risk

Investors use the correct model to value shares:

- Uses discounted cash flows
- Uses all information
- Known how to analyse the information

Investors know how to form portfolios, etc.

In short....investors have at least a **Masters in Finance!!**

Or MBA in the University of Southampton

Rational Expectations

Homo-Economicus maximizes utility over **expected** return and risk

How does Homo-Economicus form expectations?

Rational expectations:

- Investors do make systematic mistakes
- Investors use all available information to form the forecasts
- They know and use the true model to form the forecasts

$$\textit{Actual or true price}_{t+1} = \textit{Expected Price}_{t+1} + \textit{random error}_{t+1}$$

Academics and the Efficient Market Hypothesis

Michael Jensen (Journal of Financial Economics, 1978)

“...there is no other proposition in economics which has more solid empirical evidence supporting it than the Efficient Market Hypothesis”



Wall Street Journal (18.10.2004)

“Robert Shiller, a Yale University Economist, has long argued that efficient-market theorists made one huge mistake: Just because markets are unpredictable doesn’t mean they are efficient.” The leap in logic, he wrote in the 1980s, was one of “the most remarkable errors in the history of economic thought”



Conclusions

- EFH contradicts technical analysis
- Prices follow a random walk with a drift
- Behavioural finance is inconsistent with EFH
- The two paradigms remains

The Capital Budgeting Process

Generation of investment proposals

Evaluation and selection of these proposals

Approval and control of capital expenditures

Post-completion audit of investment projects

Focus here is on the evaluation and selection of investment proposals

Methods of Project Evaluation

The major methods used by managers to evaluate projects are:

- Net present value
- Internal rate of return
- Payback period

Types of Projects

The two broad categories of projects that a firm typically analyzes are

Independent projects

- These are projects that can be evaluated on their own and independently of each other

Mutually exclusive projects

- These are projects where the acceptance of one project rules out the acceptance of other (competing) projects

Which types of projects are easier to evaluate and why?

What Do Managers Do?

<i>Method Used Always or Almost Always</i>	<i>Percentage</i>
Internal rate of return	75.6%
Net present value	74.9%
Payback period	56.7%
Accounting rate of return	20.3%
Profitability index	11.9%

Source: Graham and Harvey, 2001, "The Theory and Practice of Corporate Finance: Evidence From the Field," Journal of Financial Economics. Based on survey of 392 US-based CFOs. The aggregate percentage exceeds 100 percent because most respondents used more than one method of project evaluation. Profitability index = Present value of net cash flows/Initial outlay.

The Net Present Value Method

The net present value (NPV) method involves.

- Computing the difference between the present value of the net cash flows from an investment and the initial investment outlay
- All cash flows are discounted at the **required rate of return** which reflects the project's risk

Project's net cash flows

Identify the size and timing of **incremental cash flows** as a result of the project

Net cash flows **after** corporate taxes need to be evaluated

Incremental cash flows are the cash flows earned by the firm if the project **is** undertaken **minus** cash flows earned by the firm if the project **is not** undertaken

The Net Present Value Method

The net present value is computed as

$$NPV = \frac{C_1}{(1+k)} + \frac{C_2}{(1+k)^2} + \dots + \frac{C_N}{(1+k)^N} - I_0$$

$$NPV = \sum_{t=1}^N \frac{C_t}{(1+k)^t} - I_0$$

I_0 = Initial investment

C_t = Net after-tax cash flow at the end of year t

k = Project's required rate of return or opportunity cost of capital

N = Economic life of the project in years

Decision: Accept project if $NPV \geq 0$, reject if $NPV < 0$

Note: Point of indifference when $NPV = 0$

The Net Present Value Method

Example: The net after-tax cash flows from a four-year project that costs \$1 million are as follows. Evaluate the project using the net present value method assuming that the project's required rate of return is 12% p.a. How does your decision change if the initial investment were \$1,300,000 and not \$1,000,000?

End of Year	Net Cash Flows
0	-\$1,000,000
1	\$400,000
2	\$460,000
3	\$400,000
4	\$340,000

The Net Present Value Method

The project's net present value is:

$$NPV = \frac{400}{1.12} + \frac{460}{1.12^2} + \frac{400}{1.12^3} + \frac{340}{1.12^4} - 1000 = \$224.64$$

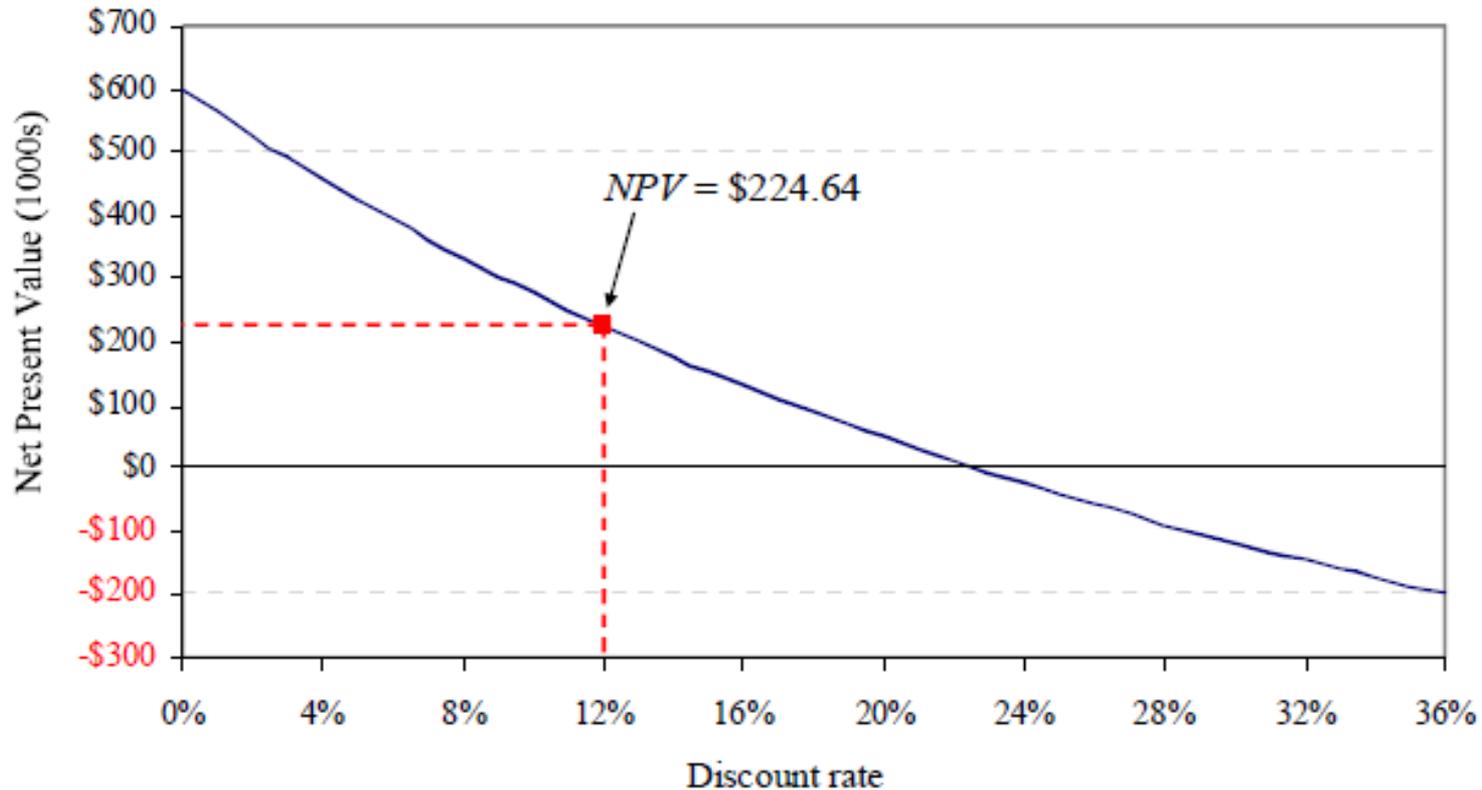
Since the NPV is positive the project should be accepted.

If the initial investment was \$1,300,000 the revised NPV is:

$$NPV = \frac{400}{1.12} + \frac{460}{1.12^2} + \frac{400}{1.12^3} + \frac{340}{1.12^4} - 1300 = -\$75.36$$

What interpretation can be associated with the net present value?

The Net Present Value Profile



Internal Rate of Return

The **internal rate of return** (IRR or r) is the rate of return that is earned by the project over its economic life

Reinvestment rate assumed in the context of the IRR?

Set NPV equal to 0 and compute the internal rate of return (r)

$$NPV \equiv 0 = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_N}{(1+r)^N} - I_0$$
$$NPV \equiv 0 = \sum_{t=1}^N \frac{C_t}{(1+r)^t} - I_0$$

Decision: Accept project if $r \geq k$, reject if $r < k$

Note: Point of indifference when $r = k$

Internal Rate of Return

The internal rate of return for .simple. projects is relatively easy to compute

Example: Consider a project which involves an initial investment of \$100,000 and yields a net cash flow of \$150,000 at the end of year 4. What is the IRR of this project?

Compute the IRR by setting the **NPV to zero** and solving for the IRR in...

$$NPV \equiv 0 = \frac{150000}{(1+r)^4} - 100000$$
$$r = \left(\frac{150000}{100000} \right)^{1/4} - 1 = 10.7\%$$

Internal Rate of Return

Example: The net cash flows from a four-year project that costs \$1,000,000 are as follows. Evaluate the project using the internal rate of return method and assuming that the project's required rate of return is 12% p.a.

End of Year	Net Cash Flows
0	-\$1,000,000
1	\$400,000
2	\$460,000
3	\$400,000
4	\$340,000

Internal Rate of Return

Recall: The net present value is of the project was.

$$NPV = \frac{400}{1.12} + \frac{460}{1.12^2} + \frac{400}{1.12^3} + \frac{340}{1.12^4} - 1000 = \$224.64$$

Internal rate of return is obtained by solving for r in...

$$NPV \equiv 0 = \frac{400}{(1+r)} + \frac{460}{(1+r)^2} + \frac{400}{(1+r)^3} + \frac{340}{(1+r)^4} - 1000$$

At r = 22%, NPV = \$10.68

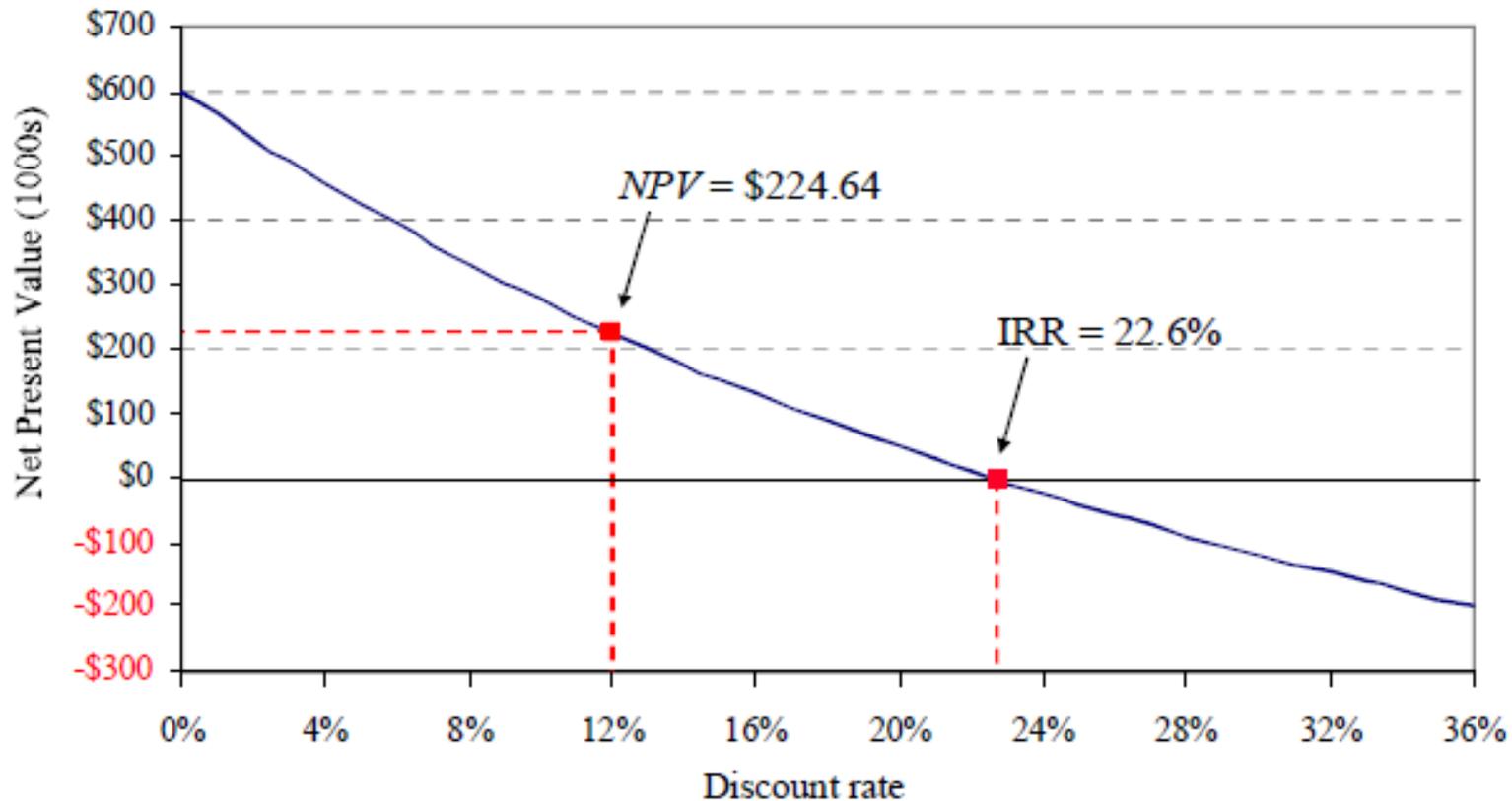
At r = 23%, NPV = -\$7.25

At r = 22.5%, NPV = \$1.65

Actual r = 22.6% > k = 12%

Both rules give the *same decision for individual projects*

Internal Rate of Return



Payback Period

A project's payback period is the time it takes for the initial cash outlay on a project to be recovered from the net after-tax cash flows

- Note that in computing the payback period we assume that the cash flows are distributed *evenly over the year (rather than at the end of each year)*

Decision rule

- A project is acceptable if its payback period is less than a pre-specified maximum payback period
- For mutually exclusive projects, the project with the shortest payback period is preferred (assuming they all meet the maximum payback period threshold)

Payback Period

Example: A firm is considering three mutually exclusive projects that require an initial outlay of \$100,000 and that generate the following pattern of cash flows. The firm typically accepts projects with a payback period less than 2 years

Project	Year 1	Year 2	Year 3	Year 4	Payback
C	\$100,000	-	-	\$10,000	1 year
D	\$50,000	\$50,000	\$50,000	\$50,000	2 years
E	\$50,000	\$30,000	\$30,000	\$90,000	2.7 years
F	\$50,000	-\$30,000	\$60,000	\$40,000	3.5 years

- Payback for project E = $2 + 20/30 = 2.7$ years
- Payback for project F = $3 + 20/40 = 3.5$ years

Decision?

Problems With Payback Period

Fails to take account of the cash flows that occur after the payback period cutoff date

Biased against projects that have longer development periods

- **Examples:** Mining and exploration projects

Ignores the time value of money

Is there any use for the payback period and accounting rate of return methods?

What method(s) should a company use?

Key Concepts

The NPV method is recommended for investment evaluation

NPV is consistent with maximization of shareholder wealth

NPV is also simple to use and gives rise to fewer problems than the IRR method

The constant chain of replacement assumption is used to evaluate and compare projects of differing lives