Financial Markets & Risk

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Session 3

Derivatives

- Binomial Trees
- Black-Scholes Call-Option-Pricing Model
- Option Greeks
- Interest Rate Swaps
- Currency Swaps
- Forward Rate Agreements

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Binomial Trees

Useful and very popular technique for pricing an option.

Diagram representing different possible paths that might be followed by the stock price over the life of an option.

Underlying assumption: stock price follows a *random walk*

In each time step:
Certain probability of moving up or down by a percentage amount

In the limit, as the time step becomes smaller, this model leads to the lognormal assumption for stock prices that underlies the Black-Scholes model.

Binomial tress can be used to value options using both no-arbitrage arguments and the risk-neutral valuation principle.

Example

Initial stock price = $20 and it is known that at the end of 3 months will be either: $22 or $18.

What is the value of an European call option to buy the stock for $21 in 3 months?

Stock price = $20

Stock Price = $22
Option price = $1

Stock Price = $18
Option price = $0

Arbitrage opportunities do not exist (assumption)
No uncertainty about the value of the portfolio at the end of the 3 months
Portfolio has no risk, the return must be equal the risk-free interest rate

Setting up a Riskless Portfolio

Long position in Δ shares of stock and a short position in one call option

What is the value of Δ that makes the portfolio riskless?

Sock prices moves up from \$20 to \$22, the value of the shares is 22 Δ
The value of the option is 1. Total value of portfolio is 22 Δ -1.

Sock prices moves up from \$20 to \$18, the value of the shares is 18 Δ
The value of the option is 0. Total value of portfolio is 18 Δ -0.

Portfolio is riskless, if the value of Δ is chosen so that the final value of the portfolio is the same (for both alternatives)

$$22\Delta - 1 = 18\Delta \Leftrightarrow \Delta = 0.25$$

Riskless portfolio: long 0.25 shares, short 1 option.

$$\text{Portfolio Value} = 22 \times 0.25 - 1 = 4.5 \text{ or } 18 \times 0.25 = 4.5$$

Riskless portfolios must, in absence of arbitrage opportunities earn the risk-free rate of interests.

Suppose risk-free rate in 12% per annum. The value of the portfolio today is the present value of 4.5:

$$4.5^{-0.12\times3/12} = 4.367$$

The value of the option today (*f*):

$$20 \times 0.25 - f = 4.367 \Leftrightarrow f = 0.633$$

If the value of the option is more than 0.633, the portfolio would cost less than 4.367 to set up and would earn more than the risk-free rate

If the value of the option is less than 0.633, shorting the portfolio would provide a way of borrowing money at less than the risk free rate.

Generalization

A derivative lasts for time T and is dependent on a stock

S_0= Stock Price
f= option price

$$\begin{tikzcd} & \substack{S_0u \\ f_u} \\ \substack{S_0 \\ f} \arrow[ur] \arrow[dr] & \\ & \substack{S_0d \\ f_d} \end{tikzcd}$$

Portfolio: Long position in Δ shares and a short position in one option

Value of the portfolio at the end of life of the option if there is an up movement in the stock price

$$S_0 u \Delta - f_u$$

Value of the portfolio at the end of life of the option if there is a down movement in the stock price

$$S_0 d \Delta - f_d$$

The two are equal when:

$$S_0 u\Delta - f_u = S_0 d\Delta - f_d$$
$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

Portfolio is riskless, no arbitrage opportunities, it must earn the risk-free interest rate.
Present value of the portfolio:

$$(S_0 u\Delta - f_u)e^{-rT}$$

The cost of setting the portfolio:

$$S_0\Delta - f$$

It follows that:

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT} \text{ or } f = S_0\Delta(1 - ue^{-rT}) + f_u e^{-rT}$$

The equation can be reduced to:

$$f = e^{-rT}[pf_u + (1-p)f_d]$$

$$p = \frac{e^{rT} - d}{u - d}$$

Option Price (one-step binomial tree):

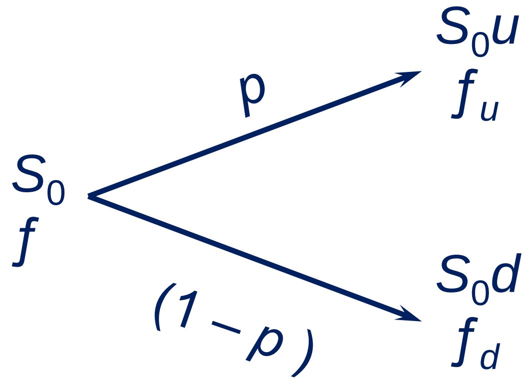
$$p = \frac{e^{0.12\times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

$$f = e^{-0.12\times 0.25}(0.6523 \times 1 + 0.3477 \times 0) = 0.633$$

p as a Probability

It is natural to interpret *p* and 1-*p* as probabilities of up and down movements

The value of a derivative is then its expected payoff in a risk-neutral world discounted at the risk-free rate



Risk-Neutral Valuation

When the probability of an up and down movements are p and 1-p the expected stock price at time T is S_0e^{rT}

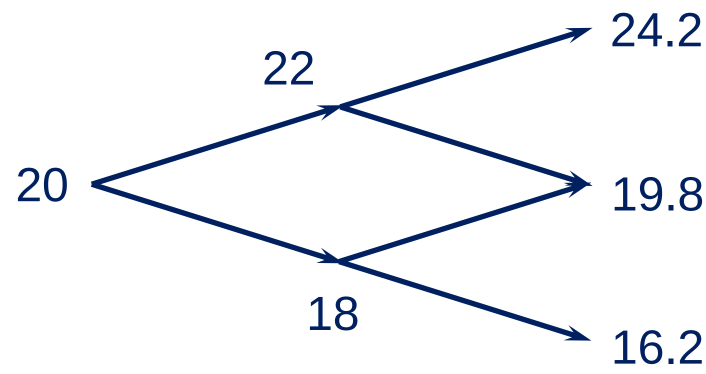
This shows that the stock price earns the risk-free rate

Binomial trees illustrate the general result that to value a derivative we can assume that the expected return on the underlying asset is the risk-free rate and discount at the risk-free rate

This is known as using risk-neutral valuation

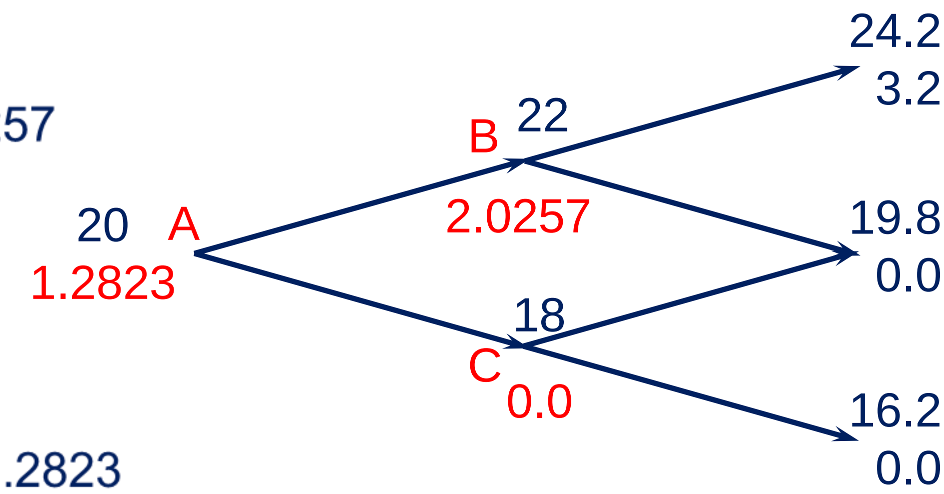
Two-Step Binomial Trees

$u = d = 10\%$
X = 21
R = 12%
Each time step is 3 months



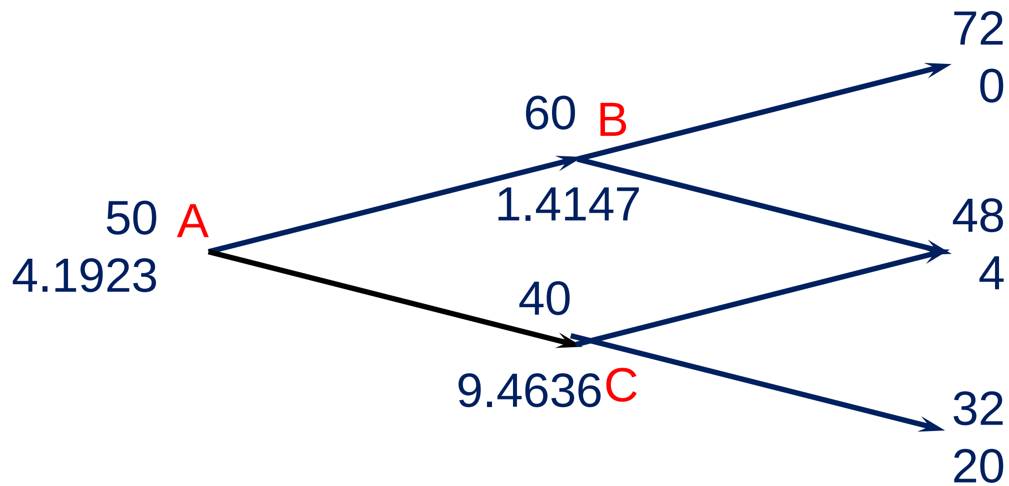
$$\textit{Value at node B}$$
$$e^{-0.12\times0.25}(0.6523\times3.2 + 0.3477\times0) = 2.0257$$

$$\textit{Value at node A}$$
$$e^{-0.12\times0.25}(0.6523\times2.0257 + 0.3477\times0) = 1.2823$$



Example with Put

$S_0 = 50$
$K = 52$
r = 5%
$u = 1.20$
$d = 0.8$
$p = 0.6282$
Each time step is one year



Value at node B

$$e^{-0.05\times1}(0.6282\times0 + 0.3718\times4) = 1.41467$$

Value at node C

$$e^{-0.05\times1}(0.6282\times4 + 0.3718\times20) = 9.46359$$

Value at node A

$$e^{-0.05\times1}(0.6282\times1.4147 + 0.3718\times9.4636) = 4.19233$$

American Options

The value of the option at the final nodes is the same as for European Options.

At earlier nodes the value of the option is the greater of:

1. The value given by:

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d]$$

2. The payoff from early exercise

$$Value\ at\ node\ B$$

$e^{-0.05\times1}(0.6282\times0 + 0.3718\times4) =$ 1.41467 since early exercice value is (-8)

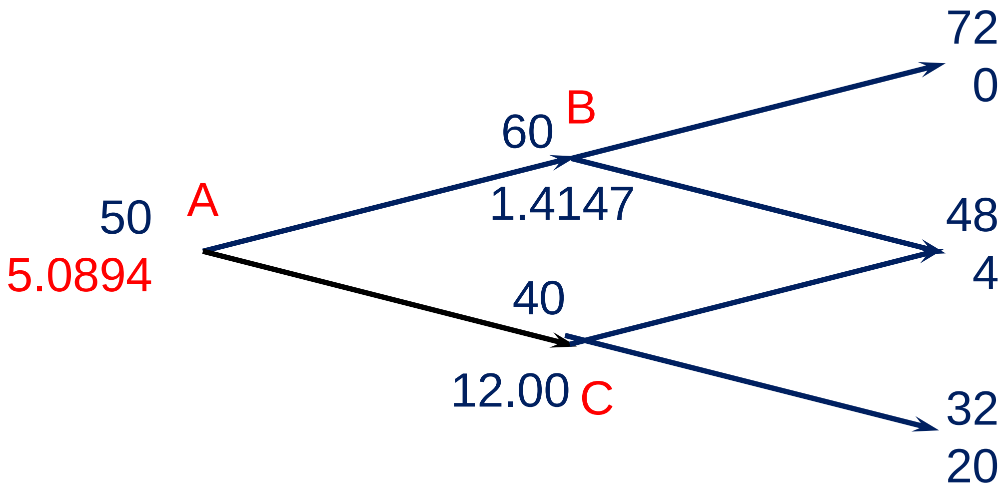
$$Value\ at\ node\ C$$

Payoff early exercise is 12

$e^{-0.05\times1}(0.6282\times4 + 0.3718\times20) =$ 9.46359

$$Value\ at\ node\ A$$

$e^{-0.05\times1}(0.6282\times1.4147 + 0.3718\times12.00) =$ 5.0894



At the initial node A, the value is 5.0894 and the payoff from early exercise is 2

Early exercise is not optimal.

The Black-Scholes Call-Option-Pricing Model

The Concepts Underlying Black-Scholes Model

The option price and the stock price depend on the same underlying source of uncertainty

We can form a portfolio consisting of the stock and the option which eliminates this source of uncertainty

The portfolio is instantaneously riskless and must instantaneously earn the risk-free rate

This leads to the Black-Scholes differential equation

Assumptions Underlying Black-Scholes

No dividends

Underlying stock returns are normally distributed

No transaction costs

Risk free interest rate for lending and borrowing

Volatility and interest rates are constant up to maturity

Black and Scholes Formulas

$$c_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$d_1 = \frac{\ln(S_0 / X) + [R_F + 0.5Var(R)]T}{\sigma\sqrt{T}};\ d_2 = d_1 - \sigma\sqrt{T}$$

N(.) is the cumulative distribution function of the standard normal distribution

$N(d_2)$ is the risk adjusted probability that the option will be exercised.

$N(d_1)$ always greater than $N(d_2)$. $N(d_1)$ must not only account for the probability of exercise as given by $N(d_2)$ but must also account for the fact that exercise or rather receipt of stock on exercise is dependent on the conditional future values that the stock price takes on the expiry date.

Example

S_0 = \$60 = market price of the underlying asset (such as the share price of an optioned stock)

X = \$50 = Exercise (strike) price

T = 0.333 = 4 months = (one third of the year) = the time until the option expires and is worthless.

R_F=7% = Risk free rate stated at an annual rate

Var (R) = 0.144 = variance of returns = The riskiness of an investment in the optioned asset.

$$d_1 = \frac{\ln(\$60/\$50) + [0.07 + 0.5 \times 0.144] \times 0.333}{0.3794 \times 0.5773} = 1.0483$$

$$d_2 = 1.0483 - 0.3794\sqrt{0.333} = 0.8293$$

Table of Standard Normal Cumulative Distribution Function Φ(z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7064	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8339
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319

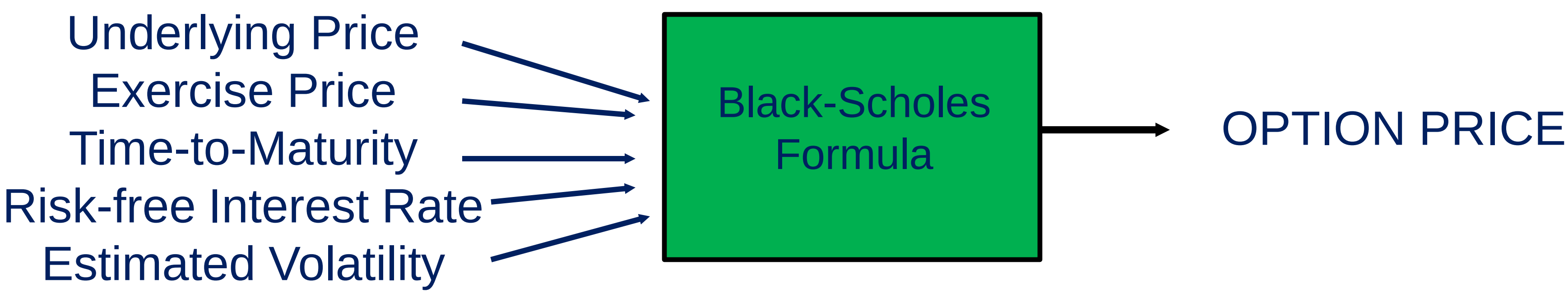
$$c_0 = S_0 N(d_1) - X e^{-rT} N(d_2)$$

$$c_0 = \$60 \times 0.8531 - \$50 e^{-0.07 \times 0.333} \times 0.7967 = \$12.29$$

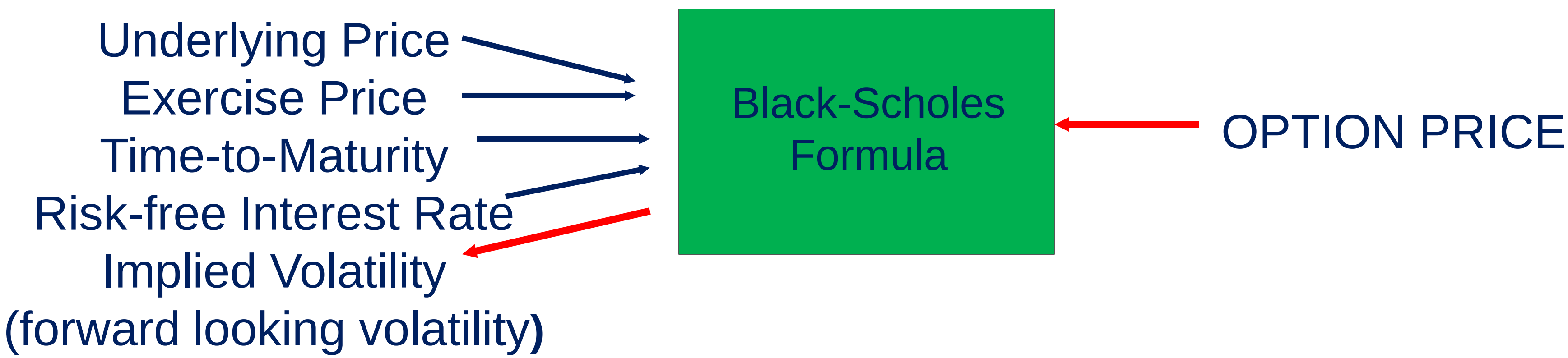
$$p_0 = C_0 + \frac{X}{(1+r)^{1/3}} - S_0 \Leftrightarrow p_0 = \$12.29 + \frac{\$50}{(1.07)^{0.333}} - \$60 = \$1.18$$

Black & Scholes in Practice

Forward Use:



Backward Use:



Example of option valuation using Black-Scholes

What is the value of a European call option with an exercise price of 6.70 and a maturity date of 276 days from now if the current share price is $12.41, standard deviation is 25% p.a. and the risk-free rate is 10.50%. Assume there to be 365 days in a year.

Use the Black-Scholes formula to derive your result.

$$c_0 = P_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$d_1 = \frac{ln(P_0/K) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\left[ln\left(\frac{12.41}{6.70}\right) + \left[0.1050 + \frac{1}{2}(0.25)^2\right]\frac{276}{365}\right]}{0.25\sqrt{\frac{276}{365}}}$$

$$d_1 = 3.3093 \qquad d_2 = 3.3093 - 0.25\sqrt{\frac{276}{365}} = 3.0919$$

$$N(d_1) = 0.9995 \qquad N(d_2) = 0.9990$$

$$\mathrm{C} = 12.41 \times 0.9995 - 6.7 \times e^{-0.105\times\left(\frac{276}{365}\right)} \times 0.9990$$

$$C = 12.4038 - 6.1824 = \$6.2214$$

Question

The stock of Cloverdale Food Processors currently sells for $40. A European Call option on Cloverdale stock has an expiration date six months in the future and a strike price of $38. The estimate of the annual standard deviation of Cloverdale stock is 45 percent, and the risk-free rate is 6 percent. What is the call worth?

Answer Question

$$d1 = \frac{\left(\frac{40}{38}\right) + \left(0.06 + \frac{0.45^2}{2}\right)\frac{1}{2}}{0.45\sqrt{\frac{1}{2}}} = \frac{0.0513 + 0.0806}{0.3182} = 0.4146$$

$$d2 = d1 - \sigma\sqrt{t} = 0.4146 - 0.45\sqrt{\frac{1}{2}} = 0.0964$$

$$N(0.4146) + 0.6608$$

$$N(0.0964) = 0.5384$$

$$C = 40(0.6608) - 38\left(2.718^{-(0.,06)(0.5)}\right)(0.5384) = \$6.58$$

Question

Stock price $S_0 = 100$
Exercise Price $X = 100$ (at the money option)
Maturity $T = 1$ year
Interest rate (continuous) $r = 5\%$
Volatility $\sigma = 0.15$

$$\ln(S_0 / X e^{-rT}) = \ln(1.0513) = 0.05$$

$$d_1 = (0.05)/(0.15) + (0.5)(0.15) = 0.4083$$

$$N(d_1) = 0.6585$$

$$d_2 = 0.4083 - 0.15 = 0.2583$$

$$N(d_2) = 0.6019$$

European call :

100×0.6585 - $100 \times 0.95123 \times 0.6019$

$= 8.60$

$$N(-d_1) = 1 - N(d_1) = 1 - 0.6585 = 0.3415$$

$$N(-d_2) = 1 - N(d_2) = 1 - 0.6019 = 0.3981$$

European put option

$$-100 \times 0.3415 + 95.123 \times 0.3981 = 3.72$$

Questions

A six-month call option with a strike price of $25.00 is selling for $3.50. Assuming the underlying stock price is also $25.00 and the risk-free rate is 6 percent APR, use the following table to determine the volatility (i.e, standard deviation of the return) implied using the option price. (Hint: Price the option using the table to determine which volatility generates a price of $3.50)

Volatility	N(d1)	N(d2)
40%	0,5799	0,4859
45%	0.6000	0.4742
50%	0.6032	0.4634

Option Greeks

Delta

Measures the exposure of option price to movement of underlying stock price

The ratio comparing the change in the price of the underlying asset to the corresponding change in the price of a derivative.

Sometimes referred to as the "hedge ratio"

For example, with respect to call options, a delta of 0.7 means that for every $1 the underlying stock increases, the call option will increase by $0.70.

Put option deltas, on the other hand, will be negative, because as the underlying security increases, the value of the option will decrease.

So a put option with a delta of -0.7 will decrease by $0.70 for every $1 the underlying increases in price.

As an in-the-money call option nears expiration, it will approach a delta of 1.00, and as an in-the-money put option nears expiration, it will approach a delta of -1.00.

For European call options on an asset paying a yield q

$$\Delta(Call) = e^{-qT}N(d_1)$$

$$\Delta(Put) = e^{-qT}[N(d_1) - 1]$$

Gamma

Measures the exposure of the option delta to the movement of the underlying stock price

The rate of change for delta with respect to the underlying asset's price. Gamma is an important measure of the convexity of a derivative's value, in relation to the underlying.

Mathematically, gamma is the first derivative of delta and is used when trying to gauge the price movement of an option, relative to the amount it is in or out of the money.

When the option being measured is deep in or out of the money, gamma is small. When the option is near or at the money, gamma is at its largest. Gamma calculations are most accurate for small changes in the price of the underlying asset.

Theta

Measures the exposure of the option price to the passage of time.

A measure of the rate of decline in the value of an option due to the passage of time.

Theta can also be referred to as the time decay on the value of an option. If everything is held constant, then the option will lose value as time moves closer to the maturity of the option.

For example, if the strike price of an option is $1,150 and theta is 53.80, then in theory the value of the option will drop $53.80 per day.

The measure of theta quantifies the risk that time imposes on options as options are only exercisable for a certain period of time. Time has importance for option traders on a conceptual level more than a practical one, so theta is not often used by traders in formulating the value of an option.

Vega

Measures the exposure of the option price to changes in volatility of the underlying

The measurement of an option's sensitivity to changes in the volatility of the underlying asset.

Vega represents the amount that an option contract's price changes in reaction to a 1% change in the volatility of the underlying asset.

Swaps

“Plain vanilla” Interest rate swap (most common type of swap).

Company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years.

In return, it receives at a floating rate on the same notional principal for the same period of time.

LIBOR (London Interbank Offered Rate): Floating rate used in most interest rate swaps agreements.
Rate of interest at which a bank is prepared to deposit money with other banks in the Eurocurrency market.
Typically, 1-month, 3-month, 6-month and 12-month (LIBOR is quoted in all major currencies)

Example

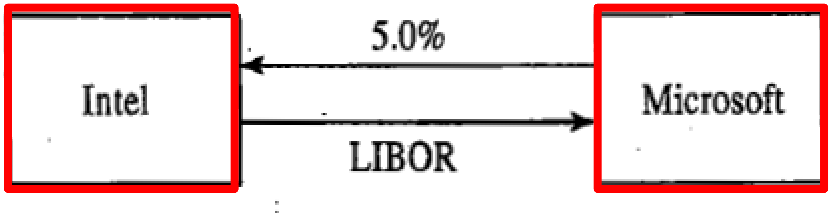
3-year swap initiated in March, 5th, 2007, between Microsoft and Intel.

Microsoft agrees to pay Intel an interest rate of 5% per annum on a principal of $100 million.

Intel agrees to pay Microsoft the 6-month LIBOR rate on the same principal.

Microsoft: fixed-rate payer
Intel: Floating rate payer

We assume that payments are to be exchanged every 6-months and that the 5% interest rate is quoted with semi-annual compounding.



Cash Flows (millions of dollars) to Microsoft in a \$100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR received

Date	Six-month Libor (%)	Floating cash flow received	Fixed cash flow paid	Net cash flow
Mar. 5, 2007	4.20			
Sept. 5, 2007	4.80	+2.10	-2.50	-0.40
Mar. 5, 2008	5.30	+2.40	-2.50	-0.10
Sept. 5, 2008	5.50	+2.65	-2.50	+0.15
Mar. 5, 2009	5.60	+2.75	-2.50	+0.25
Sept. 5, 2009	5.90	+2.80	-2.50	+0.30
Mar. 5, 2010		+2.95	-2.50	+0.45

Using the Swap to Transform a Liability

Swap can be used to transform a floating-rate loan into a fixed-rate loan.

Suppose:
Microsoft has arranged to borrow $100 million at LIBOR plus 10 basis points.
Three sets of cash flows:

1. It pays LIBOR plus 0.1% to its outside lenders
2. It receives LIBOR under the terms of the swap
3. It pays 5% under the terms of the swap

Microsoft swap have the effect of transforming borrowing at a floating rate of LIBOR plus 10 basis points into borrowings at a fixed rate of 5.1%

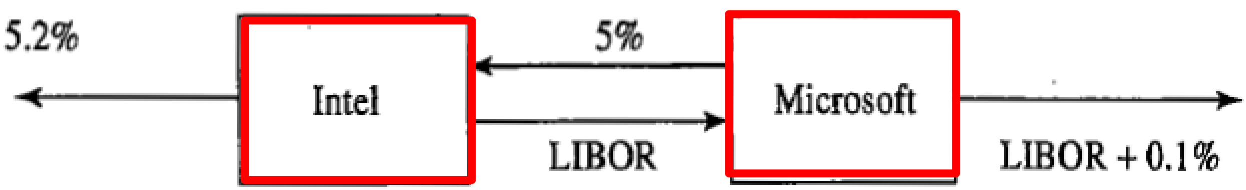
For Intel, the swap could have the effect of transforming a fixed-rate loan into a floating–rate loan.

Suppose, that Intel has a 3-year $100 million loan outstanding on which it pays 5.2%. After it has entered into the swap, it has the following three sets of cash flows:

1. It pays 5.2% to its outside lenders
2. It pays LIBOR under the terms of the swap
3. It receives 5% under the terms of the swap

For Intel, the swap have the effect of transforming borrowings at a fixed rate of 5.2% into borrowings at a floating rate of LIBOR plus 20 basis points.

Microsoft and Intel use the Swap to transform a liability



Using the Swap to Transform an Asset

Swaps can also be used to transform the nature of an asset.

Consider Microsoft in our example.

The swap could have the effect of transforming an asset earning a fixed rate of interest into an asset earning a floating rate of interest.

Suppose Microsoft owns $100 million in bonds that will provide interest at 4.7% per annum over the next 3 years.

After Microsoft has entered into a swap, it has the following three sets of cash flows:

1. It receives 4.7% on the bonds
2. It receives LIBOR under the terms of the swap
3. It pays 5% under the terms of the swap

One possible use of the swap for Microsoft is to transform an asset earning 4.7% into an asset earning LIBOR minus 30 basis points.

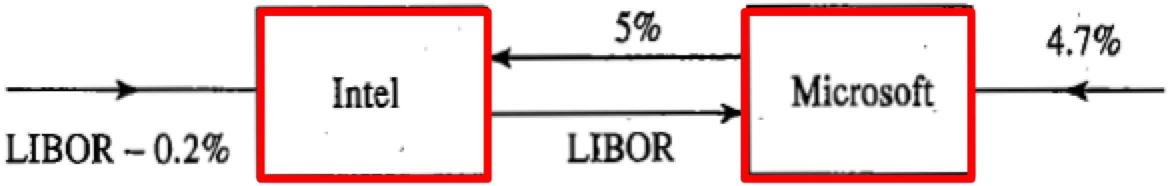
In case of Intel, the swap could have the effect of transforming an asset earning a floating rate of interest into an asset earning a fixed rate of interest.

Suppose that Intel has an investment of $100 million that yields LIBOR minus 20 basis points. After it has entered into the swap, has the following three sets of cash flows:

1. It receives LIBOR minus 20 basis points on its investment
2. It pays LIBOR under the terms of the swap
3. It receives 5% under the terms of the swap

Possible use of swap for Intel is to transform and asset earning LIBOR minus 20 basis points into an asset earning 4.8%.

Microsoft and Intel use the Swap to transform an asset



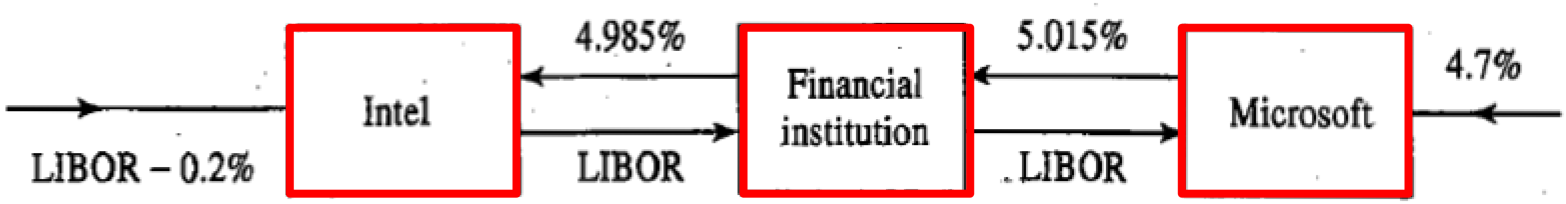
Role of Financial Intermediary

Usually two nonfinancial companies such as Intel and Microsoft do not get in touch directly to arrange a swap.

They each deal with a financial intermediary such as a bank or other financial institution.

"Plain vanilla" fixed-for-floating swaps on US interest rates are usually structured so that the financial institutions earns about 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions.

Interest rate swap when financial institution is involved



Currency Swaps

Swap that involves exchanging principal and interest payments in one currency for principal and interest in another.

A currency swap agreement requires the principal to be specified in each of the two currencies.

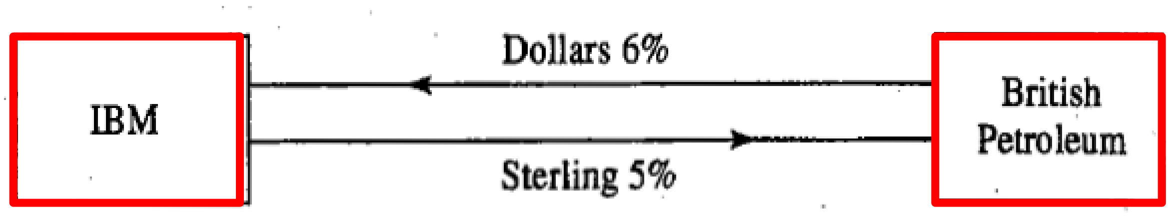
Example:

Consider a hypothetical 5-year currency swap agreement between IBM and British Petroleum entered into on February, 1, 2007.

We suppose that IBM pays a fixed rate of interest of 5% in sterling and receives a fixed rate of interest of 6% in dollars from British Petroleum.

Interest rate payments are made once a year and the principal amounts are $18 million and £10 million.

This is termed a fixed-to-fixed currency swap because the interest rate in both currencies is fixed.



Date	Dollar Cash Flow (millions)	Sterling Cash Flow (millions)
February, 1, 2007	-18.00	+10.00
February, 1, 2008	+1.08	-0.50
February, 1, 2009	+1.08	-0.50
February, 1, 2010	+1.08	-0.50
February, 1, 2011	+1.08	-0.50
February, 1, 2012	+19.08	-10.50

Use of a Currency Swap to Transform Liabilities

A swap can be used to transform borrowings in one currency to borrowings in another.

Suppose IBM can issue $18 million of US-dollar-denominated bonds at 6% interest.

The swap has the effect of transforming this transaction into one where IBM has borrowed £10 million at 5% interest.

The initial exchange of principal converts the proceeds of the bond issue from US dollars to sterling.

The subsequent exchanges in the swap have the effect of swapping the interest and principal payments from dollars to sterling

Use of a Currency Swap to Transform Assets

The swap can also be used to transform the nature of assets.

Suppose that IBM can invest £10 million in the UK to yield 5% per annum for the next 5 years.

However IBM feels the US dollar will strengthen against sterling and prefers US-dollar-denominated investment.

The swap has the effect of transforming the UK investment into a $18 million investment in the US yielding 6%.

Comparative Advantage

Currency swaps can be motivated by comparative advantage.

Suppose the 5-year fixed-rate borrowing costs to General Electric and Qantas Airways in US dollars (USD) and Australian dollars (AUD) are as below:

	USD	AUD
General Electric	5.0%	7.6%
Qantas Airways	7.0%	8.0%

Australian rates are higher than USD interest rates
General Electric is more creditworthy than Qantas Airways

From the viewpoint of a swap trader, the interesting aspect is that the spreads between the rates paid by General Electric and Qantas Airways in the two markets are not the same.

Suppose that General Electric wants to borrow 20 million AUD and Qantas Airways wants to borrow 15 million USD and the current exchange rate (USD per AUD) is 0.7500.

General Electric and Qantas Airways each borrow in the market where they have a comparative advantage.

- General Electric will borrows USD
- Qantas Airways will borrow AUD

They then use a currency swap to transform General Electric’s loan into an AUD loan and Qantas Airways’ loan into USD loan.

We expect the total gain to all parties to be 2%-0.4%=1.6% per annum

General Electric borrows USD
Qantas Airways borrows AUD

Effect of the swap is to transform the USD interest rate of 5% per annum to an AUD interest rate of 6.9% per annum for General electric.

General Electric is 0.7% per annum better off than it would be if it went directly to AUD markets.

Qantas exchanges an AUD loan at 8% per annum for a USD loan at 6.3% per annum and ends up 0.7% per annum better off than it would be if it went directly to USD markets.

The financial institution gains 1.3% per annum on its USD cash flows and loses 1.1% per annum on its AUD flows.

The financial institution makes a net gain of 0.2% per annum (ignoring the difference between the two currencies).

The total gain to all parties is 1.6% per annum.

Forward Rate Agreements

Forward Rate Agreement: is a contract that specifies a cash payment at contract maturity determined by the difference between an agreed interest rate and the realized interest rate at maturity.

There are FRAs on Eurodollar deposit rates (LIBOR) and FRAs on euro deposit rates (Euribor).

Example:

Consider a one-month FRA contract, expiring in 30 days, based on 3-month LIBOR. The underlying rate on the contract is the 3-month LIBOR that will prevail in 30 days. Suppose the two parties to the contract agree on a fixed rate of 2.5%. The buyer of the FRA will receive a payment from the seller if the actual 3-month LIBOR rate at expiration of the FRA contract is greater than 2.5%. The seller of the FRA will receive a payment from the seller if the actual 3-month LIBOR rate at expiration of the FRA contract is less than 2.5%.

Calculate and Interpret the payoff of a FRA

Two parties agree to make a loan to the other at the maturity of the FRA. They enter in a 30-day FRA contract based on 3-month LIBOR with a FRA (fixed) rate of 2.5% and a notional of $100,000,000.
This is s 1×4 FRA maturing in 1 month, at which time a 3-month loan will be exchanged (a relationship lasting a total of 4 months).

At maturity date (30 days from inception)
The seller of the FRA agrees to make a $100 million loan to the buyer at a rate of 2.5% for three months (buyer pays 2.5% interest to the FRA seller).
The buyer of the FRA agrees to loan the seller $100 million at whatever 3-month LIBOR is at maturity, again for 3 months.

No money actually changes hands at the inception of the FRA.

30 days later, 3 month LIBOR is 2.73%.

$$Interest\ Paid\ by\ FRA\ Buyer = Notional \times \left[Interest\ Rate \times \frac{t_{days}}{360}\right]$$
$$= \$100{,}000{,}000 \times \left[0.0250 \times \frac{90}{360}\right] = \$625{,}000$$

$$Interest\ Paid\ by\ FRA\ Seller = Notional \times \left[Interest\ Rate \times \frac{t_{days}}{360}\right]$$
$$= \$100{,}000{,}000 \times \left[0.0273 \times \frac{90}{360}\right] = \$682{,}500$$

The seller will simply pay the buyer the present value of the difference between the interest payments discounted at the current 3-month LIBOR rate.

$$Payment\ to\ FRA\ Buyer = PV_{LIBOR}\ of\ (Interest\ Owned\ to\ Buyer\ - Interest\ Owned\ to\ Seller)$$

$$Payment\ to\ FRA\ Buyer = \frac{\$682{,}500 - \$625{,}000}{1 + 0.0273 \times \left(\frac{90}{360}\right)} = \$57{,}110.22$$

$$FRA\ Payment = Notional\ Amount \times \frac{(Actual\ Rate - Agreed\ Rate)\left(t_{days}/360\right)}{1 + Actual\ Rate\left(t_{days}/360\right)}$$

Currency Forward Agreements

Currency forwards involve two parties who agree to exchange currencies at a future date and specified exchange rates.

Example:
Bank A agrees to buy £50,000,000 in six months from Bank B who agrees to sell the pounds at \$1.10/ £. If, in six months, the actual exchange rate is only \$1.05/ £, Bank A will suffer a loss of \$2.5 million.
Gain or loss calculation:

$$\textit{Gain or Loss on Forward Contract} =$$
$$(\textit{Actual Exchange Rate} - \textit{Contract Exchange Rate}) \times \textit{Notional}$$

$$=(\$1.05/£ - \$1.10/£) \times -50{,}000{,}000 = -\$2{,}500{,}000$$

Bank B on the other hand will make a profit of \$2.5 million, because it can buy the £50,000,000 in the spot market at \$1.05/ £ and sell it to Bank B at at \$1.10/ £.

A forward agreement is a zero-sum game.