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# FINA 1082 Financial Management

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## Session 5

The Capital Asset Pricing Model

(The Hypothesized Relationship between Risk and Return)

Fama-French (1993) Three factor Model

Carhart (1997) Four factor Model

Measuring Performance of Market-Timing Funds

Persistence of Performance

May, 17, 2014

# The Capital Asset Pricing Model

Asset Pricing: how assets are priced?

## The equilibrium concept

### *Portfolio Theory*

- ANY individual investor's optimal selection of portfolio (*partial equilibrium*)

### *CAPM*

- Equilibrium of ALL individual investors (*general equilibrium*)

Risky asset  $i$ : Its price is such that

$$E(r) = R_F + \text{Risk Premium specific to asset } i$$

$$R_F + (\text{market price of risk}) \times (\text{quantity of risk of asset } i)$$

# The Capital Asset Pricing Model

The *amount of risk* is measured by the covariance of the asset with the market portfolio

The *market price of risk* is the return above the risk-free rate that investors earn for holding the (risky) market portfolio

The *risk premium* can be thought of as a “price” times “quantity” relationship

Higher the market price of risk and/or higher the amount of risk, *greater the risk premium*

# The Capital Asset Pricing Model: What is it?

Hypothesizes that investors require higher rates of return for greater levels of relevant risk

There are no prices on the model, instead it hypothesizes the relationship between risk and return for individual securities.

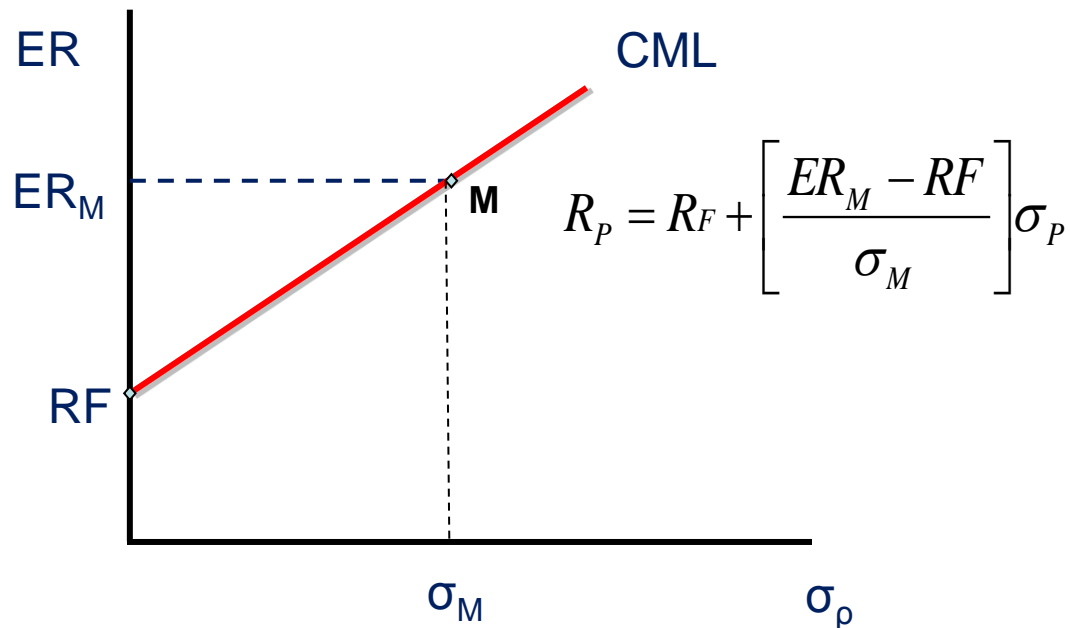
It is often used, however, to price securities and investments

# Assumptions

- one period investment horizon
- rational, risk-averse investors
- unlimited borrowing and lending is allowed at a risk free rate that is the same for all investors
- there are no taxes
- there are no transaction costs and inflation
- all assets are infinitely divisible
- free flow and instant availability of information
- there are many investors on the market
- all assets are marketable
- all investors have homogeneous expectations about expected returns, variances and covariances of assets

# Market Portfolio and Capital Market Line

The assumptions have the following implications:



The “optimal” risky portfolio is the one that is tangent to the efficient frontier on a line that is drawn from  $R_F$ . This portfolio will be the same for all investors.

This optimal risky portfolio will be the *market portfolio* (M) which contains all risky securities.

The CML has standard deviation of portfolio returns as the independent variable

The **slope of the CML** is the incremental expected return divided by the incremental risk.

$$\text{Slope of the CML} = \frac{ER_M - R_F}{\sigma_M}$$

This is called the **market price for risk**, or the **equilibrium price of risk** in the capital market

Solving for the expected return on a portfolio in the **presence of a RF asset** and **given the market price for risk**:

$$E(R_P) = R_F \left[ \frac{ER_M - R_F}{\sigma_M} \right] \sigma_P$$

$ER_M$  = expected return on the market portfolio M

$\sigma_M$  = the standard deviation of returns on the market portfolio

$\sigma_P$  = the standard deviation of returns on the efficient portfolio being considered



# The New Efficient Frontier

## The Implications – Separation Theorem – Market Portfolio

All investors will only hold individually-determined combinations of:

- The risk free asset (RF) and
- The model portfolio (market portfolio)

## The separation theorem

- The investment decision (how to construct the portfolio of risky assets) is separate from the financing decision (how much should be invested or borrowed in the risk-free asset)
- The tangent portfolio T is optimal for every investor regardless of his/her degree of risk aversion.

## The Equilibrium Condition

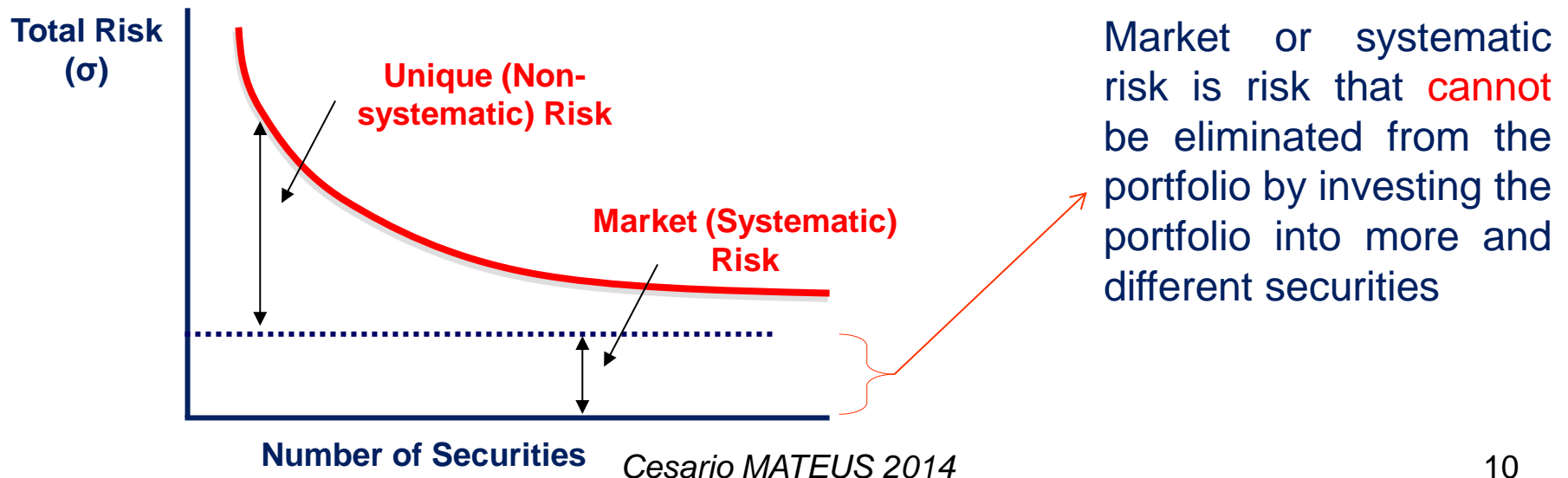
- The market portfolio must be the tangent portfolio T if everyone holds the same portfolio
- Therefore the market portfolio (M) is the tangent portfolio (T)

# Diversifiable and Non-Diversifiable Risk

CML applies to efficient portfolios

Volatility (risk) of *individual security returns* are caused by two different factors:

- **Non-diversifiable risk** (system wide changes in the economy and markets that affect all securities in varying degrees)
- **Diversifiable risk** (company-specific factors that affect the returns of only one security)



## Relevant Risk

Previous figure demonstrates that an individual securities' volatility of return comes from two factors:

- Systematic factors
- Company-specific factors

When combined into portfolios, company-specific risk is diversified away.

Since all investors are 'diversified' then in an efficient market, no-one would be willing to pay a 'premium' for company-specific risk.

Relevant risk to diversified investors then is systematic risk.

Systematic risk is measured using the Beta Coefficient

# Measuring Systematic Risk

## The Beta Coefficient

### What is the Beta Coefficient?

A measure of **systematic** (non-diversifiable) **risk**

As a '**coefficient**' the beta is a pure number and has no units of measure.

### How Can We Estimate the Value of the Beta Coefficient?

- Using a formula (and subjective forecasts)
- Use of regression (using past holding period returns)

## The Characteristic Line for Security A



The plotted points are the coincident rates of return earned on the investment and the market portfolio over past periods

## The Formula for the Beta Coefficient

Beta is equal to the covariance of the returns of the stock with the returns of the market, divided by the variance of the returns of the market

$$\beta_i = \frac{Cov_{i,M}}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i}{\sigma_M}$$

# How is the Beta Coefficient Interpreted?

The beta of the market portfolio is **ALWAYS = 1.0**

The **beta of a security** compares the volatility of its returns to the volatility of the market returns:

$$\beta_s = 1.0$$

- the security has the same volatility as the market as a whole

$$\beta_s > 1.0$$

- aggressive investment with volatility of returns greater than the market

$$\beta_s < 1.0$$

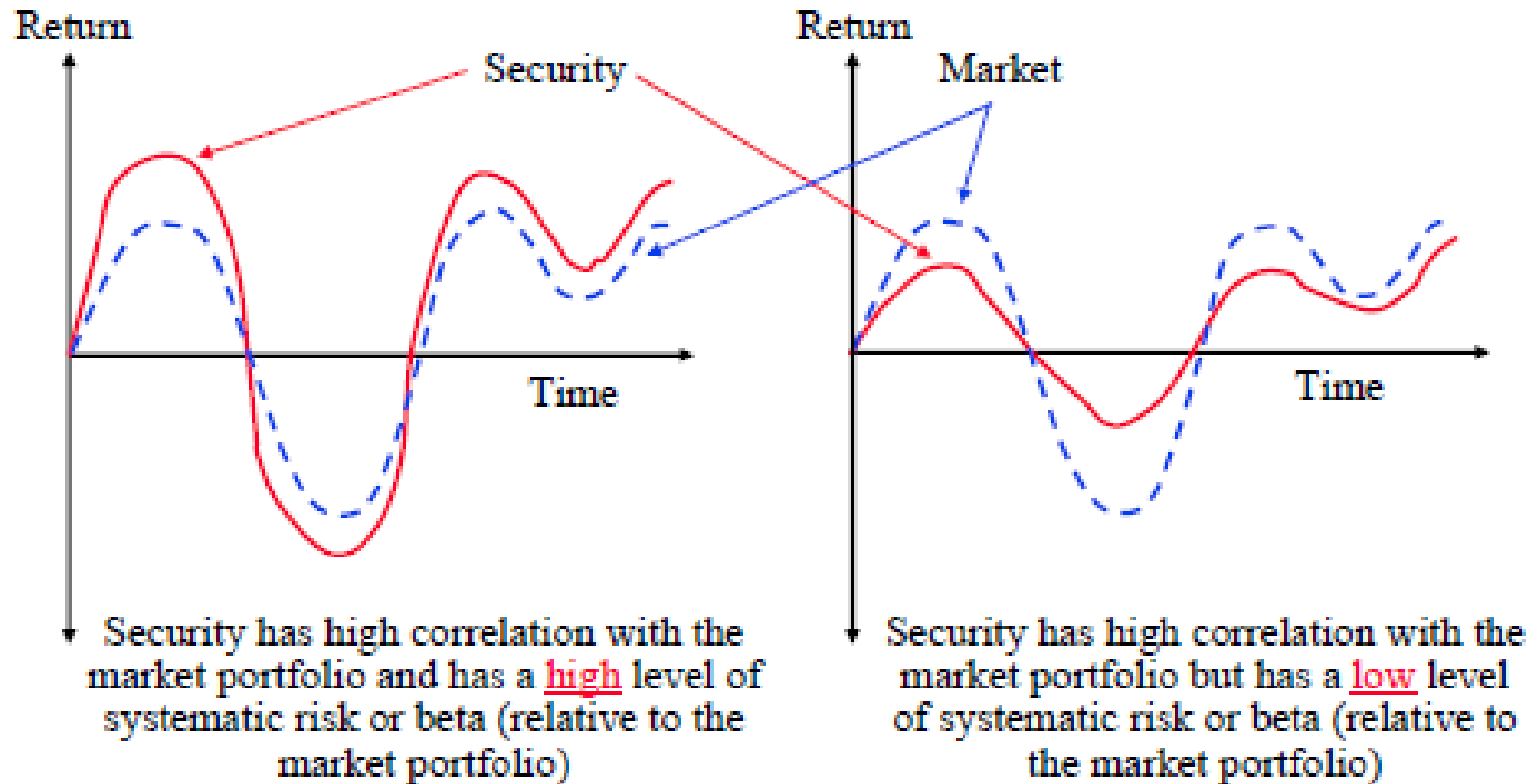
- defensive investment with volatility of returns less than the market

$$\beta_s < 0.0$$

- an investment with returns that are negatively correlated with the returns of the market

# Betas and Correlations

**Beta** is not the same as the correlation between a security (portfolio) and the market portfolio





## The Beta of a Portfolio

The beta of a portfolio is simply the weighted average of the betas of the individual asset betas that make up the portfolio

$$\beta_P = w_A\beta_A + w_B\beta_B + \cdots + w_n\beta_n$$

Weights of individual assets are found by dividing the value of the investment by the value of the total portfolio.

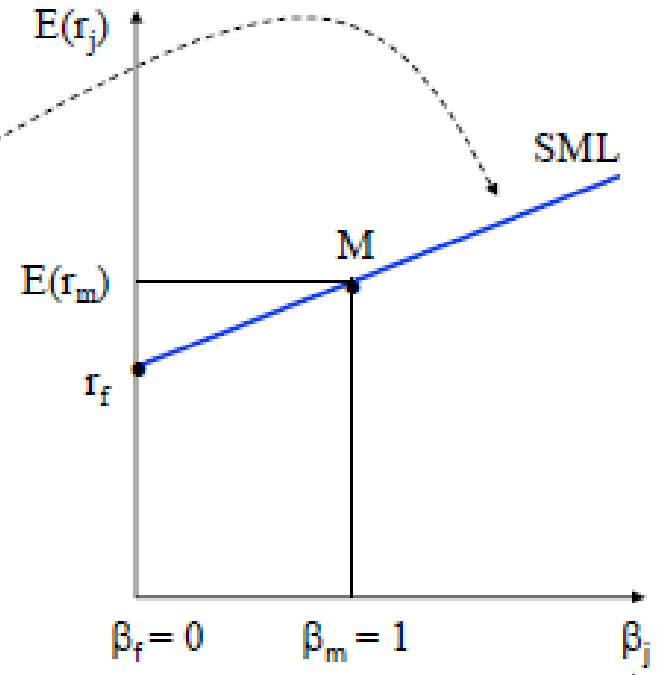
# The Security Market Line

In equilibrium, all risky securities are priced so that their expected returns plot on the SML

$$E(R_i) = R_F + \beta_j [E(R_M) - R_F]$$

Assets with  $\beta_j$  less (more) than 1 earn an expected return lower (higher) than the market Portfolio

**Note:** The x-axis of the CML (used to “price” efficient portfolios) differs from the x-axis of the SML (used to “price” individual assets)



# Prices and the CAPM

The return of an asset  $i$  is given by:

$$E(R_i) = \frac{P_1 + D_1}{P_0} - 1$$

The CAPM postulate that:

$$E(R_i) = \frac{P_1 + D_1}{P_0} - 1 = R_F + \beta_i(E(R_M) - R_F)$$

From which we have:

$$P_0 = \frac{P_1 + D_1}{1 + R_F + \beta_i(E(R_M) - R_F)} = \frac{P_1 + D_1}{1 + E(R_i)}$$

**Fundamental Price Equation:** The price of an asset is its discounted cash flow, where the discounted factor is the equilibrium rate of return.

# Relationship Between Prices and Returns

## Class Exercise 1:

Oz Ltd's dividend is expected to be \$1.00 per share next year and remain unchanged in the future (i.e.,  $g = 0$ ). The following information is given:

Oz Ltd's beta = 1.2

Riskfree rate,  $r_f = 6\%$

Expected market risk premium,  $[E(r_m) - r_f] = 7\%$

- a) What price should Oz Ltd be selling for today?
- b) What will happen to Oz price if, after a market crash, analysts change their estimate of Oz beta to 1.5 and no other change occurs? Explain
- c) What general relationship between prices and returns is being illustrated here?

# Relationship Between Prices and Returns

## a) Based on the CAPM

$$E(r) = 0.06 + 0.07(1.2) = 0.144 \text{ or } 14.4\%$$

$$P_0 = 1.00/0.144 = \$6.94$$

## b) Based on the new beta estimate of 1.5, we have

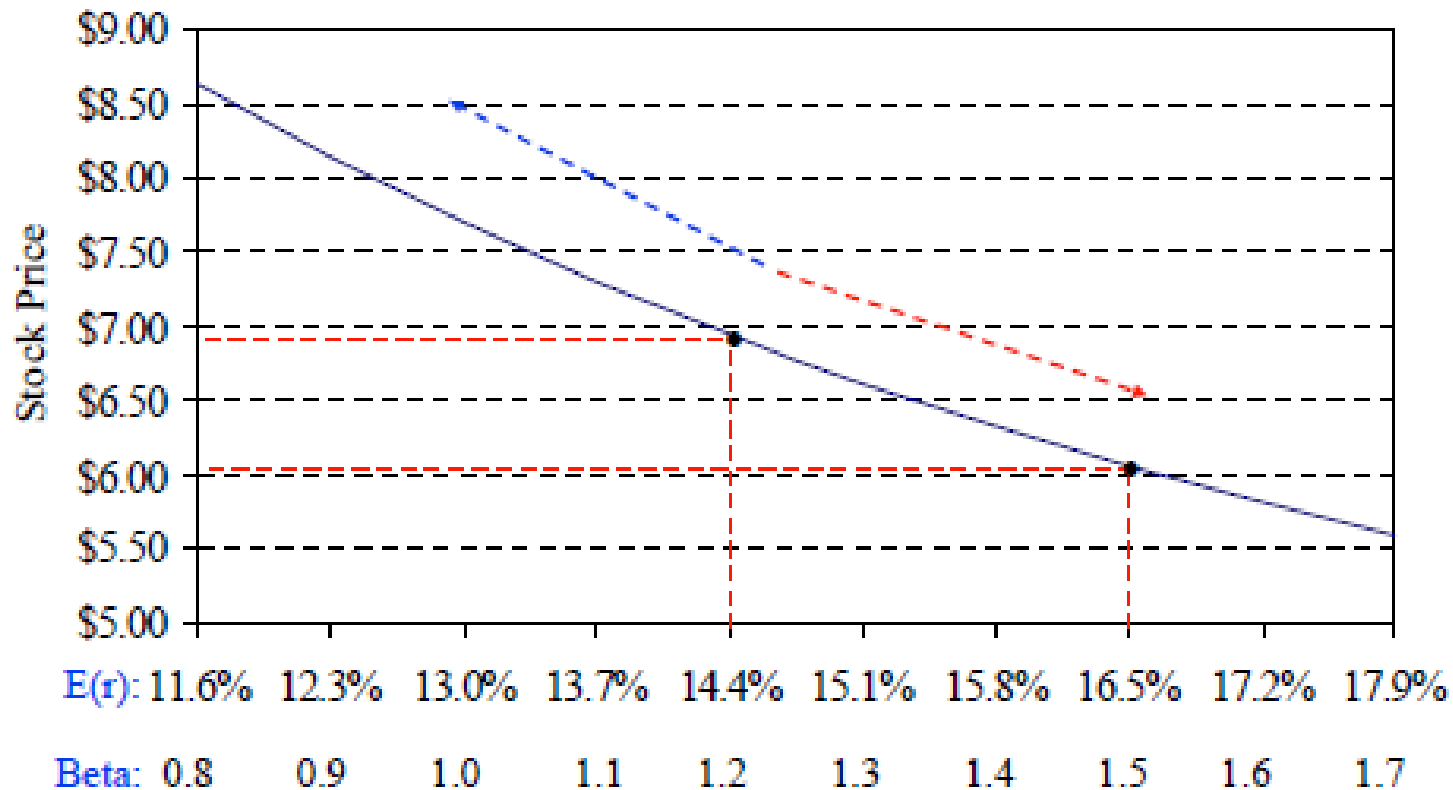
$$\text{Revised } E(r) = 0.06 + 0.07(1.5) = 0.165 \text{ or } 16.5\%$$

$E(r)$  has increased but at \$6.94 investors earn only 14.4% Investors will move funds to other similar risk securities which offer a higher expected return of 16.5%

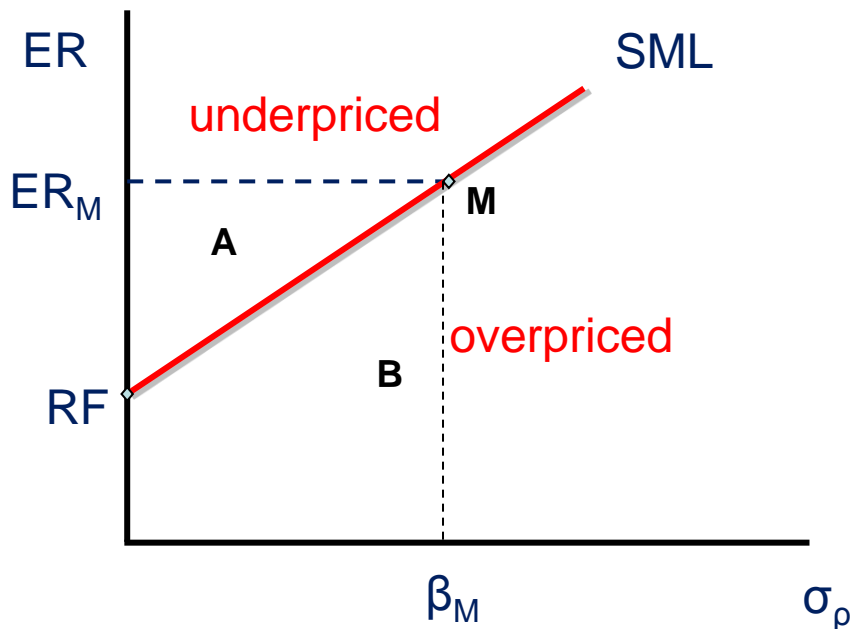
The selling pressure results in a new price

$$\text{New } P_0 = 1.00/0.165 = \$6.06$$

# Relationship Between Prices and Returns



# SML and Overvalued/Undervalued Securities



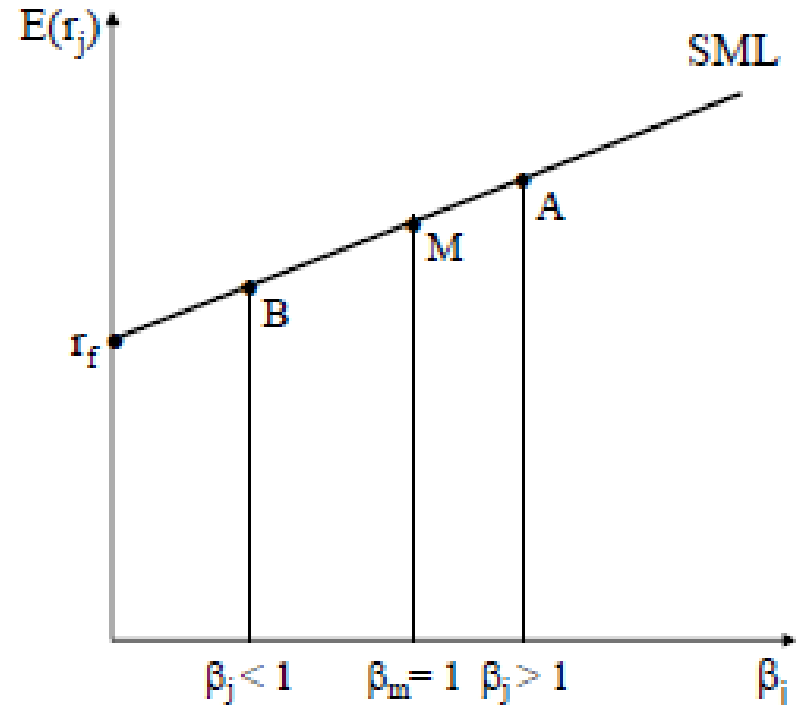
**Undervalued Securities:** plotted above the SML because they offer greater expected return for a given level of risk, implying that their prices are low. Investors will recognize the **arbitrage opportunity** and they will start buying those securities. The increase in the demand will drive prices of underpriced securities up, their returns down and the security will eventually be driven to the SML level.

The opposite process will happen to overvalued securities.

# Movements in the Security Market Line

**Application:** What happens to the SML in the following cases

- a) There is an unexpected increase in the market risk premium
- b) There is an unexpected decrease in the risk-free rate





# Movements in the Security Market Line

*a) An unexpected increase in the market risk premium*

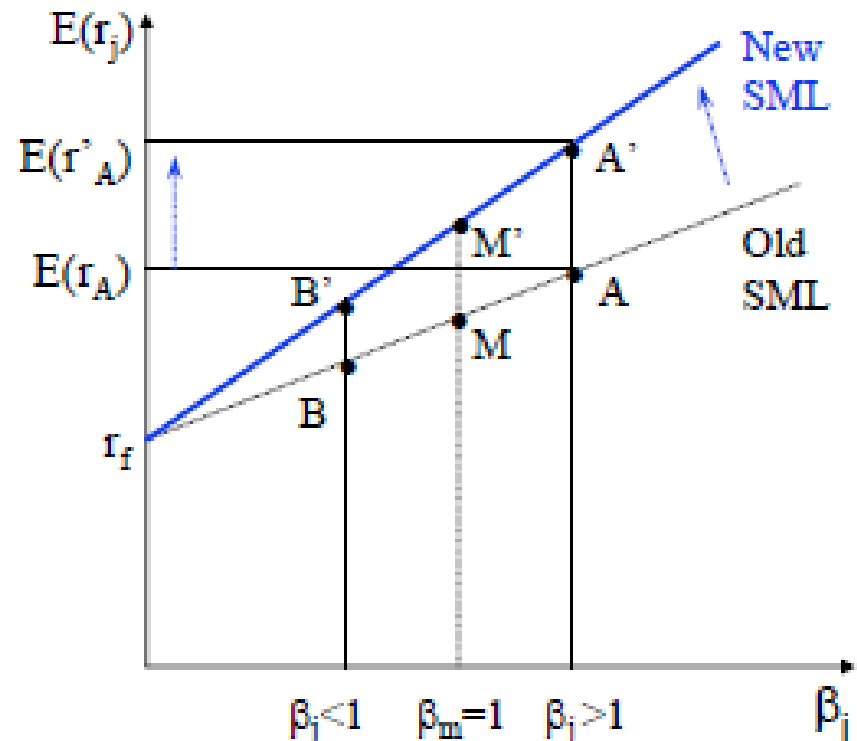
$[E(R_M) - R_F]$  *increases*

*The SML is steeper  
(assuming  $R_F$  unchanged)*

*$E(R)$  of asset A increases so A's price will fall*

*$E(R)$  of the lower risk asset B will rise less than the  $E(R)$  of the higher risk asset A*

*$E(R_M)$  also increases so the market will fall in value*



# Movements in the Security Market Line

*b) An unexpected decrease in the risk-free rate*

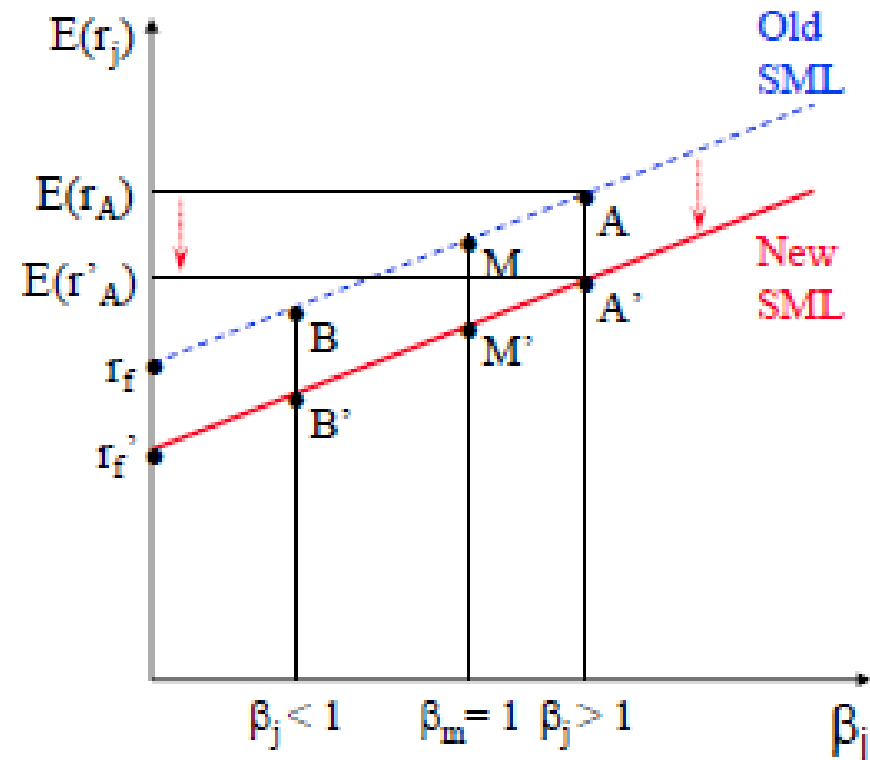
*$R_F$  decreases: assume no change in the market risk premium  $[E(R_M) - R_F]$*

*Implies a downward, parallel shift in the SML*

*$E_R$  of Asset A decreases so the price of A will rise*

*$E(R_M)$  also falls so the market will rise in value*

*Expected fall in  $E(R_B) =$  Expected fall in  $E(R_A)$*



# Using the Security Market Line

**Class Exercise 2:** Assume that the risk-free rate is 7% and the expected market return is 12%

a) Locate the expected returns for securities with the following betas on the SML

- $\beta_A = 1.5$
- $\beta_B = 0.5$
- $\beta_C = -0.5$

b) Will an investor ever invest in a security like security C?  
Why or why not?

## Answer to Class Exercise 2

a) *Given:*  $r_f = 7\%$  ,  $E(r_m) = 12\%$

$$E(r_m) - r_f = 12 - 7 = 5\%$$

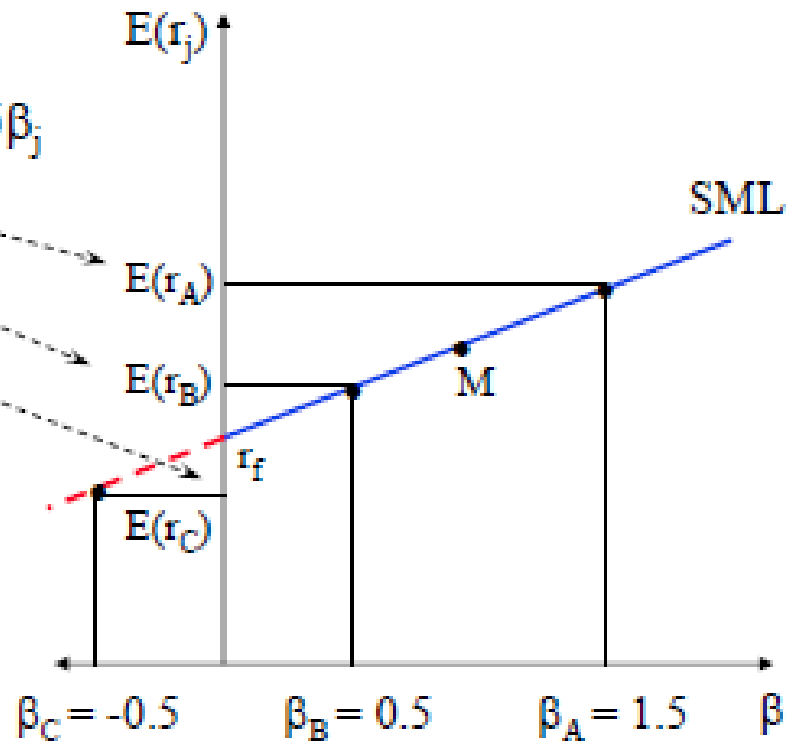
$$E(r_j) = r_f + [E(r_m) - r_f]\beta_j = 7 + 5\beta_j$$

$$E(r_A) = 7 + (5)1.5 = 14.5\%$$

$$E(r_B) = 7 + (5)0.5 = 9.5\%$$

$$E(r_C) = 7 - (5)0.5 = 4.5\%$$

b) Why would an investor ever choose security C when its expected return is below the riskfree rate?



# Using the Security Market Line

**Class Exercise 3:** You are given the following incomplete information

Security/ Portfolio	Beta	Expected Return	Standard Deviation
X	?	10%	10%
Y	1.0	12%	20%
Riskfree	?	7%	?
Market	?	?	15%

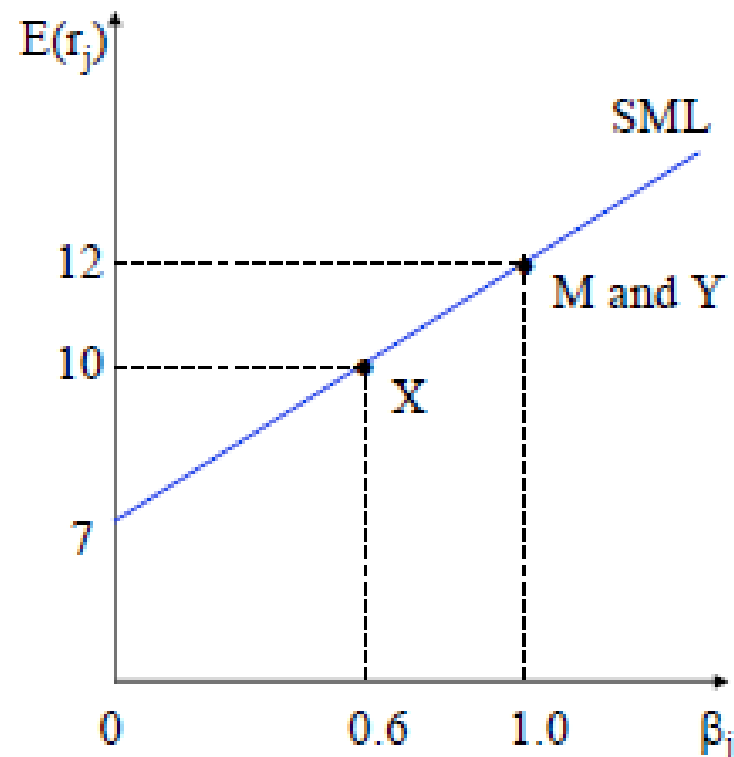
- Complete the above table and show all your calculations
- Draw a graph of the security market line and locate the above securities and portfolio on it
- Compute the beta of a portfolio with \$7,500 in X and \$2,500 in Y
- Compute the required return on this portfolio
- Evaluate the risk of this portfolio

## Answer to Class Exercise 3

Completed table

Security	Beta	E(r)	SD(r)
X	0.6	10%	10%
Y	1.0	12%	20%
Riskfree	0.0	7%	0%
Market	1.0	12%	15%

- ❖  $E(r_m) = E(r_y) = 12\%$  ( $\beta_y = \beta_m = 1$ )
- ❖  $E(r_x) = 10 = 7 + (12 - 7) \beta_x$
- ❖  $\beta_x = (10 - 7)/(12 - 7) = 0.6$
- ❖  $\beta_p = w_X \times \beta_X + w_Y \times \beta_Y$
- ❖ So,  $\beta_p = 0.75(0.6) + 0.25(1.0) = 0.7$
- ❖  $E(r_p) = 7 + (12 - 7)0.7 = 10.5\%$
- ❖ Risk evaluation of portfolio?



## Answer to Class Exercise 3

Beta of a portfolio with \$7,500 in X and \$2,500 in Y

$$w_x = 7500/(2500 + 7500) = 0.75 \quad \text{and} \quad w_y = 1 - w_x = 0.25$$

$$\beta_p = w_x \beta_x + w_y \beta_y = 0.75(0.6) + 0.25(1.00) = 0.7$$

Security X dominates portfolio beta

Required return on this portfolio

$$E(r_p) = 7 + (12 - 7) \beta_p = 7 + 5(0.7) = 10.5\%$$

Risk evaluation of this portfolio

Beta less than 1.0: portfolio is less risky than the market

Portfolio's expected return expected to change by 0.7% when the market portfolio's expected return changes by 1%

# The use of Betas

## Design portfolios suited to investors' risk preferences

- Low beta portfolios for less risk tolerant investors
- High beta portfolios for more risk tolerant investors

## Evaluate portfolio performance

- On average, high beta portfolios should outperform the market portfolio
- If market return increases by 10%, a portfolio with beta 2.0 should experience an increase in returns of 20%

## Estimate cost of equity capital – Equity Investments Analysis module

- Estimates better reflect market's risk and expected return expectations



# Questions about the CAPM

## 1. Are the CAPM's assumptions realistic?

- A model's assumptions are simplifications of reality
- The real concern is how the CAPM performs empirically

## 2. Can the CAPM be tested?

- CAPM refers to expected returns - not realized or observed returns
- Market portfolio comprises all risky assets - is it measurable?
- $\beta$  is a measure of systematic risk which is expected to be applicable in the future
  - Does past history give us a good estimate of the future beta?

## 3. Is the CAPM empirically valid?

# A Simple Test

Estimate  $\beta_j$  for a number of portfolios of securities over some time period,  $t$

- Estimates of beta are unreliable for individual securities so group securities into portfolios (e.g., 20-30 securities each) and perform tests on groups of portfolios

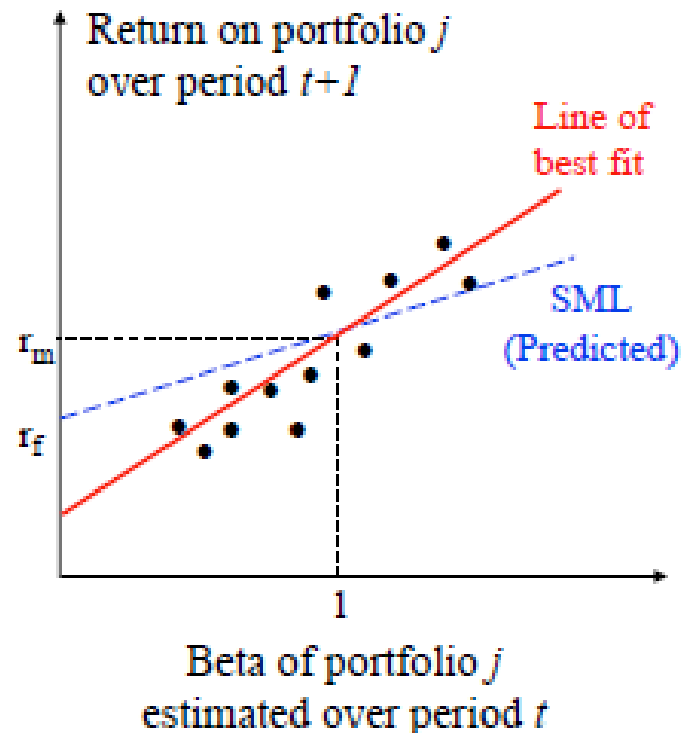
Obtain data for average  $r_j$  for each portfolio over some reasonably long *subsequent time period,  $t+1$*

Regress  $r_j$  against  $\beta_j$

- Intercept should equal  $r_f$  for the test period,  $t+1$
- Slope should equal  $(r_m - r_f)$  in the test period,  $t+1$

## Typical Results of Simple Test

- $\beta$  is an important variable...
- Intercept generally not equal to  $r_f$
- Choice of market index proxy can affect the results
- Curvilinear relationship may fit data better
- Other risk measures, such as variance, firm-size, etc may also help explain observed returns



# CAPM and Market Anomalies

The existence of market anomalies is inconsistent with the CAPM

Some findings across time

- Returns lower on Mondays than on other days
- Returns higher in January compared to other months (especially for small firms)
- Returns higher the day before a holiday
- Returns higher at the beginning and end of the trading day

Some findings across securities (holding  $\beta$  constant)

- Returns higher for firms with “low” price-earnings ratios
- Returns higher for smaller firms compared to larger firms
- Returns higher for firms with higher book-to-market value of equity ratios

## Some of the Problems With the CAPM

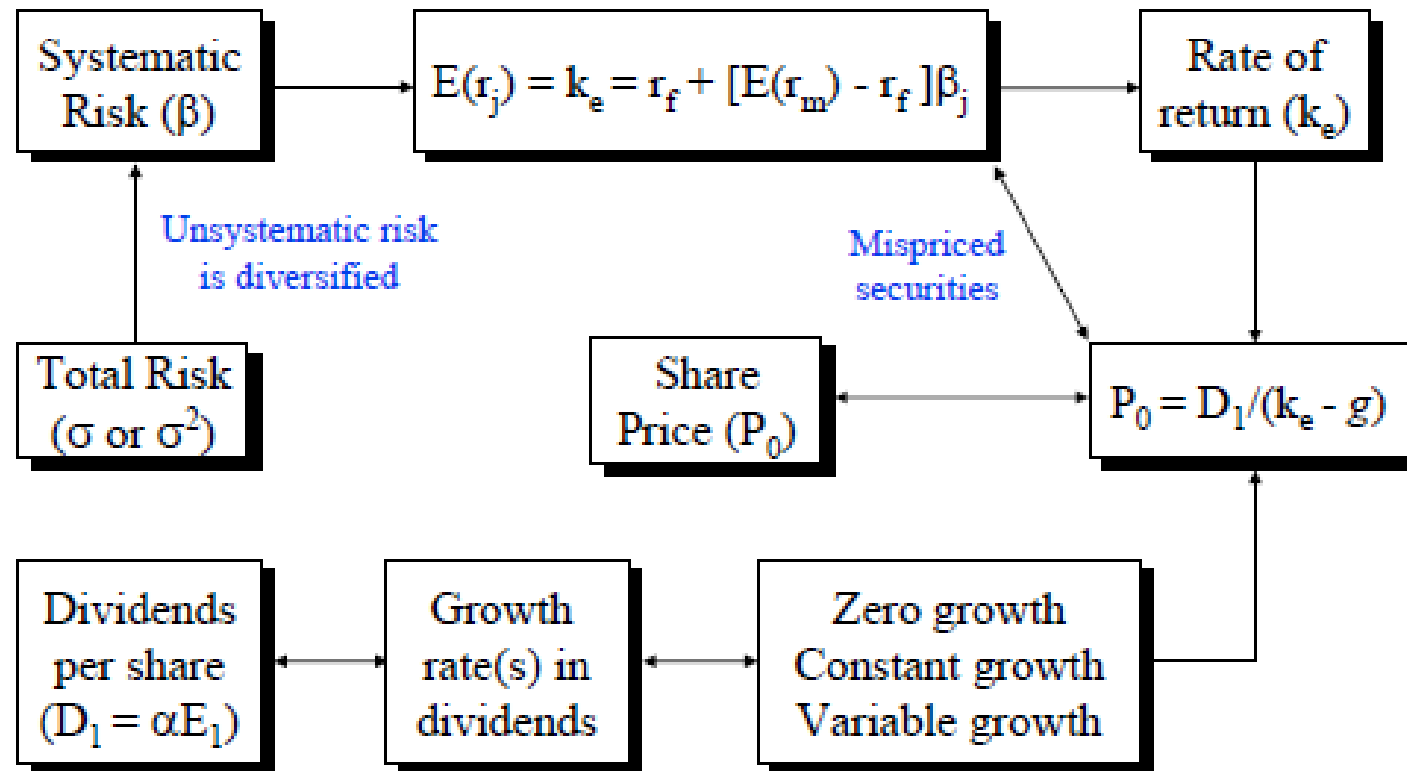
Tax systems often treat capital gains, dividends and interest differently

Other characteristics such as liquidity of an asset may be important determinants of the return asset-holders require

Firm size appears to have a bearing on realized returns, i.e. small firms generate higher returns compared to large firms

Many time series empirical regularities (anomalies) have been observed

## Putting it all Together



## Fama-French (1993) Three factor Model Alpha

Two groups of stocks consistently tended to outperform the market as a whole:

- **Small cap stocks** and **stocks with a high book-value-to price** (value vs. growth stocks)

Two factors are added to the CAPM reflecting a portfolio's exposure to these two asset classes:

$$R_{pt} - r_{ft} = \alpha_p + \beta_{p,m} (R_{mt} - r_{ft}) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \varepsilon_{pt}$$

*SMB = Small minus Big*

*HML = High (book / price) minus Low*

**One-factor CAPM:** Alpha is the amount by which an active portfolio manager outperforms a broad market index

**FF3 factor model defines alpha** (for equities) as the return an active manager achieves above the expected return **due to all three equity risk factors.**

Fama/French Benchmark Factors	January 2012	Last 3 months	Last 12 months
Rm-Rf	5.03	5.63	3.92
SMB	2.80	2.15	-0.64
HML	1.45	0.52	-7.60
Small Value	9.13	8.55	0.86
Small Neutral	6.55	7.49	1.61
Small Growth	8.30	6.13	3.69
Big Value	7.19	3.86	-4.36
Big Neutral	3.27	6.61	4.42
Big Growth	5.13	5.24	8.01



## Carhart (1997) Four factor Model Alpha

Momentum factor added (performance persistence)

$$R_{pt} - r_{ft} = \alpha_p + \beta_{p,m} (R_{mt} - r_{ft}) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{WML_t} WML_t + \varepsilon_{pt}$$

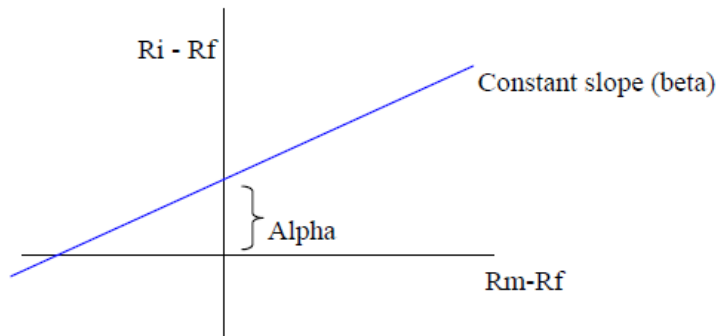
Equally weighted average of top 30% of firms with highest returns in previous 11 months minus equally weighted average of the 30% firms with the lowest returns in previous 11 months

Carhart alpha represents excess returns after market risk, small cap, value and momentum associated performance is taken into account.

# Measuring Performance of Market-Timing Funds

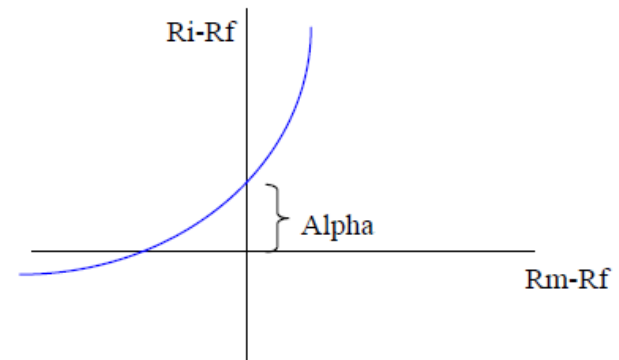
If manager does not engage in market timing, then;

- Portfolio Beta should be constant; and
- Earn excess return (alpha) if there is a stock picking skill.



If manager is engage in market timing, then;

Beta would increase as the market return increases (U-shaped quadratic relation between excess return of the market and excess return of the fund).



Treynor and Mazuy (1966) examined the timing ability of the mutual funds managers by testing for such curvature in the relationship.

$$(R_{it} - R_{ft}) = \alpha_i + b_i(R_{Mt} - R_{ft}) + c_i(R_{Mt} - R_{ft})^2 + \varepsilon_i$$

Positive  $c_i$  = superior market timing ability

No market timing ability, linear relationship between market returns and portfolio returns,  $c_i$  statistically insignificant.

Treynor and Mazuy (1966)....only *one of 57 mutual funds* exhibited a significantly positive  $c_i$ !!!!

Henriksson and Merton (1981), developed the following measure for the market....timing ability:

$$(R_{it} - R_{ft}) = a_i + b_i (R_{Mt} - R_{ft}) + c_i D(R_{Mt} - R_{ft}) + \varepsilon_i$$

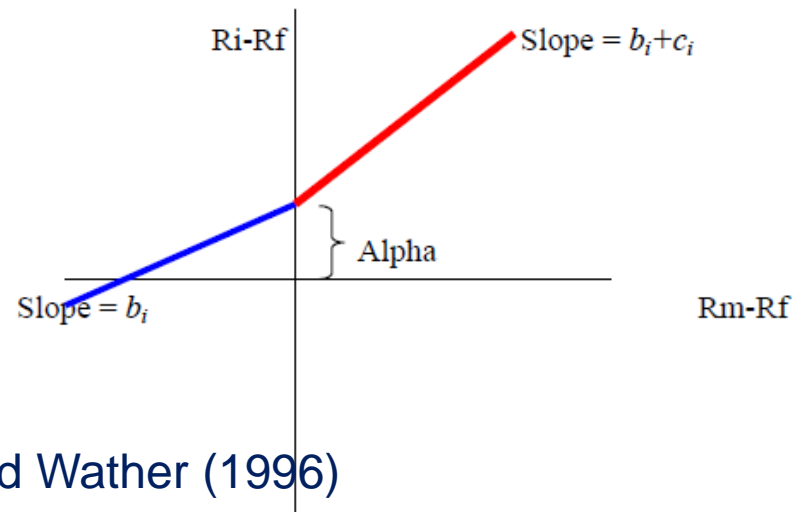
**D (dummy variable):** 1 in an up market

$b_i$  is the down market beta,

$b_i + c_i$  is the up-market beta and

$c_i$  is their difference or an indicator of the market timing ability.

If  $c_i$  is not significantly different from zero, then the up- and down- market betas are the same and we can conclude that no market timing is exhibited.



For conditional market timing see Ferson and Wather (1996)

## Fund Performance: Luck or Skill?

(Nitzsche and O'Sullivan, 2008)

- Evaluates performance of individual funds
- 935 open-ended UK equity mutual funds
- Period: April 1975 to December 2002 (surviving and non-surviving funds)
- Use of Carhart (1997) model, conditional alpha and beta model and market timing model.
- Use bootstrapping methodology (see notes)

### Main finding

Evidence of skilful picking ability only for a relatively small number of “top ranked” UK equity mutual funds

## Persistence of Performance

Try to establish if last year's winners are repeating.

Fund Name	Rank	
	1970 - 1980	1980 - 1990
Twentieth Century Growth	1	176
Templeton Growth	2	126
Quasar Associates	3	186
44 Wall Street	4	309
Pioneer II	5	136
Twentieth Century Select	6	20
Security Ultra	7	296
Mutual Shares Corp.	8	35
Charter Fund	9	119
Magellan Fund	10	1
Over-the-Counter Securities	11	242
American Capital Growth	12	239
American Capital Venture	13	161
Putnam Voyager	14	78
Janus Fund	15	21
Weingarten Equity	16	36
Hartwell Leverage Fund	17	259
Pace Fund	18	60
Acorn Fund	19	172
Stein Roe Special Fund	20	57
<b>Average annual return:</b>		
<b>Top 20 funds</b>	<b>19.00%</b>	<b>11.10%</b>
<b>All funds</b>	<b>10.40%</b>	<b>11.70%</b>

1970: 355 equity mutual funds holding broadly diversified portfolios.

More than a half **did not survived** until 2001

Of the remaining 158, **only five** produced returns 2% or more in excess of the index fund returns.

Additional studies: see notes

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Why is consistent out-performance difficult to find?

No straightforward answer to this!!

Natural answer: **markets are efficient!!**

Alternatively, excess returns “**might be moving**” from one fund to another.

Successful managers are often made offers in other companies....so when they move....the **performance leaves** with them.

Not surprising that a lot of investors choose to invest in **index tracking funds**, i.e. **follow passive investing**.

## How to measure persistence in performance?

- Contingency tables and Regression

### Contingency tables based persistence

Sort funds into one of four portfolios based on performance in year  $t$  and  $t+1$  (WW, WL, LW and LL).

Market adjusted return is looked at and defined as “annual excess return of fund – annual excess return on market index”

- **Winner** defined as a positive market adjusted return and **loser** as negative
- **Persistence**: if there is persistence one would expect to observe more WW and LL



Test for significant persistence

Brown and Goetzmann (1995) log-odds ratio

Log-odds ratio =  $\ln[(WW*LL) / (WL*LW)]$  and

Standard error =  $\sqrt{[(1/WW) + (1/WL) + (1/LW) + (1/LL)]}$

Test is standard normally distributed

Null hypothesis is: no persistence in performance

## Fletcher and Forbes (2002) results based on annual excess returns

Repeat winner tests: annual returns

	WW	WL	LW	LL	Log-odds	<i>z</i>
82–83	54	63	63	53	–0.33	–1.24
83–84	71	46	46	71	0.86	3.24 *
84–85	80	41	41	79	1.32	4.87 *
85–86	80	56	56	79	0.71	2.84 *
86–87	105	46	46	105	1.65	6.60 *
87–88	93	74	74	93	0.46	2.07 *
88–89	83	96	96	82	–0.3	–1.43
89–90	119	74	74	119	0.95	4.53 *
90–91	102	97	97	102	0.1	0.5
91–92	98	102	102	98	–0.08	–0.39
92–93	95	101	101	94	–0.13	–0.66
93–94	119	76	76	119	0.89	4.32 *
94–95	94	102	102	93	–0.17	–0.86
95–96	134	70	70	134	1.29	6.23 *
All	1327	1044	1044	1321	0.47	8.11 *

\* Significant at 5%

Significant persistence in the relative performance rankings using excess returns for both winner and loser portfolios

## Fletcher and Forbes (2002) results based on market adjusted returns

	WW	WL	LW	LL	Log-odds	z
82–83	29	35	100	69	– 0.55	– 1.89
83–84	28	105	12	89	0.68	1.82
84–85	31	14	82	114	1.12	3.18 *
85–86	92	41	65	73	0.92	3.64 *
86–87	162	14	85	41	1.72	5.09 *
87–88	52	226	9	47	0.18	0.46
88–89	2	67	11	277	– 0.28	– 0.36
89–90	7	10	79	290	0.94	1.85
90–91	14	79	51	254	– 0.12	– 0.38
91–92	18	48	82	252	0.14	0.46
92–93	54	47	152	138	0.04	0.18
93–94	104	105	49	132	0.98	4.52 *
94–95	35	119	37	200	0.46	1.76
95–96	42	34	87	245	1.24	4.75 *
All	670	944	901	2221	1.749	27.27 *

\* Significant at 5%

**Significant persistence** in the performance of the trusts relative to the benchmark index.

Persistence is driven primarily **by repeat losers** (underperformance). The number of repeat losers is **over three times higher** (2221) than the number of repeat winners (670).

## Regression Based Persistence

$$Performance_{pt} = a + bPerformance_{pt-1} + \varepsilon$$

Where 'performance' can be cumulative total returns, cumulative style adjusted returns or information ratios.

If coefficient  $b$  is positive, it is considered that period  $t-1$  performance contains information for predicting period  $t$  performance and hence, the evidence of persistence exists

US evidence, Kahn and Rudd (1995), the persistence of performance was not found among 300 equity funds in the early 1990s.

This implies that investors, unless they have another basis for choosing winners, should not base their investment decision on the past performance of funds and should invest in equity index funds.