Fixed Income Investment

Session 5
April, 26th, 2013
(morning)

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Lecture 5

Butterfly Trades – Bond Swaps

Issues in cash/risk neutral butterfly trades

Butterfly trade

Combination of a barbell (strategy that concentrates holdings in both very short-term and extremely long-term maturities) and bullet (investment in intermediate duration bonds, with no investment in long and short duration bonds) strategies.

Since mid 1980s: increase in the number of index tracking fixed income portfolio funds (generally constructed to maintain the target duration over time).

Bond index tracking portfolio: cannot take advantage of yield curves shifts or even protect the portfolio from disadvantageous shifts.

Example:

Cannot overweight long term bonds if its expected that general level of interest rates will fall.

Cannot overweight shorter maturities if its expected interest rates to rise.

Butterfly trade (also known as bond swap) is composed of:

- A sale (short sale) of one medium term fixed security (body of the butterfly, bullet portfolio) and,
- A purchase of shorter and longer maturity issues (wings of a butterfly, barbell portfolio).

Such a trade is structured so that:

- It is cash-neutral (proceeds from short-sale match the aggregate cost of the purchases);
- Same duration (price value of basis point or some other related risk measure) of the security sold and portfolio purchased.

Position value and duration unchanged

Benefits:

Yield pick-up: if the average yield on the securities purchased exceeds the yield o the security sold.

Convexity pick-up: to be discussed later

Constructing (weighting) a Butterfly

Implies calculating how much short and long maturity issue should be purchased given the amount of money available.

Two unknowns: the weight of a short and long term security.

Two conditions to be specified:

1) Aggregate cost of purchase of barbell must equal the proceeds from sale of a bullet (trade should be cash-neutral, neither generate nor require cash).

$$Q_2 P_2 = Q_1 P_1 + Q_3 P_3$$

2) Aggregate risk, price value of basis point for the barbell purchased must be equal to the aggregate value of a basis point for the bullet bond sold.

$$Q_2 V_2 = Q_1 V_1 + Q_3 V_3$$

Where V_i is the price value of a basis point expressed as the change in value of 1 million par amount of the bond for a one basis point change in yield on the bond.

Solving equations to unknown Q1 and Q3

$$Q_1 = \left(\frac{P_2V_3 - P_3V_2}{P_1V_3 - P_3V_1} \right) \times Q_2 \qquad Q_3 = \left(\frac{P_1V_2 - P_2V_1}{P_1V_3 - P_3V_1} \right) \times Q_2$$

Example:

Consider three bonds whose characteristics are in the next table.

Assume an investor owns $1 million par amount of the 5 year bond and decides to swap into a weighted portfolio of the 2 year and 10 year bonds. What would be the properly weighted butterfly trade?

Example 1

(Settlement date: 05.01.2010)

	Bond 1: 2 year	Bond 2: 5 Year	Bond 3: 10 year
Coupon	7.875%	8.25%	8.875%
Maturity	31.12.2010	15.02.2013	15.11.2018
Accrued Interest	0.1082	0.7847	1.2435
Dirty price	100.3269	100.3472	101.6185
Value of a Basis Point	$181.53	$409.29	$652.07
Conventional Yield	7.753%	8.342%	8.814%
True Yield	7.737%	8.338%	8.810%
Duration	1.897	3.595	6.7

Standard deviations of the weekly yield changes for bonds 1, 2 and 3

	Bond 1: 2 year	Bond 2: 5 Year	Bond 3: 10 year
Standard Deviation	18.3 bp	19.9 bp	20,5 bp

Correlation matrix of the weekly yield changes for bonds 1, 2 and 3

	Bond 1: 2 year	Bond 2: 5 Year	Bond 3: 10 year
Bond 1: 2 year	1.000	0.958	0.903
Bond 2: 5 Year	0.958	1.000	0.984
Bond 3: 10 year	0.903	0.984	1.000

Solution:

$$Q_1 = \left(\frac{P_2V_3 - P_3V_2}{P_1V_3 - P_3V_1} \right) \times Q_2 \qquad Q_3 = \left(\frac{P_1V_2 - P_2V_1}{P_1V_3 - P_3V_1} \right) \times Q_2$$

$$Q_1 = \left(\frac{100.3472 \times 652.07 - 101.6185 \times 409.29}{100.3269 \times 652.07 - 101.6185 \times 181.53} \right) \times 1.000,000 = \$507{,}564 = 0.507564$$

$$Q_3 = \left(\frac{100.3269 \times 409.29 - 100.3472 \times 181.53}{100.3269 \times 652.07 - 101.6185 \times 181.53} \right) \times 1.000,000 = \$486,377 = 0.486377$$

Using previous equations, weighted butterfly trade requires the purchase of $507, 564 par amount of the 2 year bond and the purchase of $486,377 par amount of the 10 year bond.

Alternative Weighting schemes for butterfly trades

Some strategies do not require a zero initial cash-flow.

In this case there is an initial cost of financing.

Three Classic strategies: Fifty-fifty weighting butterfly, the regression weighting butterfly and the maturity weighting butterfly.

Fifty-Fifty weighting butterfly

Adjust the weights so that the transaction has a zero dollar duration and 50% of dollar duration in each wing of the butterfly.

The two equations are satisfied:

$$Q_2D_2 = Q_1D_1 + Q_3D_3$$

$$\frac{Q_2D_2}{2} = Q_1D_1 = Q_3D_3$$

Aim:

- To make the trade neutral to some small steepening and flattening movements.
- In terms of YTM, if the spread change between the body and the short/long wing are equal, a fifty-fifty weighting butterfly is neutral to such curve movements.
- For a steepening scenario “-30/0/30” (short wing YTM decreases by 30 bps and the long wing YTM increases by 30 bps, while the body does not move) the trade is quasi curve neutral.
- Same apply for a flattening scenario “30/0/-30”.

Example:

From the previous equations: $Q_1 = \left(\frac{D_2}{2D_1}\right)$ *and* $Q_3 = \left(\frac{D_2}{2D_3}\right) \times Q_2$

Calculating the fifty-fifty it is obtained:

$$Q_1 = \left(\frac{3.595}{2 \times 1.879}\right) \times 1.000,000 \;\; and \;\; Q_3 = \left(\frac{3.595}{2 \times 6.7}\right) \times 1.000,000$$

$$Q_1 = 956,626 \;\; and \;\; Q_3 = 268,284$$

The fifty-fifty weighting butterfly is not cash neutral.
In the example, the portfolio manager has to pay $224,909 and if he carries the position during one day he will have to support a financing cost equal to $6 assuming a one day short rate equal to 1%.

Regression Weighting

The objective is to adjust the weights so that the transaction has a zero $duration, and so as to satisfy the following equations:

$$Q_2 D_2 = Q_1 D_1 + Q_3 D_3 \ \ and \ \ Q_1 D_1 \times \frac{1}{\beta} = Q_3 D_3$$

From there:

$$Q_1 = \left(\frac{D_2 \times \beta}{(1+\beta) D_1} \right) \times Q_2 \quad and \quad Q_3 = \left(\frac{D_2}{(1+\beta) D_3} \right) \times Q_2$$

Assuming β equal to 0.65…..

We obtain that 1 million of the 5-year bond should be swapped into:

$$Q_1 = \left(\frac{3.595 \times 0.65}{(1+0.65) \times 1.879} \right) \times 1.000,000 \quad and \quad Q_3 = \left(\frac{3.595}{(1+0.65) \times 6.7} \right) \times 1.000,000$$

$$Q_1 = 753,705 \quad and \quad Q_3 = 325,192$$

Maturity Weighting

Adjust the weights so that the transaction has a zero $duration satisfying the following equations:

$$q_1 D_1 + q_3 D_3 - q_2 D_2 = 0$$

$$q_1 D_1 = \left(\frac{M_2 - M_1}{M_3 - M_1} \right) D_2$$

$$q_3 D_3 = \left(\frac{M_3 - M_2}{M_3 - M_1} \right) D_2$$

Where M_1, M_2 and M_3 are the maturities of the short, medium and long term bonds.

Instead of searching for a regression coefficient β that is dependent on historical data….each wing of the butterfly is weighted with a coefficient depending on the maturities of the 3 bonds

In fact using previous equations:

$$q_1 D_1 = \left(\frac{M_2 - M_1}{M_3 - M_2} \right) q_3 D_3$$

The weighting butterfly is equivalent to a regression weighting butterfly with a regression coefficient equal to

$$\beta = \left(\frac{M_2 - M_1}{M_3 - M_2} \right)$$

$$q_1 D_1 + q_3 D_3 - q_2 D_2 = 0 \Leftrightarrow 1.897 \times q_1 + 6.7 \times q_3 + 3.595 = 0$$

$$q_1 D_1 = \left(\frac{M_2 - M_1}{M_3 - M_1} \right) D_2 \Leftrightarrow 1.897 \times q_1 = \frac{3}{8} \times 3.595$$

$$q_3 D_3 = \left(\frac{M_3 - M_2}{M_3 - M_1} \right) D_2 \Leftrightarrow 6.7 \times q_3 = \frac{5}{8} \times 3.595$$

$$q_1 = 710,662$$

$$q_3 = 335,354$$

Issues in cash/risk neutral butterfly trade

Butterfly trade: can be interpreted as a swap of one security (i.e. a medium term bond) for a synthetic substitute (i.e. for a pair of different securities with the same aggregate market value and the same aggregate risk).

Motivation: to increase yield and/or convexity.

Questions to answer:

1) Should one use "true" yield (Garbade,1996) or conventional yield measure?

2) How is the yield on a portfolio calculated?

3) Does convexity play a role in the relative price performance of a short term/long term bond portfolio versus a medium term bond?

4) Is keeping the aggregate value of basis points unchanged really equivalent to keeping the risk unchanged? Is the short term/long term bond portfolio really a synthetic substitute for a medium term bond?

1) Conventional versus true yields

Price of a coupon paying bond is given by:

$$P = \frac{C}{(1+y_1)^1} + \frac{C}{(1+y_2)^2} + \ldots + \frac{C}{\left(1+y_n\right)^x} + \frac{F}{\left(1+y_n\right)^x}$$

Most bonds: coupons are paid semi-annually and yield is compounded semi-annually

Unclear how *x (period)* should be computed since in most cases investors do not buy a bond on the day the coupon is paid.

Since the bond is usually bought between two coupon dates, the first x and the consequent *x_i's* are not identical 6-month periods.

Two possible ways to calculate *x_i's*:

a) Garbade (1996) proposes that each *xi* is calculated as the number of days from settlement date (SD) to each of the next coupon dates (CD) divided by 182.625. The yield calculated this way is the "true yield".

b) Conventional yield is computed by calculating the first xi as the number of days from the SD to the first CD divided by the actual number of days between the SD and CD. Then….every next *ith* payment is assumed to occur at exactly $i+x_1$ semi-annual periods aftr the SD (i=1, 2, 3…n)

Problems with this conventional yield:

1) Fails to take into account that payments cannot be made on weekends and bank holidays.
2) Length of a semi-annual period is defined as the number of days in the current coupon interval of the bond. Depending on bond and settlement date this could be 181, 182, 183 or 184 days.
3) The calculation defines all payments after the first payment as exactly one semi-annual interval apart, even though the day counts between payments are not identical.

Example:

Consider 6.25% coupon bond, with dirty price P=99.023352 and settlement date on January, 5, 2010.

Date of Payment	Cash Flow	Days to Payment	True *xi*	Conventional *xi*
30 Jun 2010	3.125	176	$\frac{176}{182.625}=0.963723$	$\frac{176}{182}=0.967033$
31 Dec 2010	103.125	360	$\frac{360}{182.625}=1.971253$	$1+xi=1.967033$

$$99.023352=\frac{3.125}{(1+y/2)^{0.963723}}+\frac{103.125}{(1+y/2)^{1.971253}} \Rightarrow \text{True yield: y=7.391\%}$$

$$99.023352=\frac{3.125}{(1+y/2)^{0.967033}}+\frac{103.125}{(1+y/2)^{1.967033}} \Rightarrow \text{Conventional yield: y=7.406\%}$$

Difference btw true and conventional yield (positive or negative).

This case True yield is 1.55 basis point lower than conventional yield.

2) How is the yield on a portfolio calculated?

Most market participants: Yield on bond portfolio equal to the market value weighted average of the yields on individual bonds in the portfolio.

Therefore yield on the butterfly trade portfolio:

$$y_p = \frac{Q_1 P_1 y_1 + Q_3 P_3 y_3}{Q_1 P_1 + Q_3 P_3} = w_1 y_1 + w_3 y_3$$

Garbade (1996): compute portfolio yield by weighting bond yields by the aggregate price value of basis points

$$y_p = \frac{Q_1 V_1 y_1 + Q_3 V_3 y_3}{Q_1 V_1 + Q_3 V_3} = w_1 y_1 + w_3 y_3$$

Example:

Consider butterfly trade on table on slide 6. Using the equation,

$$P = \frac{C}{(1+y_1)^1} + \frac{C}{(1+y_2)^2} + ... + \frac{C}{\left(1+y_n\right)^x} + \frac{F}{\left(1+y_n\right)^x}$$

The "true" yield on the 2-year/10-year portfolio is 8.572%. Per annum, compounded semi-annually.

Market value weighted portfolio yield (based on the true yields on the 2-year and 10 year bond):

$$y_P = \frac{\left(0.507567 \times 100.3269 \times 7.737\right) + \left(0.486374 \times 101.6185 \times 8.810\right)}{\left(0.507567 \times 100.3629\right) \times \left(0.486374 \times 101.6185\right)} = 8.265\%$$

Value of the basis point average (based on the true yields on the 2-year and 10 year bond):

$$y_P = \frac{\left(0.507567 \times 181.53 \times 7.737\right) + \left(0.486374 \times 652.07 \times 8.810\right)}{\left(0.507567 \times 181.53\right) \times \left(0.486374 \times 652.07\right)} = 8.568\%$$

Summary

Weighting with the price value of basis points produce a value closer to the ‘true” yield than weighting with market value.

The Butterfly trade from the example produces a yield pick-up of 23 basis points (8.568% -8.338%) rather than a yield give-up of 7.3 basis points (8.265%-8.338%)

3) Does convexity play a role in the relative price performance?

Theoretically aggregate value of a basis point of the bond portfolio purchased in a butterfly trade equal the aggregate value of a basis point of the bonds sold.

As a result

Value of portfolio of shorter and longer maturity issues is supposed to change by the same amount as the value of a medium term issue sold (for small and parallel yield curve shifts).

In reality value of short term/long term bond portfolio:

1) Rises faster than the middle security when interest rates fail
2) Falls more slowly as yields rise

Thus:

Portfolio outperforms the medium term issue regardless of the directions of movement of interest rates…reason?

GREAT CONVEXITY

Convexity pick up on a Butterfly trade is related to the difference between the maturity of the shorter and longer issues.

Wider spread between short term and long term bond in a portfolio in terms of maturities…..great the convexity!

4) Is the short term/long term bond portfolio really a synthetic substitute for a medium term bond? (Risk assessment)

Butterfly trade: Swap of one security for a synthetic substitute

Motivation: yield or/and convexity pick-up

Two constraints of a butterfly trade: Cash and risk neutral

Therefore it is required that the values of the two securities always change identically.

However, purchased portfolio is a perfect substitute for the security sold only if yields move always in parallel.

Standard deviations of medium bond (S_2) and long term-short term bond portfolio ($S_{1,3}$) bought can be defined as:

$$S_2 = Q_2 V_2 \sigma_2 \qquad S_{1,3} = \left(Q_1^2 V_1^2 \sigma_1^2 + Q_3^2 V_3^2 \sigma_3^2 + 2 Q_1 Q_3 V_1 V_3 \rho_{1,3} \sigma_1 \sigma_3 \right)^{1/2}$$

Generally ρ of Δw_2 with $\Delta w_{1,3}$ will be less than perfect positive if $\rho_{1,2}$, $\rho_{1,3}$ or $\rho_{2,3}$ is less than one.

Therefore, Butterfly trade is NOT a swap of genuine substitute securities unless the interest rates shifts are parallel and the same for all bond yields.

Example

Consider butterfly trade on table on slide 6.

$$S_2 = Q_2 V_2 \sigma_2 \Leftrightarrow S_2 = 409.29 \times 19.9 = \$8144.87$$

$$S_{1,3} = \left(Q_1^2 V_1^2 \sigma_1^2 + Q_3^2 V_3^2 \sigma_3^2 + 2 Q_1 Q_3 V_1 V_3 \rho_{1,3} \sigma_1 \sigma_3 \right)^{1/2}$$

$$S_{1,3} = \begin{bmatrix} \left(0.507564^2 \times 181.53^2 \times 18.3^2\right) + \left(0.486377^2 \times 652.07^2 \times 20.5^2\right) + \\ \left(2 \times 0.507564 \times 0.486377 \times 181.53 \times 652.07 \times 0.903 \times 18.3 \times 20.5\right) \end{bmatrix}^{1/2}$$

$$S_{1,3} = \$8056.82$$

$$\rho = correlation\ of\ \ \Delta W_2\ \ with\ \Delta W_{1,3} = \frac{Q_2 V_2 Q_1 V_1 \rho_{1,2} \sigma_1 \sigma_2 + Q_2 V_2 Q_3 V_3 \rho_{2,3} \sigma_2 \sigma_3}{S_2 S_{1,3}}$$

$$\rho = \frac{\begin{bmatrix} (1 \times 409.29 \times 0.507564 \times 181.53 \times 0.958 \times 18.3 \times 19.9) \\ +(1 \times 409.29 \times 0.486377 \times 652.07 \times 0.984 \times 19.9 \times 20.5) \end{bmatrix}}{8144.87 \times 8056.82}$$

$$\rho = 0.994548$$

Portfolio of short term and long term maturity issue is slightly less volatile than the value of the medium term issue.

$$S_{1,3} < S_2$$

However since differences in volatility is small and the correlation is close to 1:

- 2 year/10 year portfolio is a nearly perfect substitute for the 5 year bond.