Fixed Income Investment

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Lecture 6

Yield Spread Trades – Weighting, Financing and Applications

Hedging with Bonds

Conventional and Optimal hedging

Bond Spread or yield spread trade: Long-short strategy where the gains and losses are a result of a change in the yield spread ONLY and are not related to general levels of interest rates (rises and falls).

Therefore two main characteristics:

1) Its value should not vary with changes in the general level of interest rates

2) Its value should vary with change in a specified yield spread

Introduction (Change in yield vs. change in yield spread)

Yield spread changes and changes in the general level of interest rates are not linked.

Commonly known: if investors are expecting interest rates to fall they should own bonds with longer maturities, lower yields and lower coupon rates

Market participants: Not only interested in predicting whether interest rates will rise or fall but also if the spread btw yields on short and long term bonds will change.

Examples:

1) A normal upward sloping yield curve may be expected to flatten (e.g. spread between yields on short and long term bonds will narrow).
2) High coupon bonds may outperform low coupon bonds of a comparable maturity if there is a change in tax law which favors higher income.

The previous scenarios differ from simple "rise or fall in interest rates".

They involve changes in yield spreads and changes in the shape of the yield curve rather than just the yield level.

Therefore, scenarios of flattening of the yield curve can occur in the rising and falling interest rate environment.

Taking advantage (or avoiding losses) by using unexpected changes in yield spread cannot be done by buying or selling a single fixed income security.

Bond Spread or yield spread trade: Long-short strategy where the gains and losses are a result of a change in the yield spread ONLY and are not related to general levels of interest rates (rises and falls).

Weighting (constructing) a bond spread

Calculate the quantity of bonds to be bought relative to the quantity of bonds to be sold (or short-sold).

The net value of such short-long portfolio should not change with variation in the level of interest rates but it should change to investor's advantage with the changes in the differences in the yields on the bonds bought and sold.

Consider the data in next slide table.
Assume investor is expecting flattening of the yield curve (however general level of interest rates could be rising or falling).

Decline in 168 basis points yield spread between 2-year and 30-year bond.

Maturity	Coupon (%)	Price	Yield (%)	Price value of a basis point
2 yr	9.00	100 3/32	8.94	177.54
3	9.50	100 16/32	9.29	246.51
5	9.62	99 5/32	9.82	392.53
7	10.37	100 22/32	10.22	481.88
20	10.75	99 16/32	10.80	809.09
30	10.62	100	10.62	897.97

Buy long-term bond (30 yrs) and sell short-term bond (2 yr).

Short term yield is expected to increase more or decrease less relative to long term yield (if the yield curve is flattening)

Assume:

Buying \$1M principal amount of 30 year 10.62% bond.

What is the appropriate amount of the short term, 2 year issue to be shorted?

An increase (decrease) in its yield of 1bp will increase (decrease) the value of the 2 year short position by exactly \$897.97.

The gains or losses on the long position will then be matched by losses and gains on a short position whenever the yields on the two securities change by the same amount (when yields change without altering the 2-year/30 year yield spread).

$$\frac{\$897.97}{\$177.54} = \$5.06M$$

The weighting is NOT fixed for the life of any two bonds.

The weighting needs to be adjusted from time to time to maintain zero risk position in relation to the change in the level of interest rates.

Net portfolio value of the positions for different changes in interest rates.

Initial Value	
Short: \$5.06M 9% 2-year bond, yield 8.945%, quoted at 100.0937	-5 064 744
Long: \$1M 10.625% 30-year bond, yield 10.622%, quoted at 100	1 000 000
Net Value	-4 064 744
I. Value if both yields rise by 10bps	
Short: \$5.06M 9% 2-year bond, yield **9.045%**, quoted at 99.9156.	-5 055 729
Long: \$1M 10.625% 30-year bond, yield **10.722%,** quoted at 99.1064	991 064
Net Value	-4 064 665
Change	+79
II. Value if both yields fall by 10bps	
Short: \$5.06M 9% 2-year bond, yield **8.845%**, quoted at 100.2708.	-5 073 704
Long: \$1M 10.625% 30-year bond, yield **10.522%,** quoted at 100.9046.	1 009 046
Net Value	-4 064 658
Change	+86
III. Value if 30yr bond yield falls by 10bps	
Short: \$5.06M 9% 2-year bond, yield 8.945%, quoted at 100.0938.	-5 064 744
Long: \$1M 10.625% 30-year bond, yield **10.522%,** quoted at 100.9046.	1 009 046
Net Value	-4 055 698
Change	+9 046

IV. Value if 2yr bond yield rises by 10bps	
Short: $5.06M 9% 2-year bond, yield **9.045%,** quoted at 99.9156.	-5 055 729
Long: $1M 10.625% 30-year bond, yield 10.622%, quoted at 100.	1 000 000
Net Value	-4 055 729
Change	+9 015
V. Value if 30yr bond yield rises by 10bps	
Short: $5.06M 9% 2-year bond, yield 8.945%, quoted at 100.0938.	-5 064 744
Long: $1M 10.625% 30-year bond, yield **10.722%,** quoted at 99.1064.	991 064
Net Value	-4 073 680
Change	-8 936
VI. Value if 2yr bond yield falls by 10bps	
Short: $5.06M 9% 2-year bond, yield **8.845%,** quoted at 99.9156.	-5 073 704
Long: $1M 10.625% 30-year bond, yield 10.622%, quoted at 100.	1 000 000
Net Value	-4 073 704
Change	-8 960

All net values are negative: Value of a short position in a 2 year bond exceeds the value of the 30-year bond position.

Cash is generated instead of cash used.

Conversely, liquidating position uses cash because the funds needed to buy back 2 year bond exceed the amount of cash produced from the sale of 30 year bond.

Analysis of scenarios

Case I and II: If the yields on two securities rise or fall by 10bps an investor almost breaks even.
Case III and IV: If the yield curve flattens he earns around $9000 and loses around $9000 if the yield curve steepens (by 10 bps).

Conclusion: Selling $5.06M of a short term, 2 year bond and buying $1M of a long term, 30 year bond is the profitable spread position if the flattening of the yield curve is expected (vice versa if steepening is expected).

Weighting using duration

The product of bond duration and bond value should be the same for both sides of the spread position.

$$Q_1 \times (P_1 + A_1) \times D_1 = Q_2 \times (P_2 + A_2) \times D_2$$

$$\frac{Q_1}{Q_2} = \frac{D_2 \times (P_2 + A_2)}{D_1 \times (P_1 + A_1)}$$

Note: Q is quantity of a bond, P is price of a bond, D is duration and A is accrued interest

Financing Considerations

Financing position needs to be taken in account if the position is held for long periods of time.

There could be income and expenses from financing which can enhance or reduce the profits. Therefore, financing bond spreads is a very important issue in this strategy

Example

Investor holds a $1M long position in 30year, 10.62% coupon bond and $5.06 short position in 2 year 9% coupon bond, expecting the yield curve to flatten (scenarios III and IV).

Assume:

- 30 year bond can be financed in the market by REPO (repurchase agreement) at 6.5 % interest, and
- 2 year bond can be borrowed in the market for reverse repurchase agreement by accepting a rate of 6.5% on funds lent against the notes borrowed.

Question

How much does the value of the initial spread position change over an interval of one day, net of financing income and costs, assuming that bond yields don't change.

1. Increase in the value of the bonds held long due to accumulation of accrued interest (30-year bond)

$$\frac{1}{365}\times 10.625\%\times \$1M = \$291.10\ per\ day$$

2. Decrease in the value of the bonds held short due to accumulation of accrued interest (2-year bond):

$$-\left(\frac{1}{365}\times 9\%\times \$5.06M\right) = -\$1247.67\ per\ day$$

3. The cost of financing the 30-year bonds held long (i.e. the cost of borrowing the money to buy them) at REPO rate of 6.5% (30-year-bond)

$$-\left(\frac{1}{360}\times 6.5\%\times \$1M\right) = -\$180.55\ per\ day$$

4. The income from financing the 2-year bonds sold short position at a rate of 6.5% (2-year bond)

$$\left(\frac{1}{360}\times 6.5\%\times \$5.6M\right)=\$913.61\ per\ day$$

Net Financing Cost (1-4 above) = -$223.51 per day

$$\$291.10-\$1247.67-\$180.55+\$913.61=-\$223.51$$

The spread position will show a net loss unless the spread narrows by 1 bp in 4 days.

$$\left(4days\times\left(-\$223.51\right)\right)=-\$894.04\ which\ is\ still\ less\ than\ \$900\ profit$$

Problems with previous analysis:

There are some problems with the previous analysis undertaken:

1. Assumes cost of financing long position in the REPO market is the same as the rate on money lent in the reverse REPO market. In reality its is not which increases the net financing cost.

2. Assumes that yields do not change…then quoted prices do not change as the bonds were priced close to a par. However if bond is priced at a substantial premium or discount….the assumption of unchanged yields does not mean that quoted prices will remain unchanged!

3. Assumes that only the principal of the bond should be financed….it ignores the cost of <u>financing accrued interest or financing a premium bond in the REPO market.</u>

Advanced Financing Analysis (includes accrued interest and premium bond financing)

Assume:

Spread position where investors:

1) Buys 12% coupon bond expiring on 15/04/2011
2) Short-sell a weighted of 7% coupon bond expiring also on 15/04/2011
3) Transactions completed on February, 8th and settlement is February, 9th.

Coupon	7% bond	12% bond
Quoted price (Feb 9)	98	106
Yield (Feb 9)	8.6%	9.03%
Value of a basis point	$180	$190
Financing rate	6% (reverse REPO)	6.5% (REPO)
Implied quoted price (Feb 10)	98.0044 (at 8.6% yield)	105.9923(at 9.03% yield)
Accrued interest (Feb 9)	3.0458	5.0431
Accrued interest (Feb 10)	3.0660	5.0775

Relative weighting on the bond spread is 1.056 to 1 (=190/180 = 1.056)

Example:

If an investor buys $10M principal amount of the 12% bond, he will have to short-sell $10.56M principal amount of 7% bond.

Daily net financing position:

1) Change in the values of the bonds over one day interval if the yields do not change
   i. Accumulation of accrued interest
   ii. Change in bond's price as it approaches maturity

i)

Accumulation of accrued interest for 7% bond

0.0202% of principal amount (=3.0660 – 3.0458) from table

Accumulation of accrued interest for 12% bond

0.0344% of principal amount (=5.0775 –5.0431) from table ii)

Change in price for 7% bond: +0.0044% of principal amount (=98.0044-98)

from table

Change in price for 12% bond: -0.0077% of principal amount (=105.9923-106)

From table

The daily net change for 7% bond: -$2597.76 (=-$10.56M×(0.0202%+ 0.0044%)

The daily net change for 12% bond: +$2670 (=$10M×(0.0344%-0.0077%)

And,

2) Cost and income of financing long and short positions over a one day interval:

i) Cost of borrowing to finance $10M long position per day:

$$-\frac{1}{360}\times 6.5\%\times \$10M\times(106+5.0431)=\boxed{-\$2004.94\ per\ day}$$

ii) Income from financing $10.52M short position per day:

$$\frac{1}{360}\times 6.0\%\times \$10.56M\times(98+3.0458)=\boxed{\$1778.40\ per\ day}$$

Using all accumulations of accrued interest from 1) and
Financing expense and income from 2)

We obtain:

-$2597.76 + $2670 - $2004.94 + $1778.40

Overall: Net financing cost for the bond spread is: $154.3 per day

If similar analysis is undertaken but ignoring financing of accrued interest and premium bond…there is a NET FINANCIAL GAIN!!

Calculations next slide….

The increase in the value o the bonds held long due to accumulation of accrued interest:

$$\frac{1}{365} \times 12\% \times \$10M = \$3287.67\, per\ day$$

The decrease in the value of the bonds held short due to accumulation of accrued interest:

$$-(\frac{1}{365} \times 7\% \times \$10.56M) = -\$2025.21\, per\ day$$

The cost of financing the 12% coupon bond held long at REPO rate of 6.5%:

$$-(\frac{1}{360} \times 6.5\% \times \$10M) = -\$1805.56\, per\ day$$

The income from financing the 7% coupon bond short position at a rate of 6%:

$$\frac{1}{360} \times 6\% \times \$10.56M = \$1760\, per\ day$$

Applications of Bond Spread Trading

1) Yield Curve Trades

Anticipation of flattening or steepening of the yield curve.

If flattening: will buy longer maturity bonds and sell weighted amount of shorter maturity bonds

If steepening: will buy shorter maturity bonds and sell weighted amount of longer maturity bonds

Key decision: which sectors of the curve to buy or sell since the yield curve to not need to move in the parallel manner for different segments of the yield curve.

It is possible that while one segment of the curve is flattening the other one may remain unchanged or even steepening.

2) Coupon Spreads

Involves bonds with comparable maturity having very different coupon rates.

If the interest rates (yields) are expected to decrease, then we know that the lower coupon bond offers greater volatility for investor, hence they are expected to outperform higher coupon bonds.

Restrictions for Bond Spread Trading

Assumption: Investors can short-sell and that they can finance the bond purchase in the REPO market. However, in reality, many institutional investors can neither short sell bonds nor borrow money to finance bonds.

However, he can sell a bond he already owns and replace it with a weighted amount of the second bond (Bond Swap).

If he cannot finance bonds in the REPO market…he will be unable to execute a bond spread or bond swap that uses cash (unable to switch from a long to a short term bond…anticipation of the yield curve steepening).

Hedging with Bonds

Taking a strategic position (long or short) in the fixed income security (or securities) that will reduce the uncertainty about prospective future income from another fixed income security.

If, investor anticipates rising interest rates….might hedge a long position in e.g. 7 year bond by short-selling a 20 year bond.

Size of the hedge

How much of a 20 year bond one should sell given the principal amount of the 7 year issue?

Conventional approach to hedging

Is actually a yield spread trade!

Example:

Long position $100 M principal amount of 2-year bonds and wants to hedge against higher interest rates by shorting 7-year bonds

Maturity	Coupon	Quoted Price	Yield	Price value of a basis point
2 years	7%	99	7.9%	$180
7 years	9%	102	8.5%	$400
30 years	10%	106	9.3%	$1000

1 *bp* change in yield will change the value of $100 M principal amount of a 2-year bond by $18,000

Amount of a 7-year bond that should be sold to complete the hedge:

$$\frac{\$180}{\$400} = 0.45$$

Since the principal amount o a 2-year bond to hedge is \$100 M, then \$100 M×0.45 = \$45 million principal amount of 7-year bond should be sold to hedge \$100 million long position in 2-year bonds.

Or in general, the equation for a simple hedge is:

$$Q_2 = -\frac{V_1}{V_2} Q_1$$

Problems with the conventional approach to hedging (yield spread trades)

1) Assumes that the yield volatility of the two bonds used in the hedge is identical (e.g. if the yield of a short term bond changes by 1bp, the yield on the long term bond will change by the same amount).

2) Correlation of the yields change on two bonds is assumed to be perfect. Suppose the two bonds have equal yield volatility but the changes in the yields are uncorrelated.

These problems are addressed in the optimal hedging approach.

Optimal hedging with one hedge bond

The size of bond hedge is affected by three factors:

1) The volatility of each bond involved in the hedge (measured through the price value of a basis point or Duration)

2) The volatility of the yield on each bond

3) The correlation between changes in yields on pair of bonds.

	2-year	3-year	4-year	5-year	7-year	10-year	20-year	30-year
St. Deviation	20.3bp	20.5bp	21.2bp	21bp	20.8bp	20.3bp	19.2bp	18.3bp
2-year	1							
3-year	0.984	1						
4-year	0.973	0.983	1					
5-year	0.956	0.970	0.988	1				
7-year	0.927	0.945	0.972	0.985	1			
10-year	0.921	0.939	0.965	0.978	0.993	1		
20-year	0.891	0.909	0.940	0.953	0.973	0.982	1	
30-year	0.886	0.904	0.933	0.949	0.969	0.982	0.988	1

Optimal hedging with one hedge bond – Risk Minimization

Purpose of hedging: Minimize risk (Minimize the standard deviation of the value of the bond portfolio position)

Let's denote:
Bond 1 (bond that we want *to* hedge)
Bond 2 (Bond *with which* we want to wedge)

Q_1 and Q_2: principal amounts of Bond 1 and 2.
V_1 and V_2: Price value of a basis point of Bond 1 and 2.
Δy_1 and Δy_2: change in Bond 1 and 2 yields.
Δw: Net value of the long-short bond portfolio position

$$\Delta W = Q_1 V_1 \Delta y_1 + Q_2 V_2 \Delta y_2$$

Variance or uncertainty:

$$\sigma^2_{\Delta W} = Q_1^2 V_1^2 \sigma_1^2 + Q_2^2 V_2^2 \sigma_2^2 + 2 Q_1 Q_2 V_1 V_2 \sigma_1 \sigma_2 \rho_{1,2}$$

Minimizing the variance:

$$\partial \sigma^2 / \partial Q_2 = 2 Q_2 V_2^2 \sigma_2^2 + 2 Q_1 V_1 V_2 \sigma_1 \sigma_2 \rho_{1,2}$$

The amount of bond 2 Q2, that should be used to hedge Q1 is given as:

$$Q_2 = -\frac{\rho_{1,2} V_1 \sigma_1}{V_2 \sigma_2} Q_1$$

Example (from previous table)

Hedge a long position in \$100M principal of 2-year bonds by selling 7-year bonds short.

Q_1=\$100M

V_1=\$180 per \$1M principal amount

V_2=\$400

σ_1=20.3pb

σ_2=20.8bp

$\rho_{1,2}$ =0.927

Optimal amount of 7-year bond that should be sold short:

$$Q_2 = -\frac{\rho_{1,2} V_1 \sigma_1}{V_2 \sigma_2} Q_1 = -\frac{(0.927)(180)(20.3)}{(400)(20.8)} \times \$100M = -\$40.71M$$

Optimal hedge is $4.19 million smaller than the conventional hedge of $45 million.

Reasons:

- Yield on 7-year bond is more volatile than the yield on 2-year bonds

- Size of hedge depends on the level of correlation of yield changes between two bonds. Conventional hedge = perfect positive correlation. When correlation lower than that will lead to smaller hedge position.

- If securities have equal volatilities of yields and perfect correlation coefficient equal to 1 optimal hedge becomes the conventional hedge (since conventional bond hedging uses only the price value of the basis point).

Two additional implications from the above: optimal hedges are not symmetric and they may not be transitive.

Optimal Hedges are not Symmetric

As seen before:
Optimal hedge for $100 M long position in 2-year bonds was a short position in $40.71 M 7-year bonds.

Let's analyze the inverse situation:
Optimal 2-year hedge for a short position in $40.71 M of 7-year bonds.

Bond 1: 7-year bond (Q_1 = -$40.71M)
Bond 2: 2-year bond

Using the same Price Value of a Basis Point. Correlation and standard deviation data as before:

$$Q_2 = -\frac{\rho_{1,2} V_1 \sigma_1}{V_2 \sigma_2} Q_1 = -\frac{(0.927)(400)(20.8)}{(180)(20.3)} \times (-\$40.71M) = \$85.93M$$

Long position in 2-year bond will be an optimal hedge for the short position of -$40.71 M in 7-year bond

From previous example:

Optimal hedges are nor symmetric.

Long position of $85.93M in 2-year bond is the optimal hedge for the short position of -$40.71M in 7-year bond and NOT a long position $100.

Why?

Different problems….different solutions

1st Problem: Which quantity of 7-year bonds that minimizes the risk of the existing $100 M long position.

2nd Problem: Which quantity of 2-year bonds that will minimize the risk of the short position of -$40.71M in 7-year bonds.

Optimal Hedges May Not be Transitive

Consider the following example (divided in two parts)

Part 1: Hedging a short position of -$40.71M 7-year bonds with 30 year bonds.

7-year bond is bond 1 and 30-year bond is bond 2.

V_1 =$400 per $1M principal amount

V_2 =$1000

From the Table

$\sigma 1$=20.8bp

$\sigma 2$ =18.3bp

$\rho_{1,2}$ =0.969

$$Q_2 = -\frac{\rho_{1,2} V_1 \sigma_1}{V_2 \sigma_2} Q_1 = -\frac{(0.969)(400)(20.8)}{(1000)(18.3)} \times -\$40.71M = \$17.93M \text{ of 30-year bonds}$$

Part 2: Hedging a long position of $100M 2-year notes with 30 year bonds.

2-year bond is bond 1 and 30-year bond is bond 2.

V_1 =$180 per $1M principal amount
V_2 =$1000
From the Table
σ_1 =20.3pb
σ_2 =18.3bp
$\rho_{1,2}$=0.886

$$Q_2 = -\frac{\rho_{1,2} V_1 \sigma_1}{V_2 \sigma_2} Q_1 = -\frac{(0.886)(179.84)(20.3)}{(1057.29)(18.3)} \times \$100M = -\$16.72M \text{ of 30-year bonds}$$

This non-transitivity can be explained through analysis of the correlation coefficients between bond yields changes.

Less than perfect positive correlation: implies some independence in the changes in yields on two bonds.

Optimal hedge with two hedge bonds – Risk Minimization

Same rationale has in hedging with a single bond.

Aim: minimize risk i.e. minimize the standard deviation (variance) of the value of the long-short position.

Positions in three bonds: i=1, 2 and 3, Q_i denote principal amounts and V_i the price value of a basis point,

If the yields of the three securities change by Δy_1, Δy_2 and Δy_3

$$\Delta W = Q_1 V_1 \Delta y_1 + Q_2 V_2 \Delta y_2 + Q_3 V_3 \Delta y_3$$

ΔW = change in net value of short/long portfolio position

Variance of ΔW

$$\sigma^2 = Q_1^2 V_1^2 \sigma_1^2 + Q_2^2 V_2^2 \sigma_2^2 + Q_3^2 V_3^2 \sigma_3^2 + 2Q_1 Q_2 V_1 V_2 \sigma_1 \sigma_2 \rho_{1,2} + 2Q_1 Q_3 V_1 V_3 \sigma_1 \sigma_3 \rho_{1,3} + 2Q_2 Q_3 V_2 V_3 \sigma_2 \sigma_3 \rho_{2,3}$$

Bond to hedge: Q_1

Have to find values to bonds Q_2 and Q_3

$$\partial\sigma^2 / \partial Q_2 = 2Q_2 V_2^2 \sigma_2^2 + 2Q_1 V_1 V_2 \sigma_1 \sigma_2 \rho_{1,2} + 2Q_3 V_2 V_3 \sigma_2 \sigma_3 \rho_{2,3}$$

$$\partial\sigma^2 / \partial Q_3 = 2Q_3 V_3^2 \sigma_3^2 + 2Q_1 V_1 V_3 \sigma_1 \sigma_3 \rho_{1,3} + 2Q_2 V_2 V_3 \sigma_2 \sigma_3 \rho_{2,3}$$

Solving for Q_2 and Q_3

$$Q_2 = -\frac{(\rho_{1,2} - \rho_{1,3} \times \rho_{2,3})V_1\sigma_1}{(1-\rho_{2,3}^2)V_2\sigma_2} Q_1$$

$$Q_3 = -\frac{(\rho_{1,3} - \rho_{1,2} \times \rho_{2,3})V_1\sigma_1}{(1-\rho_{2,3}^2)V_3\sigma_3} Q_1$$

Example 1: Hedging with Shorter and Longer Bonds

Suppose that we want to hedge a position in \$100M of 7-year bonds (bond 1, Q_1=100) with 2-year notes (bond 2) and 30-year bonds (bond 3).

Data from previous tables (price value of basis points, standard deviation and correlations).

$V_1 = 400$

$V_2 = 180$

$V_3 = 1000$

$\sigma_1 = 20.8bp$

$\sigma_2 = 20.3bp$

$\sigma_3 = 18.3bp$

$\rho_{1,2} = 0.927$

$\rho_{1,3} = 0.969$

$\rho_{2,3} = 0.886$

$$Q_2 = -\frac{(0.927 - 0.969 \times 0.886) \times 400 \times 20.8}{(1 - 0.886^2) \times 180 \times 20.3} \times 100 = -\$72.51M \ \ principal \quad amount$$

$$Q_3 = -\frac{(0.969 - 0.927 \times 0.886) \times 400 \times 20.8}{(1 - 0.886^2) \times 1000 \times 18.3} \times 100 = -\$31.23M \ \ principal \quad amount$$

Example 2 – Hedging with Two Shorter Bonds

Find the optimal hedge position, for hedging \$10M principal amount of 30-year bonds with 2 year bonds and 7 year bonds.

Bond 1: 30 year bond, Q_1 = 10M
Bond 2: 2 year bond
Bond 3: 7 year bond

$$V_1 = 1000$$
$$V_2 = 180$$
$$V_3 = 400$$
$$\sigma_1 = 18.3bp$$
$$\sigma_2 = 20.3bp$$
$$\sigma_3 = 20.8bp$$
$$\rho_{1,2} = 0.886$$
$$\rho_{1,3} = 0.969$$
$$\rho_{2,3} = 0.927$$

$$Q_2 = -\frac{(0.886 - 0.969 \times 0.927) \times 1000 \times 18.3}{(1 - 0.927^2) \times 180 \times 20.3} \times 10 = +\$4.37M \ \ principal \ \ amount \ 2 - year \ bonds$$

$$Q_3 = -\frac{(0.969 - 0.886 \times 0.927) \times 1000 \times 18.3}{(1 - 0.927^2) \times 400 \times 20.8} \times 10 = -\$23.09M \ \ principal \ \ amount \ 7 - year \ bonds$$

Optimal hedge for a $10M long position in 30 year bond is:

Short position of -$23.09M in 7 year bonds and
Long position of $4.37M in 2 year bonds

Not a short position in both bonds as in previous examples.

This anomaly can be explained by observing the consequences of hedging a long position of $10M 30 year bonds with only 7-year bonds.

In this case…optimal hedge of a $10m 30-year bond with 7-year bond is a short position of $23.31M principal amount of the 7-year bonds.

While it is a optimal single bond hedge…it can lead to some significant yield curve risk.

Specifically, a short 7 year/long 30-year bond position will lose value if the yield curve steepens.

This yield curve risk is reduced when two bonds are used!!

If the yield curve then steepens, there will be a gain on the long 2 year / short 7 year portion of the hedge that will be used to offset the loss on the short 7 year/long 30 year portion of the hedge

These results leads to a re-interpretation of the results in example 1

In example 1:

This hedge with two bonds (2 year and 30 year bonds), can be seen as a device to limit yield curve risk.

If the hedge will be only with 2 year bond, the position would lose value if the yield curve steepened.

If the hedge will be only with 30 year bond, the position would lose value if the yield curve flattened.

By “dividing” the hedge between both 2 and 30 year bond, the hedge position is less sensitive to changes in slope of the yield curve.

General Comments on Hedging

How many different bonds are enough?

One bond hedge: can protect an existing position against changes in the general level of interest rates…but may still expose investors to losses due to changes in the slope of the yield curve.

Two bond hedge: hedge the risk of changes in the slope of the curve as well as changes in the levels of interest rates.

As one extend the number of bonds…we are able to protect against more types of changes in interest rates…!!!

A three bond hedge may protect investor from changes in:
Level, slope and curvature of the yield curve…