Portfolio Management

Session 7

Dr. Cesario Mateus
Senior Lecturer in Banking and Finance
University of Greenwich Business School, London, United Kingdom
c.mateus@greenwich.ac.uk
www.cesariomateus.com

The Term Structure of Interest Rates

Term structure

- bonds with the same characteristics, but different maturities
- focus on Treasury yields
  - same default risk, tax treatment
  - similar liquidity
  - many choices of maturity

Relationship between yield & maturity is NOT constant

- sometimes short-term yields are highest,
- most of the time long-term yields are highest

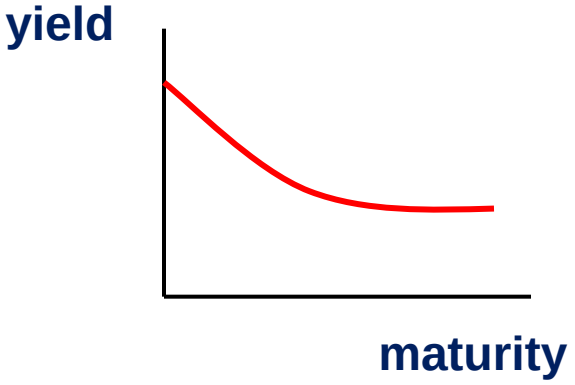
The Yield Curve

- plot of maturity vs. yield
- slope of curve indicates relationship between maturity and yield

upward sloping

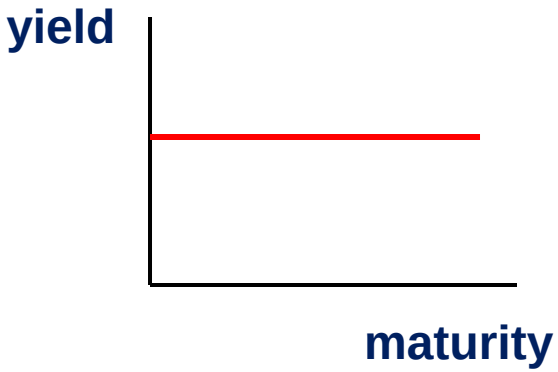
yields rise w/ maturity (common)
July 1992, currently

downward sloping (inverted)



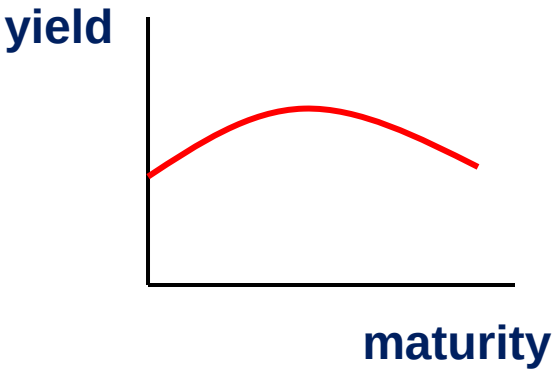
yield falls w/ maturity (rare)
April 1980

Flat



yields similar for all maturities
June 2000

humped



intermediate yields are highest
May 2000

Theories of the term structure

- explain relationship between yield and maturity
- what does the yield curve tell us?

The Pure Expectations Theory

The interest rates of longer terms reflect the market's expectation on the future interest rate movements:

Assume:

- bond buyers do not have any preference about maturity, i.e.
- bonds of different maturities are perfect substitutes

if assumption is true,

then investors care only about expected return

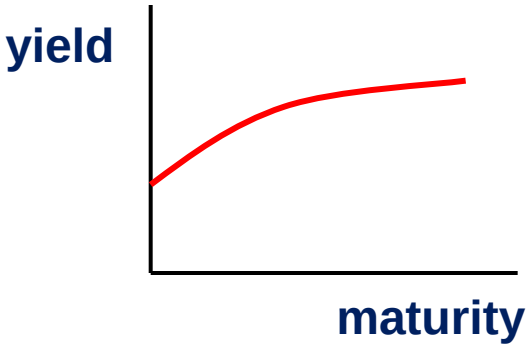
- if expect better return from ST bonds, only hold ST bonds
- if expect better return from LT bonds, only hold LT bonds

but investors hold both ST and LT bonds
so,
must EXPECT similar return:
LT yields = average of the expected ST yields

Under the Pure Expectations Theory
slope of yield curve tells us direction of expected future ST rates

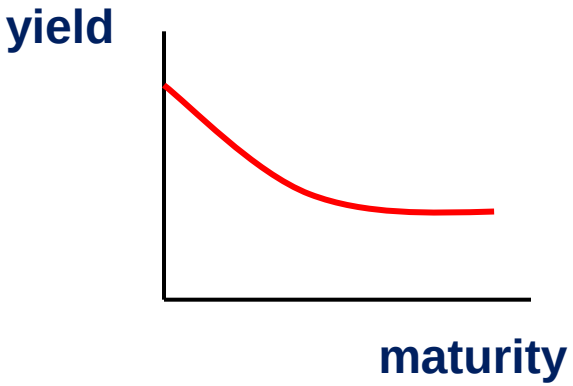
Why?
if expect ST rates to RISE,
then average of ST rates will be > current ST rate
so LT rates > ST rates and therefore, yield curve SLOPES UP

ST rates expected to rise



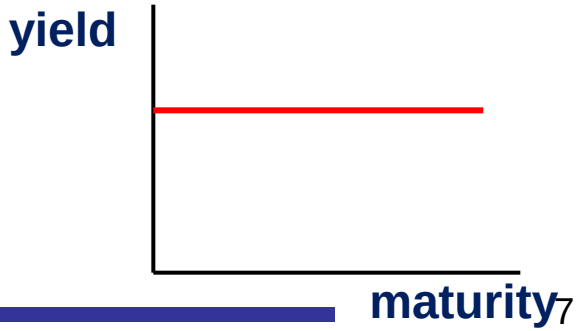
if expect ST rates to FALL,
then average of ST rates will be < current ST rate
so LT rates < ST rates and therefore yield curve SLOPES DOWN

ST rates expected to fall

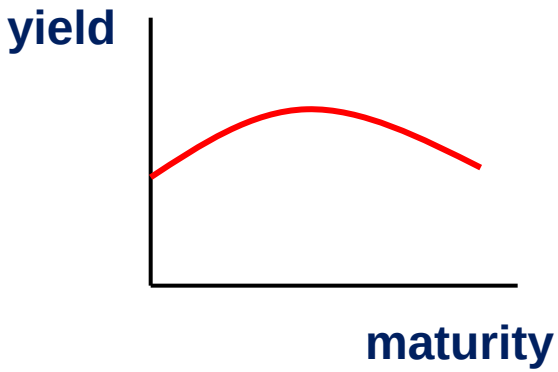


if expect ST rates to STAY THE SAME,
then average of ST rates will be = current ST rate
so LT rates = ST rates and therefore yield curve is FLAT

ST rates expected to stay the same



ST rates expected to rise, then fall



Is this theory true**?**

not quite.

FACT: yield curve usually slopes up but expectations theory would predict this only when ST rates are expected to rise

- 50% of the time

what went wrong?

back to assumption:

- bonds of different maturities are perfect substitutes

- but this is not likely
  - long term bonds have greater price volatility
  - short term bonds have reinvestment risk

- assumption is too strict
- so implication is not quite correct

Liquidity Theory

The theory asserts that market participants want to be compensated for the interest rate risk associated with holding longer term bonds, because of:

- Higher interest rate risk

Assume:
bonds of different maturities are imperfect substitutes, and investors PREFER ST bonds

so if true,

investors hold ST bonds
UNLESS LT bonds offer higher yield as incentive
higher yield = liquidity premium

IF LT bond yields have a liquidity premium,
then usually LT yields > ST yields or yield curve slopes up.

Problem

How do we interpret yield curve?

slope due to 2 things:

(1) expectation about future ST rates
(2) size of liquidity premium

Preferred Habitat Theory

Assume:

- bonds of different maturities are imperfect substitutes,

and investor preference for ST bonds OR LT bonds is not constant

- liquidity premium could be positive or negative
- yield curve very difficult to interpret
    - do not know size or sign of liquidity premium

Segmented Markets Theory

The theory argues that the bond markets should not be treated as a single market – instead, several smaller markets with different maturity terms and independent of each other

assume:

bonds of different maturities are NOT substitutes at all

if assumption is true,

- separate markets for ST and LT bonds
- slope of yield curves tells us nothing about future ST rates

unrealistic to assume NO substitution between ST and LT bonds

Different institutional investors have different maturity needs that lead them to confine their security selections to specific maturity segments

Trading Implications of the Term Structure

Information on maturities can help you formulate yield expectations by simply observing the shape of the yield curve.

Portfolio Immunization and Cash Flow Matching

- Explain strategies for managing bond portfolios to satisfy predetermined liabilities

- Two strategies will be analyzed: Immunization and Cash Flow Matching.

Immunization: hybrid strategy elements of both active and passive strategies.
Used to minimize reinvestment risk over a specified investment horizon. Can be employed to structure a portfolio designed to fund a single liability or multiple liabilities.

Cash Flow Matching: used to construct a portfolio that will fund a schedule of liabilities from a portfolio's cash flows, with the portfolio's value diminishing to zero after payment of the last liability.

Immunization Strategy for a single Liability

Classical immunization: process by which a bond portfolio is created to have an assured return for a specific time horizon irrespective of interest rate changes.

Fundamental principle: structure a portfolio that balances the change in the value of the portfolio at the end of the reinvestment horizon with the return from the reinvestment of portfolio cash flows (both capital and principal payments).

Immunization offsets interest rate risk and reinvestment risk. See exhibit 1.

Necessary condition for effectively immunized portfolios: portfolio duration equal to the desired portfolio time horizon, positive and negative incremental return sources offset one another.

General Principle of Classical Immunization

Objective: Lock in a minimum target rate of return and accumulated value regardless of how interest rates change over an investment horizon.

Risk when interest rate change: Reinvestment Risk and interest rate or price risk

Assumption: Parallel shift in the yield curve (i.e. all yields rise and fall uniformly).

Principle:

Scenario 1: Interest rate increase

Implications: 1. Reinvestment income increases and 2. value of portfolio of bonds maturities greater than the investment horizon declines in value.

Result: gain in reinvestment income $\geq$ loss in portfolio value

Scenario 2: Interest rate decline

Implications: 1. Reinvestment income decreases and 2. value of portfolio of bonds maturities greater than the investment horizon increases in value.

Result: loss in reinvestment income $\leq$ gain in portfolio value

Example

Life insurance company: sells a guaranteed investment contract (GIC). Life insurance company guarantees that a specified dollar amount will be paid to the policyholder at a specific future date.

Example: 5-year GIC that guarantees an interest rate of 7.5% per year on a bon-equivalent yield basis (or equivalent, 3.75% every six months for the next ten 6-month periods).Suppose that the payment made by the policyholder to purchase the GIC is $9,642,899.

The value that the life insurance company has guaranteed to the policyholder five years from now is $13,934,413.

When investing the $9,642,899, the target accumulated value for the life insurance company is $13,934,413 after five years, which is the same as a target yield of 7.5% on a bond-equivalent basis.

Suppose the life insurance company buys $9,642,899 par value of a bond selling at par with a 7.5% yield to maturity that matures in five years.

The life insurance company will not be assured of realizing a total return at least equal to the target return of 7.5% because to realize 7.5% the coupon interest payments must be reinvested at a minimum rate of 3.75% (reinvestment risk) every six months. Accumulated value will depend on the reinvestment rate.

Assume that immediately after investing $9,642,899 in the 7.5% 5-year bond, yields in the market change and stay at the new level for the remainder of the five years.

Exhibit 2:

Total accumulated value and total return after five years for a 5-year 7.5% Bond Selling to yield 7.5%

Investment horizon: 5 years	Price: 100.00
Coupon rate: 7.5%	Par value purchased: $9,642,899
Maturity: 5 years	Purchase price: $9,642,899
Yield to maturity: 7.5%	Target Accumulated value: $13,934,413

New yield (%)	Coupon ($)	Reinvestment income ($)	Price of bond	Accumulated Value ($)	Total return (%)
11.00	3.616,087	1,039.753	9,642,899	14,298,739	8.04
10.50	3.616,087	985,615	9,642,899	14,244,601	7.96
10.00	3.616,087	932,188	9,642,899	14,191,175	7.88
9.50	3.616,087	879,465	9,642,899	14,138,451	7.80
9.00	3.616,087	827,436	9,642,899	14,086,423	7.73
8.50	3.616,087	776,093	9,642,899	14,035,079	7.65
8.00	3.616,087	725,426	9,642,899	13,984,412	7.57
7.50	3.616,087	675,427	9,642,899	13,934,413	7.50
7.00	3.616,087	626,087	9,642,899	13,885,073	7.43
6.50	3.616,087	577,398	9,642,899	13,836,384	7.35
6.00	3.616,087	529,352	9,642,899	13,788,388	7.28
5.50	3.616,087	481,939	9,642,899	13,740,925	7.21
5.00	3.616,087	435,153	9,642,899	13,694,139	7.14
4.50	3.616,087	388,985	9,642,899	13,647,971	7.07
4.00	3.616,087	343,427	9,642,899	13,602414	7.00

1. New yield level

2. Total coupon payments

3. Reinvestment earned over the five years if the coupon payments are reinvested at the new yield

4. Price of the bond at the end of five years (par value)

5. Accumulated value from all three sources: coupon interest, reinvestment income and bond price

6. Total return

$$2\left[\left(\frac{\text{Accumulated value}}{\$9{,}642{,}899}\right)^{1/10} -1\right]$$

Suppose that instead of investing in a bond maturing in five years the life insurance company invests in a 12-year bond with a coupon of 7.5% selling at par to yield 7.5%.

See exhibit 3: presents the accumulated value and total return in five years if the market yield changes immediately after the bond is purchased and remains at the same new yield.

Note: the fourth column is the market price of a 7.5% 7-year bond (since five years have passed at the horizon date).

If the market yield increases, the portfolio will fail to achieve the target accumulated value; if the market yield decreases, the accumulated value (total return) will exceed the target accumulated value (target yield)

Exhibit 3:

Total accumulated value and total return after five years for a 12-year 7.5% Bond Selling to yield 7.5%

Investment horizon: 5 years	Price: 100.00
Coupon rate: 7.5%	Par value purchased: $9,642,899
Maturity: 12 years	Purchase price: $9,642,899
Yield to maturity: 7.5%	Target Accumulated value: $13,934,413

Exhibit 3

New yield (%)	Coupon ($)	Reinvestment income ($)	Price of bond	Accumulated Value ($)	Total return (%)
11.00	3.616,087	1,039.753	8,024,639	12,680,479	5.55
10.50	3.616,087	985,615	8,233,739	12,835,440	5.80
10.00	3.616,087	932,188	8,449,754	12,998,030	6.06
9.50	3.616,087	879,465	8.672,941	13,168,494	6.33
9.00	3.616,087	827,436	8,903,566	13,347,090	6.61
8.50	3.616,087	776,093	9,141,907	13,534,087	6.90
8.00	3.616,087	725,426	9,388,251	13,729,764	7.19
7.50	3.616,087	675,427	9,642,899	13,934,413	7.50
7.00	3.616,087	626,087	9.906163	14,148,337	7.82
6.50	3.616,087	577,398	10.178,367	14,371,852	8.14
6.00	3.616,087	529,352	10,459,851	14,605,289	8.48
5.50	3.616,087	481,939	10,750,975	14,848,992	8.82
5.00	3.616,087	435,153	11,052,078	15,103,318	9.18
4.50	3.616,087	388,985	11,363,569	15,368,642	9.54
4.00	3.616,087	343,427	11,685,837	15,645,352	9.92

Exhibit 4

Change in Reinvestment Income and Price due to interest rate change after five years for a 12-year 7.5% Bond Selling to yield 7.5%

New yield (%)	Change in Reinvestment Income ($)	Change in Price ($)	Total change in accumulated value ($)
11.00	364,326	(1,618,260)	(1,253,934)
10.50	310,188	(1,409,160)	(1,098,972)
10.00	256,762	(1,193,145)	(936,383)
9.50	204,039	(969,958)	(765,919)
9.00	152,010	(739,333)	587,323)
8.50	100,666	(500,992)	(400,326)
8.00	49,999	(254,648)	(204,649)
7.50	----	---	---
7.00	(49,340)	263,264	213,924
6.50	(98,029)	535,468	437,439
6.00	(146,075)	816,952	670,877
5.50	(193,273)	1,108,066	914,579
5.00	(240,273)	1,409,179	1,168,905
4.50	(286,441)	1,720,670	1,434,229
4.00	(331,999)	2,042,938	1,710,939

Exhibit 4 summarizes the change in reinvestment income and the change in price resulting from a change in the market yield.

Example: if the market yield rises instantaneously by 200 basis points, from 7.5% to 9.5%, reinvestment income will be $204,039 greater; however the market price of the bond will decrease by $969,958.

The net effect is that the accumulated value will be $765,919 less than the target accumulated value. The reverse will be true if the market yield decreases.

The change in the price of the bond will more than offset the decline in reinvestment income, resulting in an accumulated value that exceeds the target accumulated value.

There is a trade-off between interest rate (price) risk and reinvestment risk.

For this 12-year bond, the target accumulated value will be realized only if the market yield does not increase.

Because neither a coupon bond with the same maturity nor a bond with a longer maturity provides the target accumulated value, maybe a bond with a maturity shorter than five years will.

Consider a 7.5% bond with six months remaining to maturity selling at par.

Exhibit 5

Total accumulated Value and Total return after five years for a 6 month 7.5% Bond selling to yield 7.5%

Investment horizon: 5 years	Price: 100.00
Coupon rate: 7.5%	Par value purchased: $9,642,899
Maturity: 0.5 years	Purchase price: $9,642,899
Yield to maturity: 7.5%	Target Accumulated value: $13,934,413

Exhibit 5

New yield (%)	After one period ($)	Accumulated value ($)	Total return (%)
11.00	10,004,508	16,198,241	10.65
10.50	10,004,508	15,856,037	10.20
10.00	10,004,508	15,520,275	9.75
9.50	10,004,508	15,190,848	9.30
9.00	10,004,508	14,867,650	8.85
8.50	10,004,508	14,550,580	8.40
8.00	10,004,508	14,239,534	7.95
7.50	10,004,508	13,394,415	7.50
7.00	10,004,508	13,635,117	7.05
6.50	10,004,508	13,341,549	6.60
6.00	10,004,508	13,053,613	6.15
5.50	10,004,508	12,771,214	5.70
5.00	10,004,508	12,494,259	5.25
4.50	10,004,508	12,222,656	4.80
4.00	10,004,508	11,956,313	4.35

Column 2. Accumulated value after six months

Column 3. accumulated value after 5 years by reinvesting the value accumulated after 6 months at the yield shown in column 1.

$$\$10{,}004{,}508(1 + \text{new yield} / 2)^9$$

By investing in this 6-month bond the manager incurs no price risk, although there is reinvesting risk.

The target accumulated value will be achieved only if the market yield remains at 7.5% or rises.

Once again, the life insurance company is not assured of achieving the target accumulated value.

Question!

If one assumes there is a one-time instantaneous change in the market yield, is there a coupon bond that the life insurance company can purchase to equal the target accumulated value whether the market yield rises or falls?

The Life insurance company should look for a coupon bond so that regardless of how the market yield changes, the change in reinvestment income will offset the change in price.

Consider a 6-year 6.75% bond selling at $96.42899 to yield 7.5%. Suppose $10 million of par value of this bond is purchased for $9,642,899

Exhibit 6

Investment horizon: 5 years	Price: 100.00
Coupon rate: 6.75%	Par value purchased: $10,000,000
Maturity: 6 years	Purchase price: $9,642,899
Yield to maturity: 7.5%	Target Accumulated value: $13,934,413

Exhibit 6: provides the same information for this bond as Exhibits 2, 3 and 5 for the previous bonds considered for purchase

Exhibit 6: Total accumulated value and total return after five years form a 6-year 6.75% Bond selling to yield 7.5

New yield (%)	Coupon ($)	Reinvestment income ($)	Price of bond	Accumulated Value ($)	Total return (%)
11.00	3,375,000	970,432	9,607,657	13,953,089	7.53
10.50	3,375,000	919,903	9,652,592	13,947,495	7.52
10.00	3,375,000	870,039	9.697,846	13,942,885	7.51
9.50	3,375,000	820,831	9,743,423	13,939,253	7.51
9.00	3,375,000	772,271	9,789,325	13,936,596	7.50
8.50	3,375,000	724,350	9.835,556	13,934,906	7.50
8.00	3,375,000	677,061	9,882,119	13,934,180	7.50
7.50	3,375,000	630,395	9,929,017	13,934,413	7.50
7.00	3,375,000	584,345	9,976,254	13,935,599	7.50
6.50	3,375,000	538,902	10,023,832	13,937,734	7.50
6.00	3,375,000	494,059	10,071,755	13,940,814	7.51
5.50	3,375,000	449,808	10,120,027	13,944,835	7.52
5.00	3,375,000	406,141	10,168,650	13,949,791	7.52
4.50	3,375,000	363,051	10,217,628	13,955,679	7.53
4.00	3,375,000	320,531	10,266,965	13,962,495	7.54

Looking to the two last columns:

The accumulated value and the total return are never less than the target accumulated value and the target yield.

Next Exhibit shows why (Exhibit 7).
When the market yields rise, the change in the reinvestment income more than offsets the decline in price. When the market yield declines, the increase in price exceeds the decline in reinvestment income.

What characteristics of this bond assures that the target accumulated value will be realized regardless of how the market yield changes?

The duration of each of the four bonds is shown in Exhibit 8.

The duration of the liability is 4.82 (the duration of a zero coupon liability is equal to the number of years to maturity of the liability divided by 1 plus one-half the yield) 5 divided by (1+0.075/2)

The key should be that the bond duration has to be equal to the duration of the liability, in this case 4.82!!!!

To immunize a portfolio's target accumulated value (target yield) against a change in the market yield, the life insurance company must invest in a bond (or a bond portfolio) such that 1) the portfolio's duration is equal to the liability's duration, and 2) the initial present value of the cash flows from the bond (or bond portfolio) equals the present value of the future liability.

The two bonds with a duration shorter than the duration of the liability expose the portfolio to reinvestment risk.
The one bond with a duration greater than the investment horizon exposes the portfolio to price risk.

Exhibit 7: Change in Reinvestment Income and Price Due to Interest rate Change after five years for a 6-year 6.75% Bond Selling to Yield 7.5%.

New yield (%)	Change in Reinvestment income ($)	Change in Price ($)	Total Change in accumulated value ($)
11.0	340,036	(321,360)	18,676
10.5	289,507	(276,426)	13,081
10.0	239,643	(231,171)	8,472
9.50	190,435	(185,595)	4,840
9.00	141,875	(139,692)	2,183
8.50	93,955	(93,461)	494
8.00	46,666	(46,898)	(232)
7.50	---	---	---
7.00	(46,050)	47,237	1,187
6.50	(91,493)	94,815	3,322
6.00	(136,336)	142,738	6,402
5.50	(180,587)	191,009	10,422
5.00	(224,254)	239,632	15,378
4.50	(267,344)	288,611	21,267
4.00	(309,865)	337,947	28,082

Exhibit 8:Duration for the four Bonds Analyzed as Potential Candidates for immunization.

Bond	Duration	Risk
5-year, 7.5% coupon at par	4.11	Reinvestment
12-year, 7.5% coupon at par	7.83	Price
6-month, t.5% coupon at par	0.48	Reinvestment
6-year, 6.75% coupon at 96,42899	4.82	