## Derivatives

## Solutions

## Question 1

The Black-Scholes-Merton option pricing model assumes that the probability distribution of the stock in 1 year (or at any other future time) is lognormal. It assumes that the continuously compounded rate of return on the stock during the year is normally distributed.

## Question 2

The standard deviation of the percentage price change in time $\Delta \mathrm{t}$ is $\sigma \sqrt{\Delta t}$ where $\sigma$ is the volatility. In this problem $\sigma=0.3$ and, assuming 252 trading days in one year, $\Delta t=1 / 252=$ 0.004 so that $\sigma \sqrt{\Delta t}=0.3 \sqrt{0.004}=0.019$ or $1.9 \%$

## Question 3

The price of an option or other derivative when expressed in terms of the price of the underlying stock is independent of risk preferences. Options therefore have the same value in a risk-neutral world as they do in the real world. We may therefore assume that the world is risk neutral for the purposes of valuing options. This simplifies the analysis. In a riskneutral world all securities have an expected return equal to risk-free interest rate. Also, in a risk-neutral world, the appropriate discount rate to use for expected future cash flows in the risk-free interest rate.

## Question 4

In this case $\mathbf{S}_{0}=50, K=50, r=0.1, \sigma=0.3, \mathrm{~T}=0.25$, and,

$$
\begin{gathered}
d_{1}=\frac{\ln (50 / 50)+(0.1+0.09 / 2) \times 0.25}{0.3 \times \sqrt{0.25}}=0.2417 \\
d_{2}=d_{1}-0.3 \times \sqrt{0.25}=0.0917
\end{gathered}
$$

The European put price is,

$$
\begin{gathered}
50 \mathrm{~N}(-0.0917) e^{-0.1 \times 0.25}-50 \mathrm{~N}(-0.2417)=50 \times 0.4634 e^{-0.1 \times 0.25}-50 \times 0.4045 \\
\quad=\$ 2.37
\end{gathered}
$$

## Question 5

The implied volatility is the volatility that makes the Black-Scholes price of an option equal to its market price. It is calculated using an iterative procedure.

## Question 6

In the case $\mathrm{c}=2.5, \mathrm{~S}_{0}=15, \mathrm{~K}=13, \mathrm{~T}=0.25, \mathrm{r}=0.05$. The implied volatility must be calculated using an iterative procedure.
A volatility of 0.2 (or $20 \%$ per annum) gives $c=2.20$. A volatility of 0.3 gives $c=2.32$. A volatility of 0.4 gives $c=2.507$. A volatility of 0.39 gives $c=2.487$. By interpolation the implied volatility is about 0.397 or $39.7 \%$ per annum.

## Question 7

Answer " $b$ " is correct. The risk-neutral conclusion means that neither the stocks expected return of a wiener variable appear in the formula, whilst the cumulative normal distribution appears ONLY in the closed form solution for European options, NOT the equation itself.

## Question 8

Answer "a" is correct. Short selling is a key assumption, not NO short selling. All the other assumptions are true.

## Question 9

Answer " d " is correct. $\mathrm{d} 2=\mathrm{d} 1-\sigma V \mathrm{~T}=+0.2321-(0.2 \times \mathrm{V} 0.25)=+\mathbf{0 . 1 3 2 1}$

## Question 10

Answer " a " is correct. Option value $=\mathrm{S}$ N(d1) - X N(d2)e-rT
$=36 \times 0.2704-40 \times 0.2248 \times \mathrm{e}-0.02 \times 4 / 12$
$=9.73-8.93=\mathbf{\$ 0 . 8 0}$

## Question 11

Answer "d" is correct. Option value $=-\mathrm{S}$ N(-d1) +X N(-d2)e-rT
$=-36 \times(1-0.2704)+40 \times(1-0.2248) \times \mathrm{e}-0.02 \times 4 / 12$
$=-26.27+30.80=\$ 4.53$

## Question 12

The price of a callable bond can be expressed as follows:
Price of a callable bond = price of option-free bond - price of embedded call option
An increase in interest rates will reduce the price of the option-free bond. However, to partially offset that price decline of the option-free bond, the price of the embedded call option will decrease. This is because as interest rates rise the value of the embedded call option to the issuer is worth less. Since a lower price for the embedded call option is subtracted from the lower price of the option-free bond, the price of the callable bond does not fall as much as that of an option-free bond.

## Question 13

a. The value of the 3-year Treasury bond is 103.373, as shown below

b. The current price of the bond is 103.373 as found in part i) and the price assumed in the question. The value of the 2 -year call option is $\$ 2.5886$, as shown below:


## Question 14

Answer "c" is correct.

## Question 15

Answer " $b$ " is correct.

## Question 16

Answer "a" is correct.

## Question 17

Answer " $b$ " is correct.

## Question 18

Answer " a " is correct.
Question 19
Answer " $d$ " is correct.
Question 20
Answer "b" is correct.
Question 21
Answer "c" is correct.

