SOLUTIONS

Question 1

a. Security A is good for diversification purpose as it has negative beta and negative correlation with portfolio B. Security C is not, as its beta varies close to 1, which implies that it also resembles the market since portfolio B is similar to the market too. Its correlation coefficient is 0.88, which is also undermining diversification.

b.
Systematic risk is obtained as: \[ \beta_i^2 \sigma_m^2 \]

Total risk can be obtained as: \[ \frac{\beta_i \times \sigma_m}{\rho_{im}} \]

Correlation needed for total risk is square root of R-squared: \[ R^2 = \rho_m^2 \]

<table>
<thead>
<tr>
<th>Security</th>
<th>Systematic Risk</th>
<th>Correlation</th>
<th>total risk (standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.0003803</td>
<td>-0.61644</td>
<td>0.031633</td>
</tr>
<tr>
<td>C</td>
<td>0.0034012</td>
<td>0.883176</td>
<td>0.066034</td>
</tr>
</tbody>
</table>

c.
Covariance is given as: \[ \sigma_{ij} = \beta_i \beta_j \sigma_m^2 \]

So, it is -0.001137

d. Correlation coefficient is: \[ \rho_{ij} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j} \]

So, it is -0.54443

d.
1. Calculate expected return of A and C (using CAPM)
2. Then portfolio weights need to be calculated for A and C

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return</th>
<th>Weight</th>
<th>Expected return AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.00275</td>
<td>0.673913</td>
<td>0.026855</td>
</tr>
<tr>
<td>C</td>
<td>0.08804</td>
<td>0.326087</td>
<td></td>
</tr>
</tbody>
</table>

e.
Principle of arbitrage
If the price of the sum of two assets were not equal to the sum of the individual prices, it would be possible to make arbitrage profits.
Example:
If the combination asset (1+2) was priced lower than the sum of the individual assets 1 and 2 (selling at the higher individual price), one would be able to make a profit (and vice versa). By doing this in large quantities, it could make arbitrarily large profits. Such arbitrage opportunities are ruled out if the pricing of assets is linear.
Question 2
See excel spreadsheet.

Question 3
Style test as in the model described can be viewed as a version of Fama-French 3-factor model: take 60 consecutive observations of monthly return for a given stock/portfolio and regress them against monthly market index and style index returns differentials for the same period to generate style betas. Elaboration of this is expected in better answers.

\[ R_p = 0.025 + 0.71R_M - 1.2R_{G-V} + 0.58R_{L-S} + \varepsilon_p \]

(1.18) (5.78) (-3.55) (0.76)

- **BETA g-v** = sensitivity of a given stock’s/portfolio’s return to the difference between the growth and the value index. If the value of this beta is positive and significant - the portfolio is a growth portfolio and vice versa (in the example given the stock is a value portfolio)
- **BETA l-s** = sensitivity of a given stock’s/portfolio’s return to the difference between the large and the small index. If the value of this beta is positive and significant - the portfolio is large cap and vice versa (in the example given this beta is not significant implying that the portfolio in question is neither small not large cap but mid cap size).

Question 4
See excel spreadsheet.

b. \[ R_{pt} - r_{ft} = \alpha_p + \beta_{p,m}(R_{mt} - r_{ft}) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \varepsilon_{pt} \]

c. \[ (R_{it} - R_{ft}) = \alpha_i + b_1(R_{Mt} - R_{ft}) + c_1(R_{Mt} - R_{ft})^2 + \varepsilon_i \]
\[ (R_{it} - R_{ft}) = a_i + b_i(R_{Mt} - R_{ft}) + c_iD(R_{Mt} - R_{ft}) + \varepsilon_i \]

d. \[ Performance_{pt} = \alpha + \beta Performance_{pt-1} + \varepsilon \]