FINA 0025 –Financial Management
Portfolio Theory
Tutorial Solutions for Lecture 4 and 5

Note that detailed answers to tutorial questions will only be provided in tutorials. The following abridged answers are intended as a guide to these detailed answers. This policy is in place to ensure that you attend your tutorial regularly and receive timely feedback from your tutor. If you are unsure of your answers you should check with your tutor, a pit stop tutor or the lecturer.

A. Multiple Choice Questions

A1.

Choice "c" is the correct. The CML assumes that all the investors want to be in the efficient frontier where their potential return is consistent with the risk they are willing to take.

Choice "a" is incorrect. The CML theory assumes everyone can borrow or lend at the risk-free rate.

Choice "b" is incorrect. A CML assumption believes all investments are infinitely divisible.

Choice "d" is incorrect. Another CML assumption is that there are no unforeseen changes in inflation or interest rates.

A2.

Choice "b" is correct. Only efficient portfolios lie along the CML.

Choice "a" is incorrect. It is true that both the SML and the CML measure risk on the horizontal axis. For the SML, the measure of risk is beta or systematic risk; for the CML, the measure of risk is standard deviation or total risk.

Choice "C" is incorrect. It is true that the slopes of both the SML and CML may change over time. The slope of the SML = $(R_M - R_F)$; the slope of the CML = $(R_M - R_F)/\sigma^2$

A3.

Choice "b" is the correct. The only relevant portfolio for investors is the market portfolio. Once an investor has decided to invest, they invest in the market portfolio and then make a financing decision to reflect their risk appetite. For mandate that requires a higher than market risk, the manager must employ leverage. By borrowing an appropriate amount of money and investing in the market portfolio, the pension fund will maximize its return.

Choice "a" is incorrect. The answer achieves the risk objective but does not maximize the expected return. To maximize the expected return the pension fund must continue to invest in the market portfolio and use leverage.

Choice "c" is incorrect. This answer is similar to choice "a' because is no longer investing in the market portfolio.

Choice "d" is incorrect. This answer is only appropriate to the point that an investor wants to take risk equivalent to the market risk. Beyond that point the investor is no longer lending money (investing in T-bills) but is borrowing money at the risk free rate in order to leverage the market portfolio to the risk level desired. To achieve a 30% standard deviation when the market portfolio has a standard deviation of only 15%, you can no longer lend, you must borrow.

A4.

Choice "b" is correct. As investors become more risk averse, they require higher compensation for taking on risk. Thus, the market price of risk increases. Also, as investors become more risk averse, they prefer to hold less risky assets. Thus, the demand of risky assets (such as stocks) falls, which in turn leads to a drop in risky asset values.

Choice “a” is incorrect. As investors become more risk averse and shy away from stocks, stock values fall (not rise).
Choice “c’ is incorrect. As investors become more risk averse and shy away from stocks, stock values fall (not rise) , and the market price of risk increases (not decreases).

A5.
Choice “c” is correct. Credit risk is the risk that the bond issuer will default on interest or principal payments. This is a source of uncertainty with respect to the timing and amount of cash flows and is included in the required rate of return through the risk premium. Increasing yields under the conditions of greater uncertainty reflect investors’ demand for additional compensation to offset increased risk, which is an example of risk averse behavior. Risk averse individuals exhibit a positive relationship between expected return and expected risk.
Choice “a” is incorrect. Increasing yields under the conditions of greater uncertainty reflects a risk averse investor’s demand for additional compensation commensurate to offset the perceived increased risk. This is not a risk eliminating behavior because an investor is willing to bear some credit risk for higher yields.
Choice “b” is incorrect. Increasing yields under the conditions of greater uncertainty reflects investors’ demand for additional compensation to offset increased risk, which is an example of risk averse behavior. Risk averse individuals exhibit a positive relationship between expected return and expected risk.

A6.
Choice ‘b” is correct. Under Markowitz’s theory, risk is correctly measured by the volatility of expected returns.
Choice “a” is incorrect. The assumption is that each investment has a probability of expected return, rather than requiring a single expected return.
Choice “c” is incorrect. One of Markowitz’s key assumptions is that investors maximize their expected utility for any one period and they experience *diminishing* marginal utility of wealth.

A7.
Choice “b” is correct. Investors are assumed to only consider expected return and risk; higher returns are preferred to lower returns, if risk is constant.
Choice “a” is incorrect. If risk is constant, higher returns are preferred to lower returns
Choice “c” is incorrect. While in actually, individuals may be swayed by personal biases, Markowitz assumed that investment decisions are based only on expected return and risk.

A8.
Choice “b” is correct. The covariance is most important, as it will impact the amount of diversification benefit.
Choice “a” and “c” are incorrect; they are not the most important factors affecting risk reduction/diversification

A9.
Choice "b" is correct.

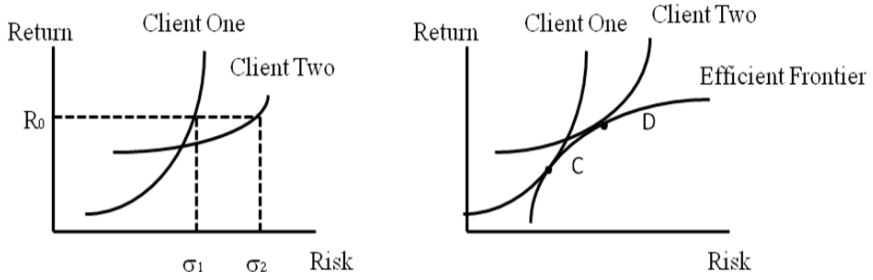
	Stock A	Stock B
Expected return	12%	15%
Standard deviation	18%	15%
Weight in portfolio	75%	25%

$r_{1,2} = 0.8$

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2r_{1,2}\sigma_1\sigma_2$$
$$\sigma_P^2 = (0.75)^2(0.18)^2 + (0.25)^2(0.15)^2 + 2(0.75)(0.25)(0.8)(0.18)(0.15) = 0.02773$$
$$\sigma_P^2 = \sqrt{0.02773} = 16.65\%$$

Choice "a" is incorrect. It incorrectly computes the simple weighted average of the standard deviations of stocks A and B.
Choice "c" is incorrect. This computation incorrectly uses 75% as the weight for B and 25% as the weight for a in the equation.

A10.
Choice "c" is correct. Risk-averse investors have steeper utility curves because they demand more return for assuming one additional unit of risk relative to less risk-averse investors. Client One is clearly the more risk averse of the two. Therefore, Client One has steeper investment utility curve than Client Two. The optimal portfolio for any investor is the point of tangency between his or her highest utility curve and the efficient frontier. Steeper utility curves will be tangent to the efficient frontier at a point below flatter utility curves. The diagram below illustrates the point graphically.



Client One's utility curves are steeper than Client's Two's. Note that Client two is willing to assume quite a bit more risk than Client One for the same increase in return. Client One's optimal portfolio is the point of tangency at point C, while Client Two's optimal portfolio is at point D. Because Client two is less risk averse than Client One, his optimal portfolio resides higher on the efficient frontier.

A11.
Choice "c" is correct. A utility curve is a series of points representing various expected return/risk combinations that give an investor equal amounts of utility

Choice "a" is incorrect. Because the utility curve is not a vertical line, the standard deviation is not constant along the curve.
Choice "b" is incorrect. All points lying along a particular utility curve offer a certain constant utility to the investor. Different utility curves offer different utilities.

B. Problems

B1.

	Risky	**Risk-free**
Expected Return	12%	5%
Weight in Portfolio	W_1	$1-W_1$

$Rp = W_1R_1 + W_2R_2$
$0.09 = [W_1 \times 0.12] + [(1- W_1) \times 0.05]$
$W_1 = 0.57$ and $(1 – W_1) = 0.43$

	Risky	**Risk-free**
Standard Deviation	15%	0%
Weight in Portfolio	W_1	$1-W_1$

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2r_{1,2}\sigma_1\sigma_2$$

$(0.06)^2 = W_1^2 (0.15)^2 + W_2^2 (0)^2 + 2 W_1W_2 r_{1,2} (0.15) (0)$

$$W_1^2 = \frac{0.0036}{0.0225} = 0.16$$

$W_1 = 0.4$ and $(1-W_1)$ 0.6

B2.

$Rp = W_1R_1 + W_2R_2 + W_3R_3 = 0.5 (18\%) + 0.3 (-15) = 0.2 (10\%) = 6.5\%$

B3.

Portfolio C is the correct answer. The efficient frontier represents the group of portfolios with the maximum returns for given levels of risk. A rational investor will always prefer a higher return for each level of risk. Similarly, you would prefer less risk for a given return. Portfolio B has a higher return than Portfolio C, but Portfolio C has a higher risk. Therefore, rational investors would always prefer Portfolio B over Portfolio C. Portfolio C could not lie on the efficient frontier because it is inferior to Portfolio B. The portfolios that represent the maximum return for each level of risk form the efficient frontier.

B4.

Using the midpoints of each of the class intervals as the single point estimates of the expected value of each class interval:
$E (R_A) = 0.30 (-5\%) + 0.30 (5\%) + 0.40 (15\%) = 6\%$
RF = 5%
$E(R_P) = W_A E(R_A) + W_F R_F = 0.6 (6\%) + 0.4 (5\%) = 5.6\%$
This is the expected return of the portfolio based on the expected return for A. Corp. stock and the risk free rate.
$\sigma_A^2 = 0.3 (-5 - 6.0)^2 + 0.30 (5 - 6.0)^2 + 0.4 (15 - 6.0)^2 = 69$
$\sigma_A = (69)^{0.5} = 8.307\%$

$\sigma_P^2 = W_A^2 \sigma_A^2 + W_F^2 \sigma_F^2 + 2 W_A \sigma_A W_F \sigma_F r_{A,F}$
Note that a risk-free asset has no variance (σ_F^2)
Therefore:
$\sigma_P^2 = W_A^2 \sigma_A^2 = 0.36\ (69) = 24.84$
$\sigma_P = (24.84)^{0.5} = 4.98\%$
This is equal to the weighted average of the standard deviations of a Corp. and the risk-free returns.
$\Sigma_{Wtd.Avg} = 0.6\ (8.307\%) + 0.4\ (0\%) = 4.98\%$

B5.
1. *If the two assets are uncorrelated:*

$$R_P = W_1 R_1 + W_2 R_2 = 0.5(10\%) + 0.5(10\%) = 10\%$$
$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2r_{1,2}\sigma_1\sigma_2 = 0.25(400) + 0.25(400) + 2(0.5)(0.5)(0)(20)(20) = 200$$
$$\sigma_P = \sqrt{200} = 14.14\%$$

Notice that the standard deviations of the rates of return for each asset that form the portfolio are 20%, but the standard deviation of the portfolio's rate of return is only 14.14%. *Diversification reduces the standard deviation of the possible rates of return of a portfolio, which is a measure of its risk* (thus, "diversification reduces risk"). Furthermore, because diversification reduces risk, it raises the portfolio's return-to-risk ratio as well.

2. *If the two assets are perfectly positively correlated:*

$$R_P = W_1 R_1 + W_2 R_2 = 0.5(10\%) + 0.5(10\%) = 10\%$$
$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2r_{1,2}\sigma_1\sigma_2 = 0.25(400) + 0.25(400) + 2(0.5)(0.5)(1)(20)(20) = 400$$
$$\sigma_P = \sqrt{400} = 20.00\%$$

The standard deviation of the portfolio is now the same as that of the stocks that compose it. *Diversification does not reduce risk when the assets that compose a portfolio are perfectly positively correlated*.

3. *If the two assets are perfectly negatively correlated:*

$$R_P = W_1 R_1 + W_2 R_2 = 0.5(10\%) + 0.5(10\%) = 10\%$$
$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2r_{1,2}\sigma_1\sigma_2 = 0.25(400) + 0.25(400) + 2(0.5)(0.5)(-1)(20)(20) = 0$$
$$\sigma_P = \sqrt{0} = 0.00\%$$

This is a special case, given these assets weights, expected returns, standard deviations, and that the assets are perfectly negatively correlated; the portfolio standard deviation is zero. In other words, the return is certain. Essentially, this is a risk-free, or fully hedged, portfolio. According to the law of one price, this portfolio should therefore earn the risk-free rate of return.

When an asset is added to a portfolio, the portfolio's standard deviation is affected. The incremental asset's variance is overwhelmed by the relative weight of the existing covariances. Therefore, *when adding an incremental asset to an existing portfolio containing a number of assets, the most important factor to consider is the new asset's average covariance with the other assets in the portfolio*.